

# Stochastic Processes, Quiz 3, 2024 Fall

## Solution and Grading

- Duration: 40 minutes
- Weight: 10% of final grade
- Closed material, No calculator
  
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- Write legibly.
- Justification is necessary unless stated otherwise.

#1. A store sells a particular brand of fresh juice. The business has the following details.

- The store sells the juice at \$10 per gallon.
- At every morning, the store gets the juice from a local producer at the cost of \$5 per gallon.
- By the end of the day, any unsold juice will be sold at a discount price of \$2 per gallon.
- Assume the daily demand for the juice is uniformly distributed between 100 and 200 gallons.

(a) What is the optimal order quantity that maximizes the expected daily profit? [5pts]

(b) If 150 gallons are ordered, what is the expected profit per day? (answer in number) [5pts]

(a) First to identify  $c_u = 5$  and  $c_o = 3$ . Then,  $F(x) = \frac{c_u}{c_o + c_u} \Rightarrow \frac{x-100}{200-100} = \frac{5}{3+5}$  leads to  $x^* = 162.5$

(b)

$$\begin{aligned}
 \mathbb{E}[\text{Profit}] &= \mathbb{E}[\text{Sales rev.}] + \mathbb{E}[\text{Salvage rev.}] - \mathbb{E}[\text{Material cost}] \\
 &= 10\mathbb{E}[\min(D, 150)] + 2\mathbb{E}[(150 - D)^+] - 5 \cdot 150 \\
 &= 10 \cdot 137.5 + 2 \cdot 12.5 - 750 = 650
 \end{aligned}$$

Difficulty: Medium



#2. Consider a random variable  $X$  that follows a uniform distribution with parameter 2 and 3. That is,  $X \sim U(2, 3)$ . What is its coefficient of variation of  $X$ ? [10pts]

Given its pdf:

$$\text{pdf } f(x) = \begin{cases} 0 & \text{if } x < 2 \\ 1 & \text{if } 2 \leq x \leq 3 \\ 0 & \text{if } x > 3 \end{cases}$$

You need to work on its moment as:

- $\mathbb{E}X = \int_{-\infty}^{\infty} xf(x) dx = \int_{-\infty}^2 xf(x) dx + \int_2^3 xf(x) dx + \int_3^{\infty} xf(x) dx$   
 $= 0 + \int_2^3 x dx + 0 = 5/2$
- $\mathbb{E}X^2 = \int_{-\infty}^{\infty} x^2 f(x) dx = \int_{-\infty}^2 x^2 f(x) dx + \int_2^3 x^2 f(x) dx + \int_3^{\infty} x^2 f(x) dx$   
 $= 0 + \int_2^3 x^2 dx + 0 = \frac{1}{3}(27 - 8) = 19/3$
- $\text{Var}(X) = \mathbb{E}X^2 - (\mathbb{E}X)^2 = 19/3 - (5/2)^2 = 1/12$
- $\text{sd}(X) = \sqrt{1/12}$

Then, it follows  $\text{cv}(X) = \text{sd}(X)/\mathbb{E}X = \frac{\sqrt{1/12}}{5/2} = \sqrt{3}/15$ .

Difficulty: Medium