# Stochastic Processes, Quiz 2, 2024 Fall

Solution and Grading

- Duration: 30 minutes
- Weight: 8% of final grade
- Closed material, No calculator

- Name: ____________________
- Student ID: ____________________
- E-mail: ____________ @seoultech.ac.kr

- Write legibly.
- Justification is necessary unless stated otherwise.

Let X be a Poisson random variable with parameter 4, and let $Y = min(X, 3)$. What is $\mathbb{P}(Y \leq 2 | Y \leq 4)$? [8pts]

You first need to identify the pmf of $Y$ as follows.

- $p(y) = \mathbb{P}(Y = y) = \mathbb{P}(min(X, 3) = y)$ therefore,
- $p(0) = \mathbb{P}(Y = 0) = \mathbb{P}(min(X, 3) = 0) = \mathbb{P}(X = 0) = \frac{4^0 e^{-4}}{0!} = e^{-4}$
- $p(1) = \mathbb{P}(Y = 1) = \mathbb{P}(min(X, 3) = 1) = \mathbb{P}(X = 1) = \frac{4^1 e^{-4}}{1!} = 4e^{-4}$
- $p(2) = \mathbb{P}(Y = 2) = \mathbb{P}(min(X, 3) = 2) = \mathbb{P}(X = 2) = \frac{4^2 e^{-4}}{2!} = 8e^{-4}$
- $$\begin{aligned} p(3) = \mathbb{P}(Y = 3) = \mathbb{P}(min(X, 3) = 3) = \mathbb{P}(X \geq 3) &= 1 - \mathbb{P}(X < 3) \\ &= 1 - (\mathbb{P}(X = 0) + \mathbb{P}(X = 1) + \mathbb{P}(X = 2)) \\ &= 1 - (e^{-4} + 4e^{-4} + 8e^{-4}) \\ &= 1 - 13e^{-4} \end{aligned}$$
- $p(4) = \mathbb{P}(Y = 4) = \mathbb{P}(min(X, 3) = 4) = 0 \qquad (\because min(X, 3) \leq 3 \;\; always)$
  also $p(y) = 0$ for all $y \geq 4$

Therefore the pmf of $Y$ is as follows:

$$p(y) = \begin{cases} e^{-4} & \text{for} \;\; y = 0 \\ 4e^{-4} & \text{for} \;\; y = 1 \\ 8e^{-4} & \text{for} \;\; y = 2 \\ 1 - 13e^{-4} & \text{for} \;\; y = 3 \\ 0 & \text{otherwise} \end{cases}$$

Then, you can continue to answer the conditional probability as:

$$\begin{aligned} \mathbb{P}(Y \leq 2 | Y \leq 4) &= \frac{\mathbb{P}(Y \leq 2 \cap Y \leq 4)}{\mathbb{P}(Y \leq 4)} = \frac{\mathbb{P}(Y \leq 2)}{\mathbb{P}(Y \leq 4)} \\ &= \frac{\mathbb{P}(Y = 0) + \mathbb{P}(Y = 1) + \mathbb{P}(Y = 2)}{\mathbb{P}(Y = 0) + \mathbb{P}(Y = 1) + \mathbb{P}(Y = 2) + \mathbb{P}(Y = 3) + \mathbb{P}(Y = 4)} \\ &= \frac{e^{-4} + 4e^{-4} + 8e^{-4}}{e^{-4} + 4e^{-4} + 8e^{-4} + 1 - 13e^{-4} + 0} \\ &= 13e^{-4} \end{aligned}$$

Difficulty: Medium
Grading Scheme: Only one minor error may earn 4pts. No partial credit if two or more errors are found.