Stochastic Processes, Quiz 4, 2024 Fall

Solution and Grading

- Duration: 40 minutes
- Weight: 10% of final grade
- Closed material, No calculator

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- Write legibly.
- Justification is necessary unless stated otherwise.

#1. Next month's production at a manufacturing company will use a certain solvent for part of its production process. The company need to prepare solvent in prior and the demand of solvent is random. Check the following assumptions:

- Assume that there is an ordering cost of \$1,500 incurred whenever an order for solvent is placed.
- The solvent costs \$50 per liter. Due to short product life cycle, unused solvent cannot be used in following months.
- There will be a \$15 disposal charge for each liter of solvent left over at the end of the month.
- If there is a shortage of solvent, the production process is seriously disrupted at a cost of \$100 per liter short.
- The demand is estimated as $\mathbb{P}(D = 300) = 0.2, \mathbb{P}(D = 500) = 0.4, \mathbb{P}(D = 700) = 0.3$, and $\mathbb{P}(D = 900) = 0.1$

After some careful analysis, the optimal (S, s) policy is obtained where $S = 500$ and $s = 444$. If the current inventory level is 450 liters, then what is the expected total operating cost for the next month? (Hint: First you need to decide whether or not to make an order.) `[10pts]`

Based on the (S, s) policy, the optimal course of action is not making an order. Thus, the expected total operating cost can be written as:

$$\begin{aligned} \mathbb{E}[\text{Total Cost}] &= \mathbb{E}[\text{Order cost}] + \mathbb{E}[\text{Shortage cost}] - \mathbb{E}[\text{Disposal cost}] \\ &= 0 + 100\mathbb{E}[(D - 450)^+] + 15\mathbb{E}[(450 - D)^+] \\ &= 0 + 100 \cdot 140 + 15 \cdot 30 = 14450 \end{aligned}$$

Difficulty: Medium-Hard

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#2. We want to decide whether i) to employ a human operator or ii) to buy a machine to paint steel beams. Consider the following facts.

- Steel beams arrive at the painting station at a constant rate of one every 100 seconds.
- A human operator takes an average time of 80 seconds to paint a steel beam, with a standard deviation of 40 seconds.
- A machine painter takes on average 90 seconds to paint a steel beam, with a standard deviation of only 18 seconds.
- Assume the hourly wage of human operator and the hourly cost of painting machine are equal.

Estimate the expected waiting time in queue of a steel beam for each type of the operators. In your opinion, which is better choice and why?`[10pts]`

By the Kingman's formula, for a $G/G/1$ queue, the expected waiting time in the queue can be found as:

- Human: $\mathbb{E}[W_q] = 80 \cdot \frac{0.8}{1-0.8}\left(\frac{0^2+0.5^2}{2}\right) = 40$
- Machine: $\mathbb{E}[W_q] = 90 \cdot \frac{0.9}{1-0.9}\left(\frac{0^2+0.2^2}{2}\right) = 16.2$

Machine is better because of less waiting time in queue. (Thus, machine has the less waiting space even though its average processing time is worse.

Difficulty: Medium