

Stochastic Processes, Mid-term #2, 2025 Spring

- Duration: 90 minutes
- Closed material, No calculator.

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- Write legibly.
- Justification is necessary unless stated otherwise.

1	10
2	10
3	10
4	10
Total	40

#1. The following statement is false. Provide a counterexample and explain why the statement is false. [10pts]

If a DTMC has a finite number of states and is aperiodic, then its stationary distribution is unique.

#2. Consider the following DTMC.

$$P = \begin{pmatrix} 0.3 & 0.7 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0.25 & 0 & 0 & 0.75 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0.5 & 0.5 & 0 \end{pmatrix}$$

(a) What is the number of classes? [5pts]

(b) For each state, identify its period. [5pts]

#3. Prove that the time to the first arrival in $PP(\lambda)$ follows $\exp(\lambda)$. [10pts]

#4. Assume that calls arrive at a customer call center between 9am ($t = 0$) and 2pm ($t = 5$) following a non-homogeneous Poisson process with the rate function of the following:

$$\lambda(t) = \begin{cases} 4 & 0 \leq t < 1 \\ 2t + 2 & 1 \leq t < 3 \\ 8 & 3 \leq t < 5 \end{cases}$$

What is the probability that the call center receives exactly 6 calls during 9am and 11am? [10pts]

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