Lecture B1. Newsvendor 1

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I. Problem and Solution

Motivation

Your brother will start part time job of selling newspapers at subway station in the morning. You are asked how many he should prepare for selling.

- What are the kind of information that you need in order to give him a good advice?
  1.
  2.
  3.
  4.

The information

1. (*demand*) How many customer?
2. (*retail price*) How much do you sell a copy at?
3. (*material cost*) How much do you pay to the wholesaler?
4. (*salvage value*) How much do you sell an unsold copy back to the wholesaler?

- Suppose your brother gave you the following information.
  1. 11, 12, 13, 14, or 15, equally likely.
  2. \$2 per copy
  3. \$1 per copy
  4. \$0.5 per copy

Exercise 1

Assume that D follows the following discrete distribution.

d	*20*	*25*	*30*	*35*
$\mathbb{P}[D=d]$	*0.1*	*0.2*	*0.4*	*0.3*
$30 \wedge d$				
$(30-d)^+$				
$24 \wedge d$				
$(24-d)^+$				

Answer the followings.

- $\mathbb{E}[30 \wedge D] =$
- $\mathbb{E}[(30-D)^+] =$
- $\mathbb{E}[24 \wedge D] =$
- $\mathbb{E}[(24-D)^+] =$

Optimal economic decision

- Your brother's goal is to earn as much money as possible.
- In other words, the newsvendor wants to make *an optimal decision* that *maximizes his expected profit*.
- In some settings, the optimal decision is related to *minimize expected cost*.
- In this case, profit, *the objective function*, is composed in the following way.

$$\begin{aligned} \text{Profit} &= \text{Revenue} - \text{Cost} \\ &= \text{Sales Rev.} + \text{Salvage Rev.} - \text{Material Cost} \\ &= (\text{from Reg. Sale}) + (\text{from unsold item}) - (\text{for preparation}) \end{aligned}$$

Solution - tabular method

Economic cost around decision making

- Your brother wants to the prepared **stock** to exactly match the customer **demand**.
- If **Stock > Demand**,
  - **Overstock cost occurs.**
  - In other words, prepared too much, excessive items, or over-prepared.
  - Overstock by how many?
  - Ex) holding cost, items lose its economic potential value
- If **Stock < Demand**,
  - **Understock cost occurs**.
  - In other words, prepared too less, lost opportunity, or lost sales.
  - Understock by how many?
  - Ex) stock-out, reputation loss, cancellation reward
- Newsvendor wants to find a optimal balance between overstock cost and understock cost.
- Just as all of the retail store you can think of.

Mathematical representation

1. (number of units) If demand D is random and you prepare X unit, then
   - # of unit for sales:
   - # of unit for overstock:
   - # of unit for understock:
2. (unit cost) If retail price is p, material cost is c, and salvage price is s, then
   - unit cost for overstock:
   - unit cost for understock:
3. Cost = (# of units) $\times$ (cost per unit)
   - overstock cost:
   - understock cost:
4. Economic cost = Understock cost + Overstock cost

Formal treatment

Remark 1

Newsvendor problem aims to find the optimal number of preparation that maximizes the expected profit formulated as:
$\mathbb{E}[\text{profit}] = \mathbb{E}(\text{sale rev.}) + \mathbb{E}(\text{salvage rev.}) - \mathbb{E}(\text{material cost})$

Theorem 1

In newsvendor problem, maximizing the expected profit is equivalent to minimizing the expected economic cost (sum of the expected overstock cost and the expected understock cost).

- *Two problems being mathematically equivalent to each other implies that a solution that solves the one problem solves the other problem, and vice versa.*

Problem and Solution

Remark 2

By the above remark and theorem, newsvendor model is to find optimal x^* that minimize total expected economic cost. That is,

$$x^* = argmin_x \; c_o\mathbb{E}[(X-D)^+] + c_u\mathbb{E}[(D-X)^+]$$

Theorem 2

The solution to the above problem can be found as:

- *If D is a continuous r.v, with cdf $F(\cdot)$, then find y s.t. $F(y) = \frac{c_u}{c_o+c_u}$*
- *If D is a discrete r.v, with cdf $F(\cdot)$, then find smallest y such that $F(y) \geq \frac{c_u}{c_o+c_u}$*

II. Exercises

Exercise 2

Find your brother's optimal stock level by the above Theorem 2. Then, find his expected profit using the Remark 1.

Exercise 3

Your brother is now selling milk. The customer demands follow $U(20, 40)$ *gallons. Retail price is \$2 per gallon, material cost is \$1 per gallon, and salvage cost is \$0.5 per gallon. Find optimal stock level and expected profit.*

Exercise 4

Lemonade sells for \$18 per gallon but only costs \$3 per gallon to make. If we run out of lemonade, it will be impossible to get more. On the other hand, leftover lemonade has a value of \$1. Express the following quantity using sale as X and demand as D.

- c_u
- c_o
- *Expected economic cost*
- *Expected profit*

Exercise 5

Prove Theorem 1. (Hint: you may use formulation from Exercise 4)

Exercise 6

Show that $(D \wedge Y) + (Y - D)^+ = Y$

Exercise 7

Let D be a continuous random variable and uniformly distributed between 5 and 10.

- $E[max(D, 8)]$
- $E[(D - 8)^{-}]$

III. Discussion

On the nature of newsvendor

- What does $F(y) = \frac{c_u}{c_o + c_u}$ imply?
  -
  -
  -
  -
  -
- Buying a suit - department store vs outlet

Newsvendor in a big picture

- Newsvendor is characterized as an *optimal decision making* problem.
- Newsvendor is characterized as a "baby model" for a *decision making under uncertainty*.
- Since it is decision making *under uncertainty*, a random variable is involved.
- It can be viewed as *decision making at time 0*, then *results come out at time 1*. It is thus called as *one-period problem*.
- *Stochasticity* is the combined notion of *time* and *randomness*.
- In sum, newsvendor is *one-period stochastic decision making* problem, where
  1. *one-period* specifies its time domain.
  2. *stochastic* tells that we make decision considering future randomness.
  3. *decision making* obviously includes optimization aspect of the problem.

"Man can learn nothing unless he proceeds from the known to the unknown. - Claude Bernard"