Stochastic Processes, Quiz 1, 2023 Spring

- Duration: 90 minutes
- Closed material, No calculator

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- Write legibly.
- Justification is necessary unless stated otherwise.

1	10
2	20
3	10
4	30
5	10
6	10
7	10
Total	100

#1. You are considering to sell a certain product. You assessed the potential demand that is 100 items with 1/3 of chance, 200 items with 1/3 of chance, and 300 items with 1/3 of chance. What is the coefficient of variation[1] of the demand? `[10pts]`

[1] Hint: $cv(X) = sd(X)/\mathbb{E}X$

#2. Let X be a Poisson random variable with parameter 4, and let $Y = min(X, 3)$.

(a) What is the pmf of Y? (i.e. Specify $\mathbb{P}(Y = i)$ for $i = 0, 1, 2, ...$) `[10pts]`

(b) What is $\mathbb{P}(Y \leq 2 | Y \leq 4)$? `[10pts]`

#3. Express S in a number. `[10pts]`

$S = 0.1 + 2 \cdot 0.1^2 + 3 \cdot 0.1^3 + 4 \cdot 0.1^4 + 5 \cdot 0.1^5 + \cdots$

#4.

(a) State the definition of the memoryless property. [10pts]

(b) State the cdf of random variable that follows exponential distribution with parameter λ. [10pts]

(c) Prove that exponential distribution possesses the memoryless property. [10pts]

#5. Suppose $X \sim U(100, 150)$. Evaluate $\mathsf{E}[(120 - X)^+]$. `[10pts]`

#6. Suppose $X \sim exp(3)$. Evaluate $\mathsf{E}[min(X, 5)]$. `[10pts]`

#7. Smith and Jones came to post office together and they are served by two clerks, server A and server B, respectively. Server A has a service time following $exp(5)$ and server B has a service time following $exp(4)$. What is the chance that Smith will be done with the service first? `[10pts]`

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