E8CascadeTest

PyR@TE 3.0

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Contents

1 Model

1.1 Gauge groups

Name	Type	Abelian	Coupling constant
U1Y	$U(1)$	True	g_{U1Y}
SU2L	$SU(2)$	False	g_{SU2L}
SU3c	$SU(3)$	False	g_{SU3c}

1.2 Fermions

Name	Generations	U1Y × SU2L × SU3c
Q	3	$(+\frac{1}{6}, \mathbf{2}, \mathbf{3})$
L	3	$(-\frac{1}{2}, \mathbf{2}, \mathbf{1})$
uR	3	$(+\frac{2}{3}, \mathbf{1}, \mathbf{3})$
dR	3	$(-\frac{1}{3}, \mathbf{1}, \mathbf{3})$
eR	3	$(-1, \mathbf{1}, \mathbf{1})$
$SigmaF$	1	$(0, \mathbf{3}, \mathbf{1})$
NR	3	$(0, \mathbf{1}, \mathbf{1})$

1.3 Scalars

Name	Complex	Expression	Generations	U1Y × SU2L × SU3c
$phiR$	False	/	1	$(0, \mathbf{1}, \mathbf{1})$
$phiI$	False	/	1	$(0, \mathbf{1}, \mathbf{1})$
H	True	$\frac{1}{\sqrt{2}}(\Pi + i\,\Sigma)$	1	$(+\frac{1}{2}, \mathbf{2}, \mathbf{1})$

2 Lagrangian

2.1 Definitions

$$Htilde_i = \epsilon_{i,j} H_j^\dagger$$

2.2 Yukawa couplings

$$\begin{aligned}-\mathcal{L}_Y = &+ Yu_{f_1,f_2}\tilde{H}_i\overline{Q}_{f_1,i,a}uR_{f_2,a} + Yd_{f_1,f_2}\overline{Q}_{f_1,i,a}H_i dR_{f_2,a} + Ye_{f_1,f_2}\overline{L}_{f_1,i}H_i eR_{f_2}\\ &+ yN_{f_1,f_2}\tilde{H}_i\overline{L}_{f_1,i}NR_{f_2} + \text{h.c.}\end{aligned}$$

2.3 Quartic couplings

$$\begin{aligned}-\mathcal{L}_Q = &+ \lambda H_i^\dagger H_i H_{i_1}^\dagger H_{i_1} + lPhi\big(2phiI^2 phiR^2 + phiIphiIphiIphiI + phiRphiRphiRphiR\big)\\ &+ lHphi\big(phiIphiI H_i H_i^\dagger + phiRphiR H_i H_i^\dagger\big)\end{aligned}$$

2.4 Scalar mass couplings

$$-\mathcal{L}_{sm} = -\mu H_i^\dagger H_i$$

3 Renormalization Group Equations

3.1 Convention

$$\beta\left(X\right) \equiv \mu\frac{dX}{d\mu} \equiv \frac{1}{\left(4\pi\right)^2}\beta^{(1)}(X)$$

3.2 Gauge couplings

$$\beta^{(1)}(g_{U1Y}) = \frac{41}{6}g_{U1Y}^3$$

$$\beta^{(1)}(g_{SU2L}) = -\frac{11}{6}g_{SU2L}^3$$

$$\beta^{(1)}(g_{SU3c}) = -7g_{SU3c}^3$$

3.3 Yukawa couplings

$$\begin{aligned}\beta^{(1)}(Yu) = &+ \frac{3}{2}YuYu^\dagger Yu - \frac{3}{2}YdYd^\dagger Yu + 3\text{Tr}\left(Yu^\dagger Yu\right)Yu + 3\text{Tr}\left(Yd^\dagger Yd\right)Yu\\ &+ \text{Tr}\left(Ye^\dagger Ye\right)Yu + \text{Tr}\left(yN^\dagger yN\right)Yu - \frac{17}{12}g_{U1Y}^2 Yu - \frac{9}{4}g_{SU2L}^2 Yu - 8g_{SU3c}^2 Yu\end{aligned}$$

$$\begin{aligned}\beta^{(1)}(Yd) = &- \frac{3}{2}YuYu^\dagger Yd + \frac{3}{2}YdYd^\dagger Yd + 3\text{Tr}\left(Yu^\dagger Yu\right)Yd + 3\text{Tr}\left(Yd^\dagger Yd\right)Yd\\ &+ \text{Tr}\left(Ye^\dagger Ye\right)Yd + \text{Tr}\left(yN^\dagger yN\right)Yd - \frac{5}{12}g_{U1Y}^2 Yd - \frac{9}{4}g_{SU2L}^2 Yd - 8g_{SU3c}^2 Yd\end{aligned}$$

$$\beta^{(1)}(Ye) = +\frac{3}{2}YeYe^\dagger Ye - \frac{3}{2}yNyN^\dagger Ye + 3\mathrm{Tr}\left(Yu^\dagger Yu\right)Ye + 3\mathrm{Tr}\left(Yd^\dagger Yd\right)Ye + \mathrm{Tr}\left(Ye^\dagger Ye\right)Ye + \mathrm{Tr}\left(yN^\dagger yN\right)Ye - \frac{15}{4}g_{U1Y}^2 Ye - \frac{9}{4}g_{SU2L}^2 Ye$$

$$\beta^{(1)}(yN) = -\frac{3}{2}YeYe^\dagger yN + \frac{3}{2}yNyN^\dagger yN + 3\mathrm{Tr}\left(Yu^\dagger Yu\right)yN + 3\mathrm{Tr}\left(Yd^\dagger Yd\right)yN + \mathrm{Tr}\left(Ye^\dagger Ye\right)yN + \mathrm{Tr}\left(yN^\dagger yN\right)yN - \frac{3}{4}g_{U1Y}^2 yN - \frac{9}{4}g_{SU2L}^2 yN$$

3.4 Quartic couplings

$$\begin{aligned}\beta^{(1)}(\lambda) = &+ 24\lambda^2 + 4lHphi^2 - 3g_{U1Y}^2\lambda - 9g_{SU2L}^2\lambda + \frac{3}{8}g_{U1Y}^4 + \frac{3}{4}g_{SU2L}^2 g_{U1Y}^2 + \frac{9}{8}g_{SU2L}^4 \\ &+ 12\lambda\mathrm{Tr}\left(Yu^\dagger Yu\right) + 12\lambda\mathrm{Tr}\left(Yd^\dagger Yd\right) + 4\lambda\mathrm{Tr}\left(Ye^\dagger Ye\right) + 4\lambda\mathrm{Tr}\left(yN^\dagger yN\right) \\ &- 6\mathrm{Tr}\left(Yu^\dagger YuYu^\dagger Yu\right) - 6\mathrm{Tr}\left(Yd^\dagger YdYd^\dagger Yd\right) - 2\mathrm{Tr}\left(Ye^\dagger YeYe^\dagger Ye\right) \\ &- 2\mathrm{Tr}\left(yN^\dagger yNyN^\dagger yN\right)\end{aligned}$$

$$\beta^{(1)}(lPhi) = +80lPhi^2 + 2lHphi^2$$

$$\begin{aligned}\beta^{(1)}(lHphi) = &+ 12lHphi\lambda + 32lHphilPhi + 8lHphi^2 - \frac{3}{2}g_{U1Y}^2 lHphi - \frac{9}{2}g_{SU2L}^2 lHphi \\ &+ 6lHphi\mathrm{Tr}\left(Yu^\dagger Yu\right) + 6lHphi\mathrm{Tr}\left(Yd^\dagger Yd\right) + 2lHphi\mathrm{Tr}\left(Ye^\dagger Ye\right) \\ &+ 2lHphi\mathrm{Tr}\left(yN^\dagger yN\right)\end{aligned}$$

3.5 Scalar mass couplings

$$\begin{aligned}\beta^{(1)}(\mu) = &- \frac{3}{2}g_{U1Y}^2\mu - \frac{9}{2}g_{SU2L}^2\mu + 12\lambda\mu + 6\mu\mathrm{Tr}\left(Yu^\dagger Yu\right) + 6\mu\mathrm{Tr}\left(Yd^\dagger Yd\right) + 2\mu\mathrm{Tr}\left(Ye^\dagger Ye\right) \\ &+ 2\mu\mathrm{Tr}\left(yN^\dagger yN\right)\end{aligned}$$

3.6 Vacuum-expectation values

Definitions:

$$H \ : \ \frac{1}{\sqrt{2}}\Pi_2 \to \frac{1}{\sqrt{2}}\left(\Pi_2 + vSM\right)$$
$$phiR \ : \ phiR \to phiR + vPQ$$

Gauge fixing:

$$\xi \to 0$$

RGEs:

$$\begin{aligned}\beta^{(1)}(vSM) = &+\frac{3}{4}g_{U1Y}^2 vSM + \frac{9}{4}g_{SU2L}^2 vSM - 3vSM\mathrm{Tr}\left(Yu^\dagger Yu\right) - 3vSM\mathrm{Tr}\left(Yd^\dagger Yd\right) \\ &- vSM\mathrm{Tr}\left(Ye^\dagger Ye\right) - vSM\mathrm{Tr}\left(yN^\dagger yN\right)\end{aligned}$$

$$\beta^{(1)}(vPQ) = 0$$