Lecture B2. Newsvendor 2

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I. Numerical approach

Analytic

Discrete distribution - try out all alternatives

- In the previous newsvendor lecture, following information was given.
  1. (*demand*) How many customer? *11, 12, 13, 14, or 15, equally likely*
  2. (*retail price*) How much do you sell a copy at? *\$2 per copy*
  3. (*material cost*) How much do you pay to the wholesaler? *\$1 per copy*
  4. (*salvage value*) How much do you sell an unsold copy back to the wholesaler? *\$0.5 per copy*

Implementation

- Following code tries the stock level $X \in \{11, 12, 13, 14, 15\}$.

```
for (X in 11:15){
  MC_N <- 10000
  D <- sample(11:15, MC_N, replace = T) # random discrete uniform
  sales_rev <- 2*pmin(D,X) # vector level minimum
  salvage_rev <- 0.5*pmax(X-D,0) # vector level maximum
  material_cost <- 1*X
  profit <- sales_rev + salvage_rev - material_cost
  print(paste0("X: ", X, ", expected profit: ", mean(profit)))
}
```

$D = \begin{bmatrix} 11 \\ 12 \\ 11 \\ \vdots \\ 12 \\ 14 \end{bmatrix}$

10000×1

```
## [1] "X: 11, expected profit: 11"
## [1] "X: 12, expected profit: 11.70015"
## [1] "X: 13, expected profit: 12.10195"
## [1] "X: 14, expected profit: 12.19445"
## [1] "X: 15, expected profit: 12.02805"
```

Continuous distribution - grid search approach

- Your brother is now selling milk. The customer demands follow $U(20, 40)$ gallons. Retail price is \$2 per gallon, material cost is \$1 per gallon, and salvage cost is \$0.5 per gallon. Find optimal stock level and expected profit.
- Notice that there are only marginal differences compared to the previous page's code.

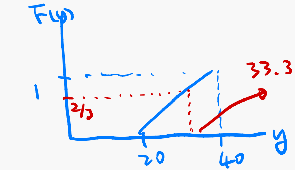
$C_o = 0.5$

$C_u = 1.0$

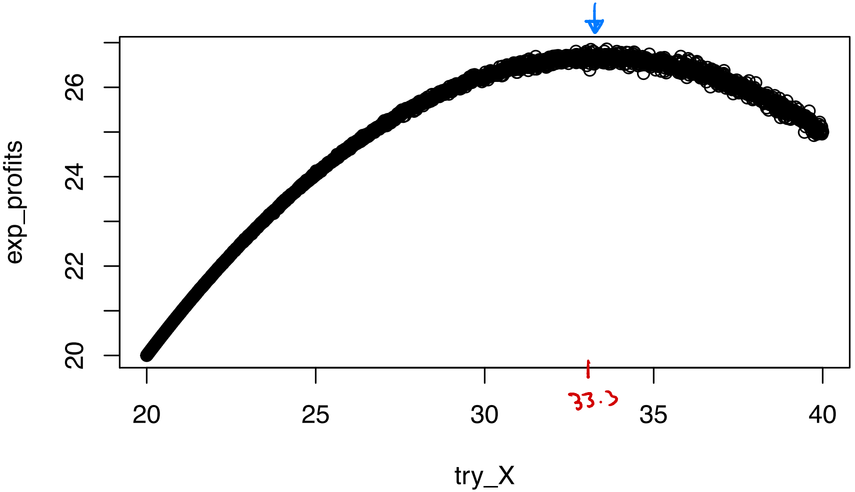
$F(y) = \frac{C_u}{C_o + C_u} = \frac{1.0}{0.5 + 1.0} = \frac{2}{3}$

discretize

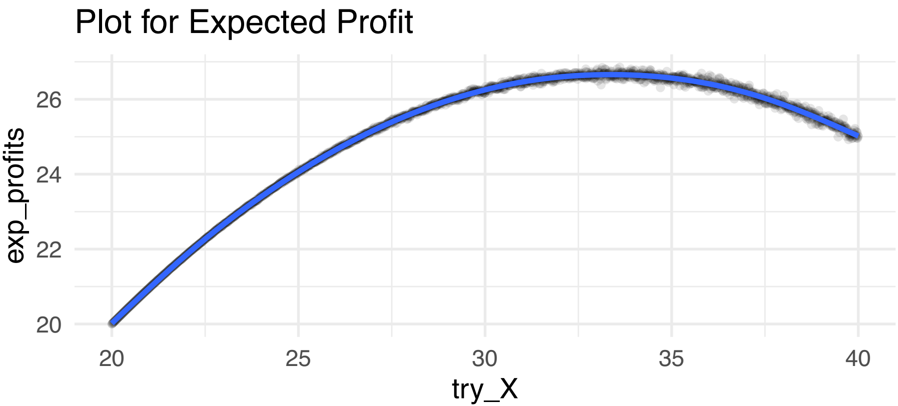
```
try_X <- seq(from = 20, to = 40, by = 0.01)
exp_profits <- NULL
for (X in try_X){
  MC_N <- 10000
  D <- runif(MC_N, min = 20, max = 40)
  sales_rev <- 2*pmin(D,X) # vector level minimum
  salvage_rev <- 0.5*pmax(X-D,0) # vector level maximum
  material_cost <- 1*X
  exp_profit <- mean(sales_rev + salvage_rev - material_cost)
  exp_profits <- c(exp_profits, exp_profit)
}
results <- data.frame(try_X, exp_profits)
```



```
plot(try_X, exp_profits)
```



```
library(tidyverse)
ggplot(results, aes(x=try_X, y=exp_profits)) +
  geom_point(alpha = 0.1, size = 1) + geom_smooth(size = 1) +
  labs(title="Plot for Expected Profit") +
  theme(element_text(size = 25)) +
  theme_minimal()
```



```
idx <- which(exp_profits==max(exp_profits)) # index for maximum profit
try_X[idx] # this is optimal quantity
```

```
## [1] 33.61
```

```
exp_profits[idx] # this is expected optimal profit
```

```
## [1] 26.85652
```

II. Advanced modeling 1 - different cost setting

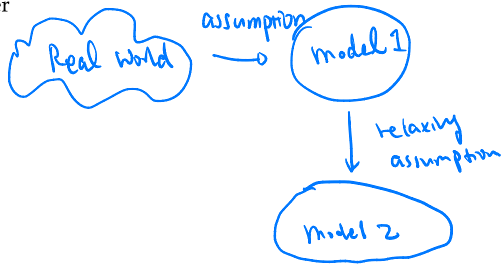
1. Various situation with different cost setting.

Assumption

- Assumptions in the basic newsvendor model
  - "positive" salvage value
  - understock cost of lost sale opportunity.

Relax

- When the above assumption is broken⋯
  - Unused stocks (overstock) may cost you in different ways. In this case, unit cost for overstock $(= c_o)$ should be evaluated accordingly.
  - disposal cost
  - holding cost
  - reputation loss
  - compensation to customer



Example - David's banana

David buys fruits and vegetables wholesale and retails them at David's Produce on La Vista Road. One of the difficult decisions is the amount of bananas to buy. Let us make some simplifying assumptions, and assume that David purchases bananas once a week at 30 cents per pound and retails them at 60 cents per pound during the week. Bananas that are more than a week old are too ripe to sell and David will pay workers to take them away. It costs 5 cent to get rid of each pound of unsold bananas. Suppose that the weekly demand for bananas is uniformly distributed between 500 and 1500 pounds.

U(500, 1500)

- Identify c_o, c_u. $C_o = 35$ $C_u = 30$
- How many pound of bananas should David order each week?
- What is the optimal expected weekly profit?

Exercise 1

Now assume that the demand for bananas is exponentially distributed with mean 1000. (Use R code to present your work.)

1. *Identify* c_o, c_u.
2. *How many pound of bananas should David order each week?*
3. *If David order 800 pounds this week, how much profit he should expect this week?*

II. Advanced modeling 2 - (S, s) policy

Inventory management

(S, s) policy. (nonzero fixed ordering cost)

- Assumptions in basic newsvendor model
  - No fixed cost for ordering
  - No existing inventory
- Variations when the above assumption is broken.
  - Fixed cost
    - You may need to travel to pick up material from wholesaler.
    - Fixed shipping cost
    - Every time you place order, it occurs f dollars
  - Inventory
    - You may already have some inventory for sale

Example - Solvent management

① $F(y) = \frac{c_u}{c_o + c_u} = \frac{(100-50)}{(50+15)+(100-50)} = \frac{50}{115}$

$\frac{900-y}{900-300}$

$y = 560$

Next month's production at a manufacturing company will use a certain solvent for part of its production process. You need to prepare solvent in prior and the demand of solvent is random. Assume that there is an ordering cost of \$1,500 incurred whenever an order for solvent is placed and the solvent costs \$50 per liter. Due to short product life cycle, unused solvent cannot be used in following months. There will be a \$15 disposal charge for each liter of solvent left over at the end of the month. If there is a shortage of solvent, the production process is seriously disrupted at a cost of \$100 per liter short. Assume that the initial inventory level is x, where x = 0, 500, 700, and 800 liters.

1. What is the **optimal ordering quantity for each case** when the demand is governed by the continuous uniform distribution varying between three hundred and nine hundred liters?

   $U(300, 900)$

2. What is the optimal ordering policy **for arbitrary initial inventory level** x?
3. Assume optimal quantity will be ordered. What is the **total expected cost** when the initial inventory $x = 0$? What is the total expected cost when the initial inventory $x = 700$?

   according to (S, s) policy

Discussion

- Motivation: What are the difference from the basic model?
  - We learned basic newsvendor model for the case when there is no **fixed** ordering cost but only **variable** ordering cost.
  - And your optimal preparation for random demand was obtained by matching $F(y)$ to the critical fraction. $\Rightarrow F(y) = \frac{C_u}{C_o + C_u}$ x^*: opt order quantity
  - However, when there exists fixed ordering cost, you need to consider more.
  - For example, suppose that x^* (the optimal order quantity that Newsvendor formula indicates) is 500 and you have 499 units currently available.
  - If there is no fixed cost, you should order 1 unit to prepare 500 units available for sale.
  - However, when fixed cost is large, then you might want to just give up some potential sale opportunity and stay with 499 level inventory.

Discussion

0 . 1 . 2 . . . 14.15

$x^* = y =$ opt. order quantity

- Opt. decision when nonzero inventory and nonzero fixed cost
- Still, y provides "the order up to quantity", which means if you are going to ever order, then you should order up to y. (If you are paying fixed cost, then why not order up to optimal?)
- Therefore, at any level of your inventory, your decision is either.
  - i) order up to y
  - ii) order nothing.

if you have x unit of inventory you should order

- Without having the newsvendor optimal equality (or inequality for discrete distribution), you don't know whether you need to order 0 unit or 1 unit or 2 unit, ⋯ And you can't consider all possible cases!
- Basic Newsvendor solution narrows down your action set to two.
- At some level of inventory, i) is optimal choice, and at some level, ii) can be optimal choice.

order up to y

order nothing

0
1
2
3
⋮
$y-x-1$
$y-x$
$y-x+1$
⋮

Back to the example

- The example asked you to find which action is optimal when you have different inventory level $x = 0, 500, 700, 800$
- You need to evaluate total expected costs for two possible actions and make decision based on the comparison.
- Solution to 1. **optimal ordering quantity for each case** ($x = 0 . 500 . 700 . 800$)
    - What is y? $F(y) = \frac{C_u}{C_o + C_u} \Rightarrow \frac{y - 300}{900 - 300} = \frac{50}{65 + 50} \Rightarrow y = 560$
    - What are the two possible actions at inventory x?
        - i) order up to y (i.e. order 560−x)
        - ii) " nothing (i.e. order 0)
    - Formula for total cost?

total cost = $\underset{\text{(fixed + var.)}}{\text{order cost}}$ + $\underset{\text{(shortage)}}{\text{disruption cost}}$ + disposal cost

- Solution to 1. **optimal ordering quantity for each case** (continued)
  1. What would you do when $x = 0$?
  2. What would you do when $x = 500$?
  3. What would you do when $x = 700$?
  4. What would you do when $x = 800$?

$D \sim U(300, 900)$

Case 1 : $x=0$

i) $\mathbb{E}[TC \mid \text{if order } 560 \text{ at } x=0] = (1500 + 50 \cdot 560) + 100 \cdot \mathbb{E}[(D-560)^+] + 15\mathbb{E}[(560-D)^+]$

$= 29500 + 100 \cdot 96 + 15 \cdot 56 = 39940$

ii) $\mathbb{E}[TC \mid \text{if order } 0 \text{ at } x=0] = 0 + 100\mathbb{E}[(D-0)^+] + 15\mathbb{E}[(0-D)^+]$

$= 100 \cdot \mathbb{E}D = 60000$

$\therefore$ We should order up to 560 liters if $x=0$!

Case 2 : $x=500$

$\therefore$ We should order zero if $x=500$!

i) $\mathbb{E}[TC \mid \text{if order } 60 \text{ at } x=500] = (1500 + 50 \cdot 60) + 100\mathbb{E}[(D-560)^+] + 15\mathbb{E}[(560-D)^+]$

$= 4500 + 100 \cdot 96 + 15 \cdot 56 = 14940$

ii) $\mathbb{E}[TC \mid \text{if order } 0 \text{ at } x=500] = 0 + 100\mathbb{E}[(D-500)^+] + 15\mathbb{E}[(500-D)^+]$

$= 13795$

Case 3: $x = 700$

* order 0 because you have more than order up to quantity $(y, 560)$
  (i.e. $700 > 560$)

Case 4: $x = 800$

* order 0 for the same reason.

Case : $x = 530$

* order 0 because we would not even order at $x = 500$

2. Opt. ordering policy for arbitrary initial inventory level x?
   - You should make an order when inventory level is low despite of fixed cost.
   - You should not make an order when inventory level is high.
   - Then there is a critical 'threshold' where you should order up to y if inventory level below 'the threshold' and order nothing if above.
   - At the threshold, the two actions should result same total expected cost.

i) order $560-x$
ii) " 0

x
0 ... order $560-x$

↑ ordering is optimal

s ... equivalent ... threshold

↓ not ordering is optimal

500 ... Order 0
530 ... "
700 ... "
800 ... "

Case $x=s$

① $\mathbb{E}[TC \mid \text{order } 560-s \text{ if } x=s]$

=

② $\mathbb{E}[TC \mid \text{order } 0 \text{ if } x=s]$

=

the quantity s will make ① and ② equal!

(S, s) policy?

560 to be found

- Definition: (S, s) policy is the inventory policy where you make an order up to point S if and only if your inventory level is below s.
  - How would you describe S and s?
    - S: order up to quantity (= y from basic newsvendor model)
    - s: threshold of inventory levels where two actions lead to equivalent result.
- Once S and s are found, what's the advantage?
  - You can make very quick and automated decision whether or not to place an order.
  - You can regard this **policy** as a readily available solution for any inventory level.
  - Policy vs solution

Sol'n to every contingency

contingent cases of

⊛ Under (S, s) policy, if your current inventory level is x, your optimal action is

$$\begin{cases} \text{if } x \le s & \text{order } S - x \\ \text{if } x > s & \text{order } 0 \end{cases}$$

Solution to 2. **for arbitrary initial inventory level**

- At the threshold level s, the two actions are equivalent.

$\mathbb{E}[TC \mid \text{if order } 560-s, \text{ at } x=s] = \mathbb{E}[TC \mid \text{if order } 0 \text{ at } x=s]$

$LHS = (1500 + 50(560-s)) + 100\,\mathbb{E}[(D-560)^+] + 15\,\mathbb{E}[(560-D)^+]$

$= 1500 + 50 \cdot 560 - 50s + 100 \cdot 96 + 15 \cdot 56 = 39940 - 50s$

$RHS = (\ 0\ ) + 100\,\mathbb{E}[(D-s)^+] + 15\,\mathbb{E}[(s-D)^+]$

$= \frac{100}{1200}(900-s)^2 + \frac{15}{1200}(s-300)^2$

cf) $\mathbb{E}[(D-s)^+] = \int_{300}^{900} (y-s)^+ \frac{1}{600}\,dy = \frac{1}{1200}(900-s)^2$

Setting LHS = RHS gives

$$\Rightarrow 39940 - 50s = \frac{100}{1200}(900-s)^2 + \frac{15}{1200}(s-300)^2$$

$$\Rightarrow 115s^2 - 129000s + 34422000 = 0$$

$$s = \frac{-(-129000) \pm \sqrt{129000^2 - 4 \cdot 115 \cdot 34422000}}{2 \times 115}$$

$$= \cancel{684} \text{ or } \boxed{437}$$

($\because$ $s < S$ must be met)

cf) $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$(S, s) = (560, 437)$$

If
- $x = 400$, order 160
- $x = 430$, " 130
- $x = 470$, " 0

3. **total expected cost**

Exercise 2

What is the total expected cost when the initial inventory is $x = 0$? What is the total expected cost when the initial inventory $x = 700$?

Summary for (S, s)-policy

∃ inventory
∃ fixed ordering cost

1. Departure from basic newsvendor to (S, s)-policy.

   ① Find S (=y) from newsvendor equation
   ② On x = s, two actions are equivalent

2. What would you do at particular inventory level x?

   $\mathbb{E}\left[\text{TC (or profit)} \,\middle|\, \text{order } S-x \text{ from } x \text{ get to } S\right]$ vs $\mathbb{E}\left[\text{TC (or. profit)} \,\middle|\, \text{order } 0 \text{ from } x \text{ get to } x\right]$

3. What is the policy for an arbitrary inventory level?

   $\mathbb{E}\left[\text{TC (or, profit)} \,\middle|\, \text{order } S-s \text{ from } s \text{ get to } S\right] = \mathbb{E}\left[\text{TC (or profit)} \,\middle|\, \text{order } 0 \text{ from } s \text{ get to } s\right]$

   (set equal)

   then, solve for s

Exercise 3

in page 16

Repeat the problem 1.,2.,3, when the demand is following:
$\mathbb{P}(D = 300) = .2, \mathbb{P}(D = 500) = .4, \mathbb{P}(D = 700) = .3$*, and* $\mathbb{P}(D = 900) = .1$. *The fixed order cost is ~~still~~ now \$1000.*

```
"An idea is always a generalization, and generalization is a property of thinking. To generalize means to think. - Georg Wilhelm Friedrich Hegel"
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