Lecture B1. Newsvendor 1

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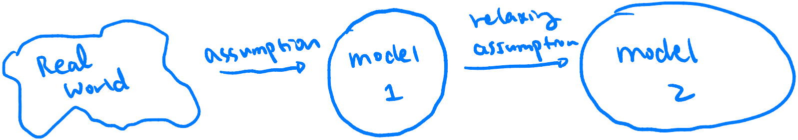

I. Problem and Solution

Motivation

Your brother will start part time job of selling newspapers at a *subway station in the morning. You are asked how many he should prepare for selling.*

- What are the kind of information that you need in order to give him a good advice?
  1. (Demand) How many customer?
  2. (Retail Price) How much do you sell a copy at?
  3. (Material Price) " pay to wholesaler?
  4. (Salvage price) What happens to unsold newspaper?

유동인구수.

어제는 신문 몇부?

신문의 소매가격?

최근에 issue

신문의 도매가격?

unsold item

The information

1. (*demand*) How many customer?
2. (*retail price*) How much do you sell a copy at?
3. (*material cost*) How much do you pay to the wholesaler?
4. (*salvage value*) How much do you sell an unsold copy back to the wholesaler?

- Suppose your brother gave you the following information.
  1. 11, 12, 13, 14, or 15, equally likely.
  2. \$2 per copy
  3. \$1 per copy
  4. \$0.5 per copy

Exercise 1

Assume that D follows the following discrete distribution.

d	20	25	30	35
$\mathbb{P}[D=d]$	0.1	0.2	0.4	0.3
$30 \wedge d$				
$(30-d)^+$				
$24 \wedge d$				
$(24-d)^+$				

Answer the followings.

- $\mathbb{E}[30 \wedge D] =$
- $\mathbb{E}[(30-D)^+] =$
- $\mathbb{E}[24 \wedge D] =$
- $\mathbb{E}[(24-D)^+] =$

Optimal economic decision

- Your brother's goal is to earn as much money as possible.
- In other words, the newsvendor wants to make *an optimal decision* that *maximizes his expected profit*.
- In some settings, the optimal decision is related to *minimize expected cost*.
- In this case, profit, *the objective function*, is composed in the following way.

$$\begin{aligned}
\text{Profit} &= \text{Revenue} - \text{Cost} \\
&= [\text{Sales Rev.} + \text{Salvage Rev.}] - \text{Material Cost} \\
&= (\text{from Reg. Sale}) + (\text{from unsold item}) - (\text{for preparation})
\end{aligned}$$

Solution - tabular method

preparation (stock) \ demand	11 (20%)	12 (20%)	13 (20%)	14 (20%)	15 (20%)	Exp. Profit
11	$11 \cdot 2 + 0 - 11 \cdot 1 = 11$	11	11	11	11	11
12	10.5	12	12	12	12	11.7
13	10	11.5	13	13	13	12.1
14	9.5	11	12.5	14	14	12.2
15	9	10.5	12	13.5	15	12

stock < demand "understock" "lost sale"

Stock > Demand "overstock" "too many prep"

Economic cost around decision making

Demand : D
Stock : X

- Your brother wants to the prepared **stock** to exactly match the customer **demand**.
- If **Stock > Demand**,
  - **Overstock cost occurs.**
  - In other words, prepared too much, excessive items, or over-prepared.
  - Overstock by how many? $\max(X-D, 0)$
  - Ex) holding cost, items lose its economic potential value
- If **Stock < Demand**,
  - **Understock cost occurs**.
  - In other words, prepared too less, lost opportunity, or lost sales.
  - Understock by how many? $\max(D-X, 0)$
  - Ex) stock-out, reputation loss, cancellation reward
- Newsvendor wants to find a optimal balance between overstock cost and understock cost.
- Just as all of the retail store you can think of.

Mathematical representation

1. (number of units) If demand D is random and you prepare X unit, then
   - # of unit for sales: $\min(X, D)$
   - # of unit for overstock: $(X-D)^+$
   - # of unit for understock: $(D-X)^+$
2. (unit cost) If retail $\boxed{p}$rice is p, material $\boxed{c}$ost is c, and $\boxed{s}$alvage price is s, then
   $=2$ $=2$ $=0.5$
   - unit cost for overstock: $(C_o) = c-s$
   - unit cost for understock: $(C_u) = p-c$
3. Cost = (# of units) × (cost per unit)
   - overstock cost: $(c-s)(X-D)^+$
   - understock cost: $(p-c)(D-X)^+$
4. Economic cost = Understock cost + Overstock cost

$$= (p-c)(D-X)^+ + (c-s)(X-D)^+$$

$$= C_u(D-X)^+ + C_o(X-D)^+$$

Formal treatment

Remark 1

Newsvendor problem aims to find the optimal number of preparation that maximizes the expected profit formulated as:
$\mathbb{E}[\text{profit}] = \mathbb{E}(\text{sale rev.}) + \mathbb{E}(\text{salvage rev.}) - \mathbb{E}(\text{material cost})$

Theorem 1

In newsvendor problem, maximizing the expected profit is equivalent to minimizing the expected economic cost (sum of the expected overstock cost and the expected understock cost).

- *Two problems being mathematically equivalent to each other implies that a solution that solves the one problem solves the other problem, and vice versa.*

Problem and Solution

Remark 2

By the above remark and theorem, newsvendor model is to find optimal x^* that minimize total expected economic cost. That is,

$$x^* = argmin_x \; c_o\mathbb{E}[(X-D)^+] + c_u\mathbb{E}[(D-X)^+]$$

Theorem 2

The solution to the above problem can be found as:

- *If D is a continuous r.v, with cdf $F(\cdot)$, then find y s.t. $\boxed{F(y) = \frac{c_u}{c_o+c_u}}$*
- *If D is a discrete r.v, with cdf $F(\cdot)$, then find smallest y such that $F(y) \geq \frac{c_u}{c_o+c_u}$*

II. Exercises

Exercise 2

Find your brother's optimal stock level by the above Theorem 2. Then, find his expected profit using the Remark 1.

x^* : smallest y s.t. $F(y) \geq \frac{C_u}{C_o + C_u} = \frac{1}{0.5+1} = \frac{2}{3}$

✓ $F(y) = \mathbb{P}(D \leq y)$

	y	11	12	13	14	15
pmf:	$\mathbb{P}(D=y)$	0.2	0.2	0.2	0.2	0.2
cdf	$\mathbb{P}(D\leq y)$	0.2	0.4	0.6	0.8	1.0

$F(11) = 0.2$

$F(12) = 0.4$

$F(13) = 0.6$

$F(14) = 0.8$ → x^*

$F(15) = 1.0$

Exercise 3

Your brother is now selling milk. The customer demands follow $U(20, 40)$ *gallons. Retail price is \$2 per gallon, material cost is \$1 per gallon, and salvage cost is \$0.5 per gallon. Find optimal stock level and expected profit.*

Exercise 4

Lemonade sells for \$18 per gallon but only costs \$3 per gallon to make. If we run out of lemonade, it will be impossible to get more. On the other hand, leftover lemonade has a value of \$1. Express the following quantity using sale as X and demand as D.

- c_u
- c_o
- *Expected economic cost*
- *Expected profit*

Exercise 5

Prove Theorem 1. (Hint: you may use formulation from Exercise 4)

Exercise 6

Show that $(D \wedge Y) + (Y - D)^+ = Y$

Exercise 7

Let D be a continuous random variable and uniformly distributed between 5 and 10.

- $E[max(D, 8)]$
- $E[(D-8)^{-}]$

III. Discussion

On the nature of newsvendor

- What does $\underset{\mathbb{P}(D\le y)}{\overset{}{F(y)}} = \frac{c_u}{c_o+c_u}$ imply?
  - $\mathbb{P}(D \le y)$: Demand가 y보다 작거나 같을 확률
  - $\boxed{\mathbb{P}(\text{meeting all demand})}$
  - $\mathbb{P}$(readiness)
  - $\mathbb{P}$ (overstocked situation)
  - fill rate
- Buying a suit - department store vs outlet

$$\frac{C_u\uparrow}{C_o + C_u\uparrow}\uparrow \qquad \frac{C_u\downarrow}{C_o + C_u\downarrow}\downarrow$$

$$F(y) = \frac{C_u}{C_o + C_u} \qquad \begin{cases} C_u: \text{understock cost} \\ C_o: \text{overstock cost} \end{cases}$$

i) $\underset{10}{C_u} \gg \underset{1}{C_o}$ $\qquad F(y) = \frac{10}{1+10} = \frac{10}{11}$

ii) $\underset{1}{C_u} \ll \underset{10}{C_o}$ $\qquad F(y) = \frac{1}{10+1} = \frac{1}{11}$

Newsvendor = tradeoff bet'n C_o & C_u

Optimization = tradeoff

Newsvendor in a big picture

deterministic opt
stochastic opt

- Newsvendor is characterized as an *optimal decision making* problem.
- Newsvendor is characterized as a "baby model" for a *decision making under uncertainty*.
- Since it is decision making *under uncertainty*, a random variable is involved.
- It can be viewed as *decision making at time 0*, then *results come out at time 1*. It is thus called as *one-period problem*.

  $0 \longrightarrow 1$

- *Stochasticity* is the combined notion of *time* and *randomness*.
- In sum, newsvendor is *one-period stochastic decision making* problem, where
  1. *one-period* specifies its time domain.
  2. *stochastic* tells that we make decision considering future randomness.
  3. *decision making* obviously includes optimization aspect of the problem.

one-period stoc. dec making (Newsvendor)
↓
multi period stoc. process (DTMC)
↓
multi-period stoc dec. making (MDP. Reinforcement learning)

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"Man can learn nothing unless he proceeds from the known to the unknown. - Claude Bernard"
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