Stochastic Processes, Mid-term, 2025 Spring

Solution and Grading

- Duration: 90 minutes
- Closed material, No calculator.

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- Write legibly.
- Justification is necessary unless stated otherwise.

1	10
2	10
3	10
4	10
5	10
Total	50

#1. Clearly explain the meaning of each component in Kendall's queuing notation $M/D/2/3$.[10pts]

- M:
- D:
- 2:
- 3:

1) Interarrival times follow an exponential distribution, 2) Service times are deterministic, 3) There are two servers, and 4) Size of wating area is three.

Grading Scheme:

- Each 2.5 pts. No partial point.

#2. Consider a queuing system with a single server and infinite waiting space. If the arrival rate and the service rate are equal, state the conditions under which the system can be stable.[10pts]

Both interarrival times and services times must be deterministic.

Grading Scheme:

- No partial point.

#3. A free legal clinic has a single lawyer who provides one-on-one consultations. Clients arrive at an average rate of 2 per hour, and the interarrival time is known to follow an exponential distribution. Each consultation takes an average of 25 minutes, with a standard deviation of 10 minutes.

(a) What is the expected waiting time in a queue?`[5pts]`

(b) What is the expected number of customers in the system?`[5pts]`

- $\rho = \frac{1/\mathbb{E}U}{1/\mathbb{E}V} = \frac{1/30}{1/25} = 5/6$, $c_a = 1$, and $c_s = 10/25 = 2/5$
- $\mathbb{E}W_q = \mathbb{E}V \cdot \frac{\rho}{1-\rho} \cdot \frac{c_a^2+c_s^2}{2} = 25 \cdot \frac{5/6}{1-5/6} \cdot \frac{1^2+(2/5)^2}{2} = 25 \cdot 5 \cdot 29/50 = 145/2 = 72.5$ minutes
- By Little's law, $L_{sys} = \lambda W_{sys} = \lambda(W_q + W_{svc}) = 1/30 \cdot (72.5 + 25) = \frac{97.5}{30} = 3.25$

Grading scheme: No partial point

#4. Consider a queuing system with two servers, X and Y. (Customers can receive service from either server.) Server X provides service at an average rate of 1 customer every 20 minutes, and Server Y provides service at an average rate of 1 customer every 30 minutes. Service times follow an exponential distribution. Each arriving customer is assigned to the first available server.

There are currently four customers, A, B, C, and D. A is currently being served by server X, and B is being served by server Y. When either A or B finishes service, the next customer, C, will be assigned to the available server. After C, the next customer is D.

(a) What is the probability that A will be the last to leave the system? `[5pts]`

(b) What is the probability that C will be the last to leave the system? `[5pts]`

(a) This is the case that server Y finishes a service faster than the server X three times in a row. Thus, the probability is $0.4 \times 0.4 \times 0.4 = 0.064 (= 8/125)$.

Grading scheme: If the presented answer is $0.6 \times 0.6 \times 0.6$, then 2.5 pts

(b) We need to consider the following two cases where C is served by X or Y.

1. If A finishes before B (with prob. 0.6),
   - then C starts service at server X immediately after A leaves, and B continues service at server Y.
   - If server Y finishes service faster than server X twice in a row (with prob. 0.4×0.4),
   - then C will be the last to leave the system.
   - The probability of this happening is $0.6 \times 0.4 \times 0.4 = 0.096$
2. If B finishes before A (with prob. 0.4),
   - then C starts service at server Y immediately after B leaves, and A continues service at server X.
   - If server X finishes service faster than Y twice in a row (with prob. 0.6×0.6),
   - then C will be the last to leave the system.
   - The probability of this happening is $0.4 \times 0.6 \times 0.6 = 0.144$
3. Thus, the answer is 0.24(=0.096+0.144)(=6/25).

Grading scheme: No partial point

#5. A travel agency can pre-purchase airline tickets for a Hawaii route at $600 each. Unsold tickets has a salvage value of $400 each. The customer demand varies according to the retail price that the agency offers. The following is the demand information.

- If the retail price for a ticket is offered at $1,000, then customer demand is estimated to follow a discrete uniform distribution between 71 and 75. (i.e. 71, 72, 73, 74, or 75, each with probability of 20%)
- If the retail price for a ticket is offered at $900, then customer demand is estimated to follow a discrete uniform distribution between 81 and 85. (i.e. 81, 82, 83, 84, or 85, each with probability of 20%)

How many tickets should the travel agency purchase, and what should be the retail price? `[10pts]`

- If retail price is set at $1000,
  - $c_o = 200, c_u = 400, F(x) = 2/3$, it follows that the optimal order quantity is 74.
  - $\mathbb{E}min(D, 74) = 72.8$
  - $\mathbb{E}(74 - D)^+ = 1.2$
  - Thus, the expected profit is $72.8 * 1000 + 1.2 * 400 - 74 * 600 = 28,880$
- If retail price is set at $900,
  - $c_o = 200, c_u = 300, F(x) = 3/5$, it follows that the optimal order quantity is 83.
  - $\mathbb{E}min(D, 83) = 82.4$
  - $\mathbb{E}(83 - D)^+ = 0.6$
  - Thus, the expected profit is $82.4 * 900 + 0.6 * 400 - 83 * 600 = 24,600$
- Therefore, the agency should buy 74 tickets and sell at $1,000.

Grading scheme: Your work must involve the three step process: i) finding an optimal order quantity for each retail price, ii) finding the corresponding expected profits, and iii) comparing the retail price policy. If your work involves the three steps and reasonable efforts are made, then 5pts are given.

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