

Stochastic Processes, Quiz 3, 2024 Spring

- Duration: 120 minutes
- Closed material, No calculator

- Name: _____
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- Write legibly.
- Justification is necessary unless stated otherwise.

1	15
2	15
3	15
4	15
5	10
Total	70

#1. This problem concerns whether the following statement is true or false.

If a DTMC has a finite number of states and is irreducible, then its stationary distribution is unique.

(a) Circle one of the following: [5pts] (**True/False**)

(b) If you answered "true", then explain why it is true and provide an example. If you answered "false", then provide a counterexample and explain it. [10pts]

#2. This problem concerns whether the following statement is true or false.

If a DTMC has a finite number of states and is aperiodic, then its stationary distribution is unique.

(a) Circle one of the following: [5pts] (**True/False**)

(b) If you answered "true", then explain why it is true and provide an example. If you answered "false", then provide a counterexample and explain it. [10pts]

#3. This problem concerns whether the following statement is true or false.

If a DTMC is aperiodic and irreducible, then there exists a stationary distribution.

(a) Circle one of the following: [5pts] (**True/False**)

(b) If you answered "true", then explain why it is true and provide an example. If you answered "false", then provide a counterexample and explain it. [10pts]

#4. For a DTMC with the following transition matrix, calculate \mathbf{P}^{100} [15pts]

$$\mathbf{P} = \begin{pmatrix} 0 & 1/2 & 1/6 & 1/3 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

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#5. You either drink coffee or not on any given day. The following is the transition rule:

- If you drank coffee yesterday and today, the chance of you drinking coffee tomorrow is 0.2.
- If you did not drink coffee yesterday but drank coffee today, then the chance of drinking coffee tomorrow is 0.4.
- If you drank coffee yesterday but not today, then the chance of drinking coffee tomorrow is 0.6.
- If you did not drink coffee yesterday and today, then you will drink coffee tomorrow with the probability 0.8.

Consider a stochastic process $\{X_n, n \in \{0, 1, 2, \dots\}\}$, where $X_n = C$ implies drinking coffee on the n -th day and $X_n = NC$ implies not drinking coffee on the n -th day.

We know that the stochastic process $\{X_n\}$ is not a DTMC. However, one can use $\{X_n\}$ to construct a stochastic process that is a DTMC. Please do so (construct a DTMC using $\{X_n\}$) and present a transition matrix of the constructed DTMC. Make sure to mark the state index in the transition matrix. [10pts]