Turbulent pair dispersion as a continuous-time random walk

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Relative dispersion beyond the t^3 law

• The trajectories of tracers solve the advection equation

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{x}(t) = \mathbf{v}(\mathbf{x}(t), t) \text{ where } \mathbf{v} \text{ is the turbulent velocity field.}$$

Modeling of relative dispersion for tracers is relevant to describe spatial fluctuations of passive scalar field, which relate to the two-point transition probabilities $\lim_{\mathbf{r}_0\to 0}\lim_{\nu\to 0} p_\nu(\mathbf{x}, \mathbf{x}+\mathbf{r}, t|\mathbf{x}_0, \mathbf{x}_0+\mathbf{r}_0)$.

• Standard phenomenology focuses on the second-order moment, namely $\langle|\mathbf{r}(t)-\mathbf{r}_0|^2\rangle$, to discriminate between :

- The Batchelor regime, for $t \ll \tau_0 = \epsilon^{-1/3} r_0^{2/3}$:

$$\langle|\mathbf{r}(t)-\mathbf{r}_0|^2\rangle \propto \epsilon^{2/3} r_0^2\, t^2 \ \rightarrow \textbf{ballistic separation}$$

- The Richardson regime, for $\tau_0 \ll t \ll T_L$:

$$\langle|\mathbf{r}(t)-\mathbf{r}_0|^2\rangle \propto g\epsilon\, t^3 \ \rightarrow \textbf{explosive separation}$$

(*e.g.* Salazar & Collins, 2009)

• Long established modeling has focused on reproducing the t^3 law, based on the assumption that the distances between tracers undergo a scale-dependent diffusion (Richardson, 1926).

$$\frac{\partial}{\partial t}p(r,t) = \frac{\partial}{\partial r}\left(r^{d-1}K(r,t)\frac{\partial}{\partial r}\frac{p(r,t)}{r^{d-1}}\right)$$

The Markovian description has some some shortcomings :

- It is justified only if the velocity has vanishing Lagrangian correlation time τ_0 (Falkovich et al, 2001)
- It does not describe the ballistic/explosive crossover.

• The t^3 law *per se* does not prescribe univocally the *full* distribution. DNS exhibit systematic deviations from Richardson distribution :

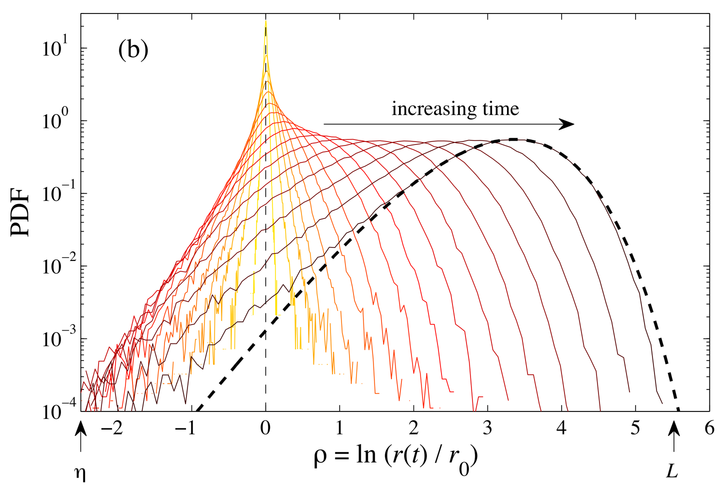


Figure 1: Evolution of the separation pdf for $r_0 = 12\eta$ in a DNS at $R_\lambda \simeq 800$. In dash : Richardson pdf.

Memory effects due to Lagrangian correlations?

- Direct approach (Eyink & Benveniste, 2013)
- Levy walk phenomenology using a distribution of waiting times (Shlesinger et al, 1987 ; Sokolov et al ,2000).

A piecewise-ballistic toy model

• We revisit the Levy walk approach to turbulent dispersion with a simple non-Markovian model for the joint time-evolution of the distance $r(t) = |\mathbf{r}|$ and of the correlation time $\tau(\mathbf{r})$. In the following first-order stochastic *ballistic model*, the separations undergo a multiplicative sequence of "ballistic" events [1].

• In terms of the separation vector $\mathbf{r}$:

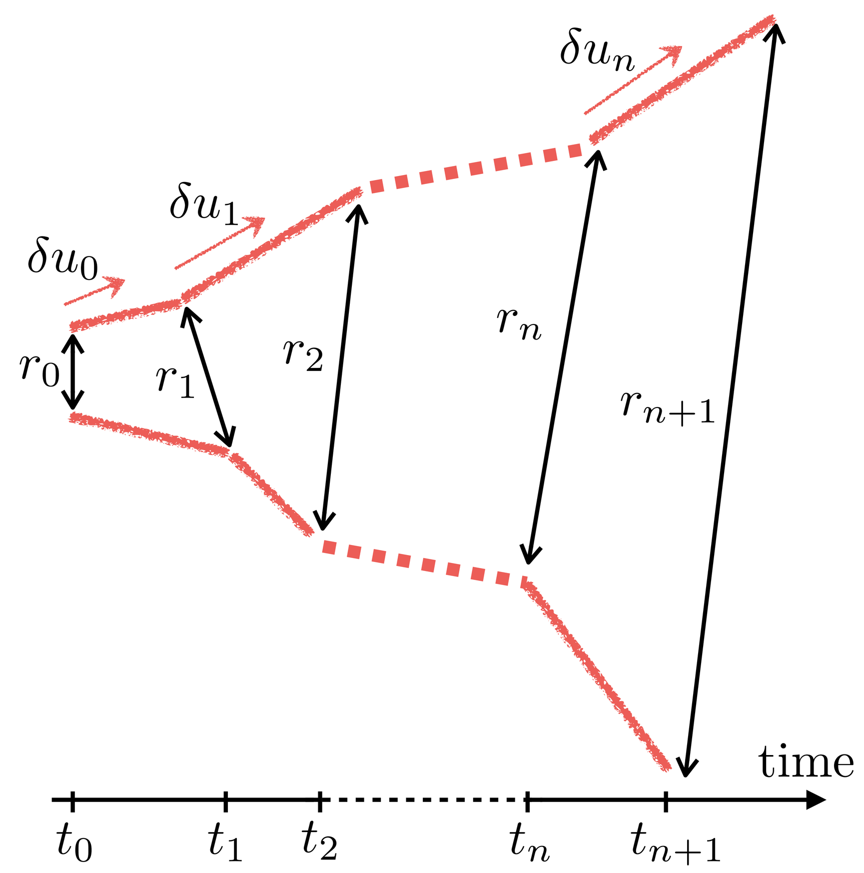
$$\mathbf{r}_{n+1} = \mathbf{r}_n + \tau_n\,\delta\mathbf{v}_n \text{ and } t_{n+1} = t_n + \tau_n$$

In terms of the pairwise distance : $r = |\mathbf{r}|$

$$r_{n+1} = r_n\left(1 + 2\alpha_n\beta_n + \beta_n^2\right)^{1/2} \text{ and } t_{n+1} = t_n + (2\epsilon)^{-1/3}(\beta_n r_n)^{2/3}$$

• The "turbulent inputs" are the joint statistics of $\alpha = \frac{\delta\mathbf{v}_\parallel}{|\delta\mathbf{v}|}$ and $\beta = \frac{|\delta\mathbf{v}|^3}{2\epsilon r}$.

$\longrightarrow$ e.g. *K41 modelling* : take α_n and β_n independent of the r_n's.



Some ballistic references

[1] Simon Thalabard, Giorgio Krstulovic, and Jérémie Bec. Turbulent pair dispersion as a continuous-time random walk. *Journal of Fluid Mechanics*, 755:R4, 2014.

[2] Mickaël Bourgoin. Turbulent pair dispersion as a ballistic cascade phenomenology. *Journal of Fluid Mechanics*, 772:678–704, 2015.

[3] IM Sokolov, J Klafter, and A Blumen. Ballistic versus diffusive pair dispersion in the richardson regime. *Physical review E*, 61(3):2717, 2000.

Qualitative features of the model

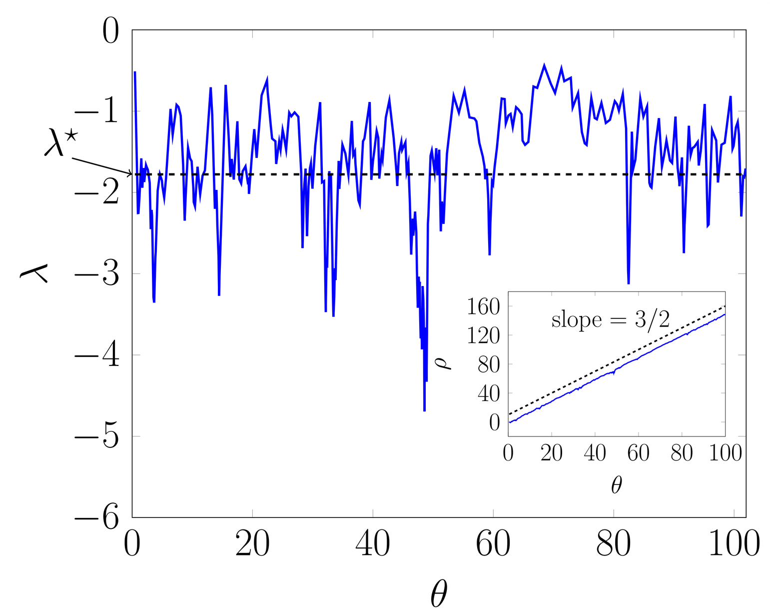
• Change of variables :

$$\lambda = \log\frac{r}{r_0} - \frac{3}{2}\log\frac{t}{t_0}\ ,\ \ \theta = \log\frac{t}{t_0}$$

$\rightarrow$ Closed equation for λ :

$$\lambda_n + 1 = \lambda_n + \frac{3}{2}\log\frac{(1+2\alpha_n\beta_n+\beta_n^2)^{1/3}}{1+\beta_n^{2/3}e^{2\lambda_n/3}}$$

$\rightarrow$ For "reasonable" choices of α, and β, the λ's become **stationary**.



$\rightarrow$ **Explosive separation :**

$$\langle\log r/r_0\rangle - \frac{3}{2}\log t/t_0 \rightarrow \langle\lambda\rangle_\infty = \text{cst}$$

$$\text{Var}(\log r) \rightarrow \text{constant}$$

$\rightarrow$ **Self-similar regime :**

$$p(\log r, t) \rightarrow \Psi(\log r - 3/2\log t)$$

• Those essential features are indeed seen in DNS :

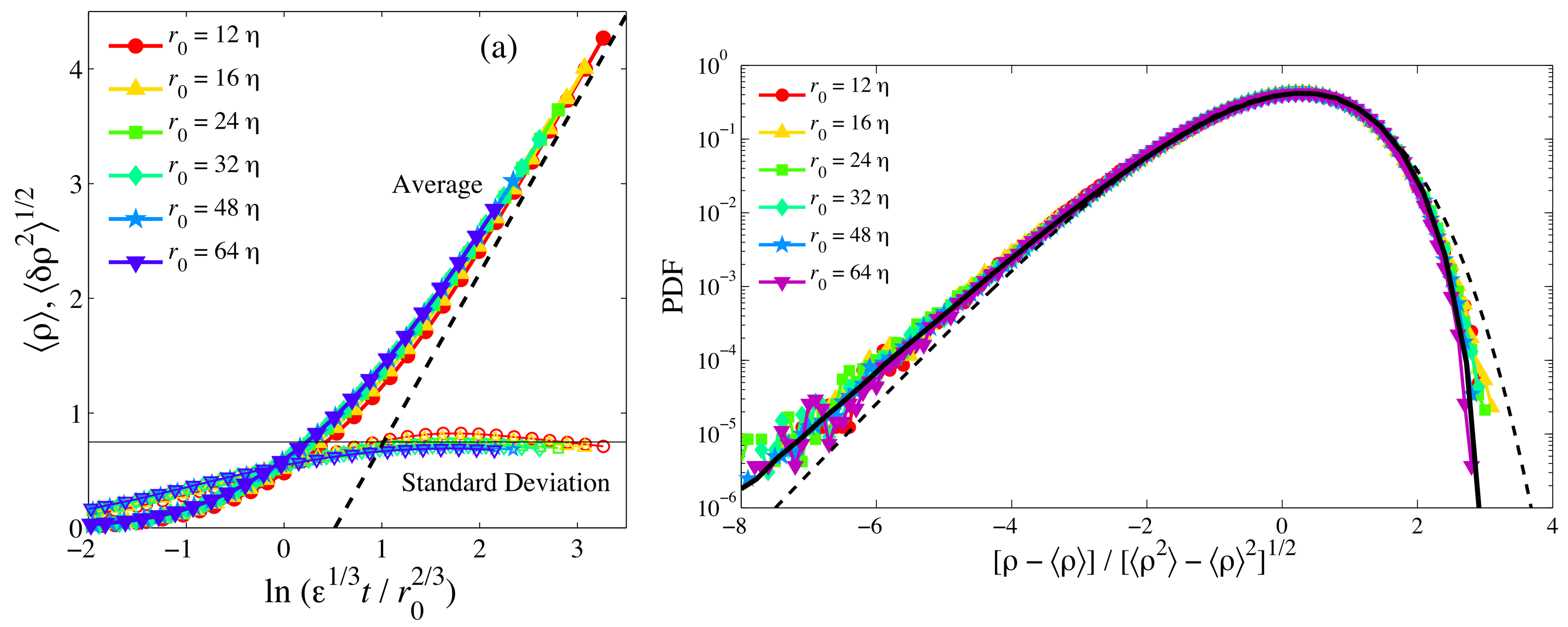


Figure 2: Statistics of tracer separations from a 3D DNS with $R_\lambda \simeq 800$. Left : convergence towards the explosive regime. Right : pdf of λ at time $t = 9\tau_0$. For the ballistic fit (in solid black), we arbitrarily use $p(\alpha) = \frac{5}{6}\left(\frac{\alpha+1}{2}\right)^{2/3}$ and $p(\beta) = \frac{e^{-(1/2)\ln^2\beta}}{\sqrt{2\pi}\beta}$. The dashed line represents Richardson pdf.

Perspectives

$\longrightarrow$ Our piecewise ballistic model provides a qualitative picture *(i)* in which the explosive separation emerges from the ballistic regime, and *(ii)* that reproduces the essential features of pair statistics.

$\longrightarrow$ Some perspectives could include *(i)* the inclusion of intermittent statistics, and *(ii)* a ballistic view on the Backward / Forward asymmetry (cf [2]).