Stochastic Processes, Final Exam, 2024 Spring

- Duration: 120 minutes

- Closed material, No calculator

- Name: _____

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- Write legibly.

- Justification is necessary unless stated otherwise.

1	30
2	40
3	20
Total	90

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#1. Suppose that you are running a barbershop with a single barber whose service time is exponentially distributed with the mean of 1/4 hour. Currently, the arrival process is given as a Poisson process with the rate of 2 customers per hour. There is only one extra waiting space in the shop. Suppose that the shop operates for 10 hours per day and each customer generates revenue of 20 dollars on average. The barber gets paid 200 dollars per day.

(a) What is the expected waiting time for each customer in the queue? [10pts]

(b) What is the expected daily profit of this barber shop? [10pts]

(c) You are planning to expand this barbershop by hiring another barber and by adding another waiting area. Describe the new service system by presenting the rate matrix <u>or</u> the rate diagram.[1] [10pts]

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[1] Present only one. If both are presented, then one with more error will be graded.

#2. Consider the modified version of M/M/2/2 queuing model of the following, where each server has different service rate and the server J is the priority server.[2] The following figure is the rate diagram for the CTMC $X(t)$.[3] (All numbers at diagram are "per minute" rate, i.e. the rate from state 0 to state $1J$ is 10/min)
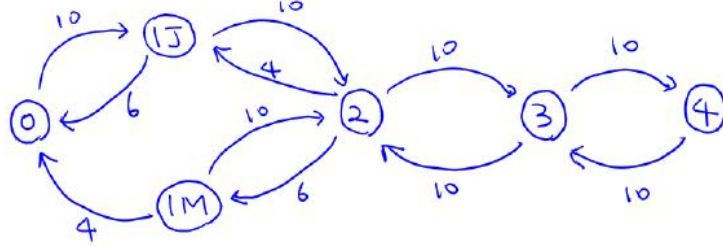
(a) Present the rate matrix G. [5pts]

Assume now that you have obtained $\pi = \{\pi_0, \pi_{1J}, \pi_{1M}, \pi_2, \pi_3, \pi_4\} = \{0.2, 0.15, 0.05, 0.2, 0.2, 0.2\}$ and use this stationary distribution π to answer following more questions.

(b) What is the long run fraction of time that the server J is busy? [5pts]

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[2] An arrival finding the server J free will begin receiving service by the server J. An arrival finding the server J busy and the server M free will join the server M

[3] For example, $X(t) = 3$ means that there are 3 customers in the system at time t; $X(t) = 1J$ means there is one customer in the middle of the service by server J at time t

(c) What is the throughput (TH)? [5pts]

(d) What is the expected number of customers in the system? (L_{sys}) [5pts]

(e) What is the expected total time spent in the system for a customer? (W_{sys}) [5pts]

(f) What is the expected waiting time in the queue for a customer? (W_q) [5pts]

(g) (Difficult) What is the probability that a customer is served by server J? (Justification is necessary) [4] [10pts]

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[4] Hint: The correct answer is between 0 and 1. If your answer is not in this range, then it means you did something wrong in the previous questions or on this question. You probably need to use your answer to (f).

#3. Consider a factory with two machines and a repairman. The time until machine A breaks down follows an exponential distribution with mean 3 hours, and the time until machine B breaks down follows an exponential distribution with mean 2 hours. A repairman can work on one broken machine at a time. Repair times for a machine is exponentially distributed. It takes 1 hour on average to fix the machine A and 2 hours on average to fix the machine B. What is the average number of machine under repair?[20pts]

"Examinations are formidable even to the best prepared, for the greatest fool may ask more than the wisest man can answer." - Charles Caleb Colton.