Stochastic Processes, Quiz 2, 2024 Spring

- Duration: 120 minutes
- Closed material, No calculator

- Name: _____
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- Write legibly.
- Justification is necessary unless stated otherwise.

1	10
2	10
3	10
4	15
5	20
Total	65

#1. Consider a $M/M/1/1$ queue such as a single doctor's office. Both the inter-arrival time and the service time are exponentially distributed, and the current expected waiting time in the queue for a customer is 80 minutes. You are proposing a strict appointment policy in order to change the inter-arrival time from a random variable with exponential distribution to a constant. This action eliminates all variation in inter-arrival time while keeping other parameters unchanged. What would be the expected waiting time per customer after your suggested policy applied to the system? (Answer in a number and justify it) `[10pts]`

#2. Suppose that we have the following transition matrix for a discrete time Markov chain $\{X_n : n \geq 0,\, n \in \mathbb{N}\}$. Suppose $\mathbb{P}(X_0 = 3) = 1$, then what is $\mathbb{P}(X_2 = 2)$? `[10pts]`

$$\mathbf{P} = \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} \begin{pmatrix} 0.7 & ? & 0 \\ 0.5 & ? & 0 \\ 0 & ? & 0.8 \end{pmatrix}$$

#3. In post office, there are two servers, A and B. A's service time follows an exponential distribution with mean 3 minutes and B's service time follows a distribution of $exp(1/5)$. Their service times are independent to each other. Alice and Betty came to post office at noon and services started. (Alice being served by A and Betty being served by B)

(a) What is the chance that Betty will leave the post office before Alice? `[5pts]`

(b) What is the chance that both of them will be still in the service at 12:05 pm? `[5pts]`

#4. A small bank is staffed by a single server. During a normal business day, the inter-arrival times of customers to the bank follow an exponential distribution with mean 3 minutes. On the other hand, the service time for a customer follows normal distribution with mean 2 minutes and standard deviation of 1 minute. Answer following questions in a number.

It is OK to use previous answer. For example, if you didn't solve problem (a) but know that the answer for (b) is [the answer for (a)] times 5, then you may answer question (b) as "5 × [ans in (a)]"

(a) What is the long-run fraction of times that the server is busy? Is this system stable? `[5pts]`

(b) What is the long-run average waiting time of each customer in the queue? `[5pts]`

(c) What is the long-run average number of customers waiting for service? `[5pts]`

#5. You are selling lemonade. You have collected the following information.

- The demand is uniformly distributed between 20 gallons and 35 gallons.
- The selling price is 5 dollars per gallon.
- The value of unsold lemonade is 0.5 dollars per gallon.
- Every time you make an order, it costs fixed 25 dollars plus 2 dollars per gallon of lemonade.
- You have 10 gallons of inventory
- Assume that we know $s = 12$ in the (S, s) policy.

(a) What is S? (answer in a number) `[5pts]`

Now you have figured out (S, s) policy. (The big S is from your answer to (a) and the small s is equal to 12 as given in the question. In case that you have not found an answer to (a), then you may use 27 as big S for the rest of this problem.)

(b) Remind that you have 10 gallons of inventory. What is the optimal order amount? Answer in a number. `[5pts]`

(c) What is the expected profit if you make an optimal order? Answer in a number. (In case that you have not found an answer to (b), then you may use 15 as the optimal order quantity.) [10pts]