Stochastic Processes, Quiz 3, 2023 Spring

- Duration: 120 minutes

- Closed material, No calculator

- Name: _____

- Student ID: _____

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- Write legibly.

- Justification is necessary unless stated otherwise.

1	50
2	25
3	25
Total	100

#1. Fill in the blanks by using the following terms. [Each 5pts]

periodic	aperiodic	class	recurrent	transient	absorbing
finite	infinite	irreducible	reducible	reach	communicate

(a)	
(b)	
(c)	
(d)	
(e)	
(f)	
(g)	
(h)	
(i)	
(j)	

- An absorbing state is a special case of a [(a)   ] state.

- A DTMC is said to be [(b)   ] if there exists a state whose period is higher than one.

- A DTMC is said to be [(c)   ] if there are more than one class.

- If all diagonal elements of transition matrix in DTMC are positive, then the DTMC has to be [(d)   ].

- A DTMC is said to be [(e)   ] if all states communicate.

- A group of states that can reach each other is said to be a [(f)   ].

- In a DTMC, the limiting probability is same as the unique stationary distribution regardless of initial states if following <u>three</u> conditions of the DTMC are met: [(g)   ] [(h)   ] [(i)   ].

- State i is said to be [(j)   ] if, starting from state i, the probability of getting back to state i in some future is equal to 1.

#2. You either drink coffee or not on any given day. The following is the transition rule:

- If you drank coffee yesterday and today, the chance of you drinking coffee tomorrow is 0.2.

- If you did not drink coffee yesterday but drank coffee today, then the chance of drinking coffee tomorrow is 0.4.

- If you drank coffee yesterday but not today, then the chance of drinking coffee tomorrow is 0.6.

- If you did not drink coffee yesterday and today, then you will drink coffee tomorrow with the probability 0.8.

(a) Suppose you drank coffee yesterday and today, then what is the probability that you will not drink coffee tomorrow but will drink coffee the day after tomorrow? (Answer in number, justification is necessary, and very limited partial points given) [10pts]

(b) In the long run, what is the probability that you will be drinking coffee on a given day? (Answer in number, justification is necessary, and very limited partial points will be given) [15pts]

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#3. Consider a DTMC with the following transition matrix.

$$\mathbf{P} = \begin{pmatrix} .25 & .5 & & .25 & & & & \\ .5 & & .5 & & & & & \\ & .5 & .25 & & .25 & & & \\ & & & 1 & & & & \\ & & .5 & .5 & & & & \\ & & & & & .3 & .7 & \\ & & & & & .7 & & .3 \\ & & & & & .3 & .7 & \end{pmatrix}$$

(a) How many classes does it have? [5pts]

(b) Calculate $\mathbf{P}^{100}$ (Show your work with proper justification.) [20pts]

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Write your name before detaching. Your Name: