Stochastic Processes, Final Exam, 2025 Spring

- Duration: 120 minutes
- Closed material, No calculator.

- Name: ____________________
- Student ID: ____________________
- E-mail: ______________ `@seoultech.ac.kr`

- Write legibly.
- Justification is necessary unless stated otherwise.

1	16
2	20
3	20
4	14
Total	70

#1. Consider one-server service system of the following.

- Interarrival times follows an exponential distribution with mean of 0.5 minutes.
- Service time follows an exponential distribution with mean of 20 seconds.
- The service system can have maximum of four customers at the same time. (One in the service and other three in the waiting space).
- Assume that the stationary distribution is found as $\pi = (\pi_0, \pi_1, \pi_2, \pi_3, \pi_4) = (\frac{3}{18}, \frac{6}{18}, \frac{4}{18}, \frac{3}{18}, \frac{2}{18})$.

Answer the following questions. `[Each 2pts]`

(i) What is the long run fraction of time the system is empty?

(ii) What is the long run fraction of time the server is busy?

(iii) What is the probability that a customer is not accepted to system?

(iv) What is the expected # of customer in the system?

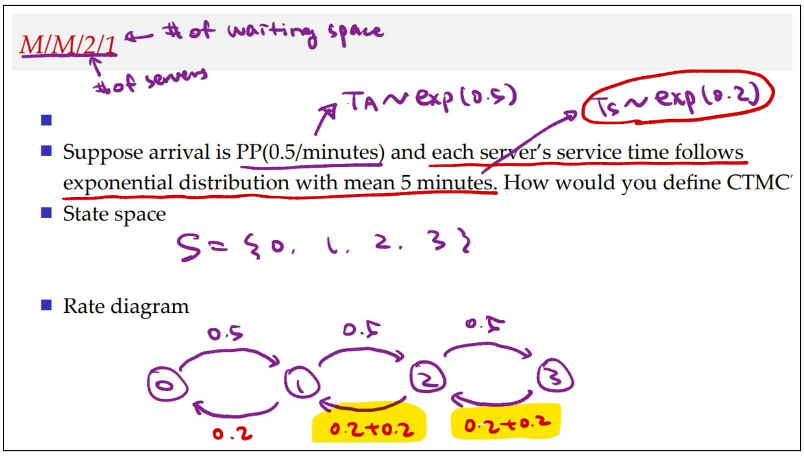
(v) What is the expected # of customer in the queue?

(vi) What is expected total time spent in the system for a customer? (Waiting + Service time)

(vii) What is expected waiting time in a queue for a customer?

(viii) What is TH(throughput)?

#2. Consider the following lecture note content. To define the $M/M/2/1$ queue as a CTMC, the state is defined as the number of customers in the system, yielding the state space $S = \{0, 1, 2, 3\}$. A corresponding rate diagram has also been constructed.



(a) In the rate diagram, the highlighted portions are labeled as "0.2 + 0.2". This indicates that when there are 2 or 3 customers in the system, both servers are busy, and thus the rate at which the number of customers decreases is the sum of the service rates of the two servers. Formally state the relevant theorem that justifies this modeling approach. (Hint: The theorem is related to the exponential distribution.) `[10pts]`

(b) Prove the theorem you stated above. `[10pts]`

(intended blank)

#3. Consider the following transition matrix for a DTMC.

$$P = \begin{pmatrix} .4 & .6 & 0 & 0 & 0 & 0 \\ .7 & .3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & .5 & .5 \\ .6 & 0 & .4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

(a) What is the number of classes? For each class, identify its period and identify whether the class is recurrent, transient, or absorbing. `[10pts]`

(b) Find P^{100}. `[10pts]`

(intended blank)

#4. A dormitory is planning to install a set of shared washing machines for its residents. The system operates under the following conditions.

- The interarrival times of residents follow an exponential distribution, with an average arrival rate of 2 residents per hour.
- The washing time for each machine also follows an exponential distribution, with each machine completing 1 wash per hour on average.
- If all machines are in use, an arriving resident does not wait and immediately leaves.

The dormitory director wants to ensure that a resident can use a washing machine with at least 90% probability upon arrival. To meet this requirement, how many machines need to be installed? [14pts]

(intended blank)