E8Cascade2LoopGravity

PyR@TE 3.0

2 July 2025, 19:56

Contents

1 Model

1.1 Gauge groups

Name	Type	Abelian	Coupling constant
U1Y	$U(1)$	True	g_1
SU2L	$SU(2)$	False	g_2
SU3c	$SU(3)$	False	g_3

1.2 Fermions

Name	Generations	U1Y $\times$ SU2L $\times$ SU3c
Q	3	$(+\frac{1}{6}, \mathbf{2}, \mathbf{3})$
L	3	$(-\frac{1}{2}, \mathbf{2}, \mathbf{1})$
uR	3	$(+\frac{2}{3}, \mathbf{1}, \mathbf{3})$
dR	3	$(-\frac{1}{3}, \mathbf{1}, \mathbf{3})$
eR	3	$(-1, \mathbf{1}, \mathbf{1})$
$SigmaF$	1	$(0, \mathbf{3}, \mathbf{1})$
NR	3	$(0, \mathbf{1}, \mathbf{1})$

1.3 Scalars

Name	Complex	Expression	Generations	U1Y $\times$ SU2L $\times$ SU3c
$phiR$	False	/	1	$(0, \mathbf{1}, \mathbf{1})$
$phiI$	False	/	1	$(0, \mathbf{1}, \mathbf{1})$
R_3	False	/	1	$(0, \mathbf{1}, \mathbf{1})$
H	True	$\frac{1}{\sqrt{2}}(\Pi + i\,\Sigma)$	1	$(+\frac{1}{2}, \mathbf{2}, \mathbf{1})$

2 Lagrangian

2.1 Definitions

$$Htilde_i = \epsilon_{i,j} H_j^\dagger$$

2.2 Yukawa couplings

$$-\mathcal{L}_Y = + Yu_{f_1,f_2}\tilde{H}_i\overline{Q}_{f_1,i,a}uR_{f_2,a} + Yd_{f_1,f_2}\overline{Q}_{f_1,i,a}H_i dR_{f_2,a} + Ye_{f_1,f_2}\overline{L}_{f_1,i}H_i eR_{f_2}$$
$$+ yN_{f_1,f_2}\tilde{H}_i\overline{L}_{f_1,i}NR_{f_2} + \text{h.c.}$$

2.3 Quartic couplings

$$-\mathcal{L}_Q = + \lambda H_i^\dagger H_i H_{i_1}^\dagger H_{i_1} + lPhi\big(2phiI^2phiR^2 + phiIphiIphiIphiI + phiRphiRphiRphiR\big)$$
$$+ lHphi\big(phiIphiIH_iH_i^\dagger + phiRphiRH_iH_i^\dagger\big)$$

2.4 Trilinear couplings

$$-\mathcal{L}_T = + cR_3 R_3 H_i^\dagger H_i$$

2.5 Scalar mass couplings

$$-\mathcal{L}_{sm} = -\mu_2 H_i^\dagger H_i + MPhi\big(phiIphiI + phiRphiR\big)$$

3 Renormalization Group Equations

3.1 Convention

$$\beta\left(X\right) \equiv \mu\frac{dX}{d\mu} \equiv \frac{1}{(4\pi)^2}\beta^{(1)}(X) + \frac{1}{(4\pi)^4}\beta^{(2)}(X)$$

3.2 Gauge couplings

$$\beta^{(1)}(g_1) = \frac{41}{6}g_1^3$$

$$\beta^{(2)}(g_1) = + \frac{199}{18}g_1^5 + \frac{9}{2}g_1^3 g_2^2 + \frac{44}{3}g_1^3 g_3^2 - \frac{17}{6}g_1^3\text{Tr}\left(Yu^\dagger Yu\right) - \frac{5}{6}g_1^3\text{Tr}\left(Yd^\dagger Yd\right) - \frac{5}{2}g_1^3\text{Tr}\left(Ye^\dagger Ye\right)$$
$$- \frac{1}{2}g_1^3\text{Tr}\left(yN^\dagger yN\right)$$

$$\beta^{(1)}(g_2) = -\frac{11}{6}g_2^3$$

$$\beta^{(2)}(g_2) = + \frac{3}{2}g_1^2 g_2^3 + \frac{163}{6}g_2^5 + 12g_2^3 g_3^2 - \frac{3}{2}g_2^3\text{Tr}\left(Yu^\dagger Yu\right) - \frac{3}{2}g_2^3\text{Tr}\left(Yd^\dagger Yd\right) - \frac{1}{2}g_2^3\text{Tr}\left(Ye^\dagger Ye\right)$$
$$- \frac{1}{2}g_2^3\text{Tr}\left(yN^\dagger yN\right)$$

$$\beta^{(1)}(g_3) = -7g_3^3$$

$$\beta^{(2)}(g_3) = +\frac{11}{6}g_1^2g_3^3 + \frac{9}{2}g_2^2g_3^3 - 26g_3^5 - 2g_3^3\text{Tr}\left(Yu^\dagger Yu\right) - 2g_3^3\text{Tr}\left(Yd^\dagger Yd\right)$$

3.3 Yukawa couplings

$$\beta^{(1)}(Yu) = +\frac{3}{2}YuYu^\dagger Yu - \frac{3}{2}YdYd^\dagger Yu + 3\text{Tr}\left(Yu^\dagger Yu\right)Yu + 3\text{Tr}\left(Yd^\dagger Yd\right)Yu$$
$$+ \text{Tr}\left(Ye^\dagger Ye\right)Yu + \text{Tr}\left(yN^\dagger yN\right)Yu - \frac{17}{12}g_1^2 Yu - \frac{9}{4}g_2^2 Yu - 8g_3^2 Yu$$

$$\beta^{(2)}(Yu) = +\frac{3}{2}YuYu^\dagger YuYu^\dagger Yu - \frac{1}{4}YuYu^\dagger YdYd^\dagger Yu - YdYd^\dagger YuYu^\dagger Yu$$
$$+ \frac{11}{4}YdYd^\dagger YdYd^\dagger Yu - \frac{27}{4}\text{Tr}\left(Yu^\dagger YuYu^\dagger Yu\right)Yu - \frac{27}{4}\text{Tr}\left(Yu^\dagger Yu\right)YuYu^\dagger Yu$$
$$+ \frac{15}{4}\text{Tr}\left(Yu^\dagger Yu\right)YdYd^\dagger Yu + \frac{3}{2}\text{Tr}\left(Yu^\dagger YdYd^\dagger Yu\right)Yu$$
$$- \frac{27}{4}\text{Tr}\left(Yd^\dagger Yd\right)YuYu^\dagger Yu - \frac{27}{4}\text{Tr}\left(Yd^\dagger YdYd^\dagger Yd\right)Yu$$
$$+ \frac{15}{4}\text{Tr}\left(Yd^\dagger Yd\right)YdYd^\dagger Yu - \frac{9}{4}\text{Tr}\left(Ye^\dagger Ye\right)YuYu^\dagger Yu + \frac{5}{4}\text{Tr}\left(Ye^\dagger Ye\right)YdYd^\dagger Yu$$
$$- \frac{9}{4}\text{Tr}\left(Ye^\dagger YeYe^\dagger Ye\right)Yu + \frac{1}{2}\text{Tr}\left(Ye^\dagger yNyN^\dagger Ye\right)Yu - \frac{9}{4}\text{Tr}\left(yN^\dagger yN\right)YuYu^\dagger Yu$$
$$+ \frac{5}{4}\text{Tr}\left(yN^\dagger yN\right)YdYd^\dagger Yu - \frac{9}{4}\text{Tr}\left(yN^\dagger yNyN^\dagger yN\right)Yu - 12\lambda YuYu^\dagger Yu + 6\lambda^2 Yu$$
$$+ 2lHphi^2 Yu + \frac{223}{48}g_1^2 YuYu^\dagger Yu + \frac{135}{16}g_2^2 YuYu^\dagger Yu + 16g_3^2 YuYu^\dagger Yu$$
$$- \frac{43}{48}g_1^2 YdYd^\dagger Yu + \frac{9}{16}g_2^2 YdYd^\dagger Yu - 16g_3^2 YdYd^\dagger Yu + \frac{85}{24}g_1^2\text{Tr}\left(Yu^\dagger Yu\right)Yu$$
$$+ \frac{45}{8}g_2^2\text{Tr}\left(Yu^\dagger Yu\right)Yu + 20g_3^2\text{Tr}\left(Yu^\dagger Yu\right)Yu + \frac{25}{24}g_1^2\text{Tr}\left(Yd^\dagger Yd\right)Yu$$
$$+ \frac{45}{8}g_2^2\text{Tr}\left(Yd^\dagger Yd\right)Yu + 20g_3^2\text{Tr}\left(Yd^\dagger Yd\right)Yu + \frac{25}{8}g_1^2\text{Tr}\left(Ye^\dagger Ye\right)Yu$$
$$+ \frac{15}{8}g_2^2\text{Tr}\left(Ye^\dagger Ye\right)Yu + \frac{5}{8}g_1^2\text{Tr}\left(yN^\dagger yN\right)Yu + \frac{15}{8}g_2^2\text{Tr}\left(yN^\dagger yN\right)Yu + \frac{1187}{216}g_1^4 Yu$$
$$- \frac{3}{4}g_1^2g_2^2 Yu + \frac{19}{9}g_1^2g_3^2 Yu - \frac{19}{4}g_2^4 Yu + 9g_2^2g_3^2 Yu - 108g_3^4 Yu$$

$$\beta^{(1)}(Yd) = -\frac{3}{2}YuYu^\dagger Yd + \frac{3}{2}YdYd^\dagger Yd + 3\text{Tr}\left(Yu^\dagger Yu\right)Yd + 3\text{Tr}\left(Yd^\dagger Yd\right)Yd$$
$$+ \text{Tr}\left(Ye^\dagger Ye\right)Yd + \text{Tr}\left(yN^\dagger yN\right)Yd - \frac{5}{12}g_1^2 Yd - \frac{9}{4}g_2^2 Yd - 8g_3^2 Yd$$

$$\beta^{(2)}(Yd) = + \frac{11}{4}YuYu^\dagger YuYu^\dagger Yd - YuYu^\dagger YdYd^\dagger Yd - \frac{1}{4}YdYd^\dagger YuYu^\dagger Yd$$
$$+ \frac{3}{2}YdYd^\dagger YdYd^\dagger Yd - \frac{27}{4}\mathrm{Tr}\left(Yu^\dagger YuYu^\dagger Yu\right)Yd + \frac{15}{4}\mathrm{Tr}\left(Yu^\dagger Yu\right)YuYu^\dagger Yd$$
$$- \frac{27}{4}\mathrm{Tr}\left(Yu^\dagger Yu\right)YdYd^\dagger Yd + \frac{3}{2}\mathrm{Tr}\left(Yu^\dagger YdYd^\dagger Yu\right)Yd + \frac{15}{4}\mathrm{Tr}\left(Yd^\dagger Yd\right)YuYu^\dagger Yd$$
$$- \frac{27}{4}\mathrm{Tr}\left(Yd^\dagger YdYd^\dagger Yd\right)Yd - \frac{27}{4}\mathrm{Tr}\left(Yd^\dagger Yd\right)YdYd^\dagger Yd + \frac{5}{4}\mathrm{Tr}\left(Ye^\dagger Ye\right)YuYu^\dagger Yd$$
$$- \frac{9}{4}\mathrm{Tr}\left(Ye^\dagger Ye\right)YdYd^\dagger Yd - \frac{9}{4}\mathrm{Tr}\left(Ye^\dagger YeYe^\dagger Ye\right)Yd + \frac{1}{2}\mathrm{Tr}\left(Ye^\dagger yNyN^\dagger Ye\right)Yd$$
$$+ \frac{5}{4}\mathrm{Tr}\left(yN^\dagger yN\right)YuYu^\dagger Yd - \frac{9}{4}\mathrm{Tr}\left(yN^\dagger yN\right)YdYd^\dagger Yd - \frac{9}{4}\mathrm{Tr}\left(yN^\dagger yNyN^\dagger yN\right)Yd$$
$$- 12\lambda YdYd^\dagger Yd + 6\lambda^2 Yd + 2lHphi^2 Yd - \frac{79}{48}g_1^2 YuYu^\dagger Yd + \frac{9}{16}g_2^2 YuYu^\dagger Yd$$
$$- 16g_3^2 YuYu^\dagger Yd + \frac{187}{48}g_1^2 YdYd^\dagger Yd + \frac{135}{16}g_2^2 YdYd^\dagger Yd + 16g_3^2 YdYd^\dagger Yd$$
$$+ \frac{85}{24}g_1^2 \mathrm{Tr}\left(Yu^\dagger Yu\right)Yd + \frac{45}{8}g_2^2 \mathrm{Tr}\left(Yu^\dagger Yu\right)Yd + 20g_3^2 \mathrm{Tr}\left(Yu^\dagger Yu\right)Yd$$
$$+ \frac{25}{24}g_1^2 \mathrm{Tr}\left(Yd^\dagger Yd\right)Yd + \frac{45}{8}g_2^2 \mathrm{Tr}\left(Yd^\dagger Yd\right)Yd + 20g_3^2 \mathrm{Tr}\left(Yd^\dagger Yd\right)Yd$$
$$+ \frac{25}{8}g_1^2 \mathrm{Tr}\left(Ye^\dagger Ye\right)Yd + \frac{15}{8}g_2^2 \mathrm{Tr}\left(Ye^\dagger Ye\right)Yd + \frac{5}{8}g_1^2 \mathrm{Tr}\left(yN^\dagger yN\right)Yd$$
$$+ \frac{15}{8}g_2^2 \mathrm{Tr}\left(yN^\dagger yN\right)Yd - \frac{127}{216}g_1^4 Yd - \frac{9}{4}g_1^2 g_2^2 Yd + \frac{31}{9}g_1^2 g_3^2 Yd - \frac{19}{4}g_2^4 Yd + 9g_2^2 g_3^2 Yd$$
$$- 108g_3^4 Yd$$

$$\beta^{(1)}(Ye) = + \frac{3}{2}YeYe^\dagger Ye - \frac{3}{2}yNyN^\dagger Ye + 3\mathrm{Tr}\left(Yu^\dagger Yu\right)Ye + 3\mathrm{Tr}\left(Yd^\dagger Yd\right)Ye$$
$$+ \mathrm{Tr}\left(Ye^\dagger Ye\right)Ye + \mathrm{Tr}\left(yN^\dagger yN\right)Ye - \frac{15}{4}g_1^2 Ye - \frac{9}{4}g_2^2 Ye$$

$$\beta^{(2)}(Ye) = + \frac{3}{2}YeYe^\dagger YeYe^\dagger Ye - \frac{1}{4}YeYe^\dagger yNyN^\dagger Ye - yNyN^\dagger YeYe^\dagger Ye$$
$$+ \frac{11}{4}yNyN^\dagger yNyN^\dagger Ye - \frac{27}{4}\mathrm{Tr}\left(Yu^\dagger YuYu^\dagger Yu\right)Ye - \frac{27}{4}\mathrm{Tr}\left(Yu^\dagger Yu\right)YeYe^\dagger Ye$$
$$+ \frac{15}{4}\mathrm{Tr}\left(Yu^\dagger Yu\right)yNyN^\dagger Ye + \frac{3}{2}\mathrm{Tr}\left(Yu^\dagger YdYd^\dagger Yu\right)Ye - \frac{27}{4}\mathrm{Tr}\left(Yd^\dagger YdYd^\dagger Yd\right)Ye$$
$$- \frac{27}{4}\mathrm{Tr}\left(Yd^\dagger Yd\right)YeYe^\dagger Ye + \frac{15}{4}\mathrm{Tr}\left(Yd^\dagger Yd\right)yNyN^\dagger Ye - \frac{9}{4}\mathrm{Tr}\left(Ye^\dagger YeYe^\dagger Ye\right)Ye$$
$$- \frac{9}{4}\mathrm{Tr}\left(Ye^\dagger Ye\right)YeYe^\dagger Ye + \frac{5}{4}\mathrm{Tr}\left(Ye^\dagger Ye\right)yNyN^\dagger Ye + \frac{1}{2}\mathrm{Tr}\left(Ye^\dagger yNyN^\dagger Ye\right)Ye$$
$$- \frac{9}{4}\mathrm{Tr}\left(yN^\dagger yN\right)YeYe^\dagger Ye - \frac{9}{4}\mathrm{Tr}\left(yN^\dagger yNyN^\dagger yN\right)Ye + \frac{5}{4}\mathrm{Tr}\left(yN^\dagger yN\right)yNyN^\dagger Ye$$
$$- 12\lambda YeYe^\dagger Ye + 6\lambda^2 Ye + 2lHphi^2 Ye + \frac{129}{16}g_1^2 YeYe^\dagger Ye + \frac{135}{16}g_2^2 YeYe^\dagger Ye$$

$$-\frac{45}{16}g_1^2 yNyN^\dagger Ye + \frac{9}{16}g_2^2 yNyN^\dagger Ye + \frac{85}{24}g_1^2 \text{Tr}\left(Yu^\dagger Yu\right) Ye + \frac{45}{8}g_2^2 \text{Tr}\left(Yu^\dagger Yu\right) Ye$$

$$+ 20g_3^2 \text{Tr}\left(Yu^\dagger Yu\right) Ye + \frac{25}{24}g_1^2 \text{Tr}\left(Yd^\dagger Yd\right) Ye + \frac{45}{8}g_2^2 \text{Tr}\left(Yd^\dagger Yd\right) Ye$$

$$+ 20g_3^2 \text{Tr}\left(Yd^\dagger Yd\right) Ye + \frac{25}{8}g_1^2 \text{Tr}\left(Ye^\dagger Ye\right) Ye + \frac{15}{8}g_2^2 \text{Tr}\left(Ye^\dagger Ye\right) Ye$$

$$+ \frac{5}{8}g_1^2 \text{Tr}\left(yN^\dagger yN\right) Ye + \frac{15}{8}g_2^2 \text{Tr}\left(yN^\dagger yN\right) Ye + \frac{457}{24}g_1^4 Ye + \frac{9}{4}g_1^2 g_2^2 Ye - \frac{19}{4}g_2^4 Ye$$

$$\beta^{(1)}(yN) = -\frac{3}{2}YeYe^\dagger yN + \frac{3}{2}yNyN^\dagger yN + 3\text{Tr}\left(Yu^\dagger Yu\right) yN + 3\text{Tr}\left(Yd^\dagger Yd\right) yN$$

$$+ \text{Tr}\left(Ye^\dagger Ye\right) yN + \text{Tr}\left(yN^\dagger yN\right) yN - \frac{3}{4}g_1^2 yN - \frac{9}{4}g_2^2 yN$$

$$\beta^{(2)}(yN) = +\frac{11}{4}YeYe^\dagger YeYe^\dagger yN - YeYe^\dagger yNyN^\dagger yN - \frac{1}{4}yNyN^\dagger YeYe^\dagger yN$$

$$+ \frac{3}{2}yNyN^\dagger yNyN^\dagger yN - \frac{27}{4}\text{Tr}\left(Yu^\dagger YuYu^\dagger Yu\right) yN + \frac{15}{4}\text{Tr}\left(Yu^\dagger Yu\right) YeYe^\dagger yN$$

$$- \frac{27}{4}\text{Tr}\left(Yu^\dagger Yu\right) yNyN^\dagger yN + \frac{3}{2}\text{Tr}\left(Yu^\dagger YdYd^\dagger Yu\right) yN$$

$$- \frac{27}{4}\text{Tr}\left(Yd^\dagger YdYd^\dagger Yd\right) yN + \frac{15}{4}\text{Tr}\left(Yd^\dagger Yd\right) YeYe^\dagger yN$$

$$- \frac{27}{4}\text{Tr}\left(Yd^\dagger Yd\right) yNyN^\dagger yN - \frac{9}{4}\text{Tr}\left(Ye^\dagger YeYe^\dagger Ye\right) yN + \frac{5}{4}\text{Tr}\left(Ye^\dagger Ye\right) YeYe^\dagger yN$$

$$- \frac{9}{4}\text{Tr}\left(Ye^\dagger Ye\right) yNyN^\dagger yN + \frac{1}{2}\text{Tr}\left(Ye^\dagger yNyN^\dagger Ye\right) yN + \frac{5}{4}\text{Tr}\left(yN^\dagger yN\right) YeYe^\dagger yN$$

$$- \frac{9}{4}\text{Tr}\left(yN^\dagger yNyN^\dagger yN\right) yN - \frac{9}{4}\text{Tr}\left(yN^\dagger yN\right) yNyN^\dagger yN - 12\lambda yNyN^\dagger yN + 6\lambda^2 yN$$

$$+ 2lHphi^2 yN - \frac{81}{16}g_1^2 YeYe^\dagger yN + \frac{9}{16}g_2^2 YeYe^\dagger yN + \frac{93}{16}g_1^2 yNyN^\dagger yN$$

$$+ \frac{135}{16}g_2^2 yNyN^\dagger yN + \frac{85}{24}g_1^2 \text{Tr}\left(Yu^\dagger Yu\right) yN + \frac{45}{8}g_2^2 \text{Tr}\left(Yu^\dagger Yu\right) yN$$

$$+ 20g_3^2 \text{Tr}\left(Yu^\dagger Yu\right) yN + \frac{25}{24}g_1^2 \text{Tr}\left(Yd^\dagger Yd\right) yN + \frac{45}{8}g_2^2 \text{Tr}\left(Yd^\dagger Yd\right) yN$$

$$+ 20g_3^2 \text{Tr}\left(Yd^\dagger Yd\right) yN + \frac{25}{8}g_1^2 \text{Tr}\left(Ye^\dagger Ye\right) yN + \frac{15}{8}g_2^2 \text{Tr}\left(Ye^\dagger Ye\right) yN$$

$$+ \frac{5}{8}g_1^2 \text{Tr}\left(yN^\dagger yN\right) yN + \frac{15}{8}g_2^2 \text{Tr}\left(yN^\dagger yN\right) yN + \frac{35}{24}g_1^4 yN - \frac{9}{4}g_1^2 g_2^2 yN - \frac{19}{4}g_2^4 yN$$

3.4 Quartic couplings

$$\beta^{(1)}(\lambda) = +24\lambda^2 + 4lHphi^2 - 3g_1^2\lambda - 9g_2^2\lambda + \frac{3}{8}g_1^4 + \frac{3}{4}g_1^2 g_2^2 + \frac{9}{8}g_2^4 + 12\lambda\text{Tr}\left(Yu^\dagger Yu\right)$$

$$+ 12\lambda\text{Tr}\left(Yd^\dagger Yd\right) + 4\lambda\text{Tr}\left(Ye^\dagger Ye\right) + 4\lambda\text{Tr}\left(yN^\dagger yN\right) - 6\text{Tr}\left(Yu^\dagger YuYu^\dagger Yu\right)$$

$$- 6\text{Tr}\left(Yd^\dagger YdYd^\dagger Yd\right) - 2\text{Tr}\left(Ye^\dagger YeYe^\dagger Ye\right) - 2\text{Tr}\left(yN^\dagger yNyN^\dagger yN\right)$$

$$\beta^{(2)}(\lambda) = -312\lambda^3 - 40lHphi^2\lambda - 32lHphi^3 + 36g_1^2\lambda^2 + 108g_2^2\lambda^2 + \frac{629}{24}g_1^4\lambda + \frac{39}{4}g_1^2g_2^2\lambda + \frac{7}{8}g_2^4\lambda$$

$$- \frac{379}{48}g_1^6 - \frac{559}{48}g_1^4g_2^2 - \frac{353}{48}g_1^2g_2^4 + \frac{241}{16}g_2^6 - 144\lambda^2\mathrm{Tr}\left(Yu^\dagger Yu\right) - 144\lambda^2\mathrm{Tr}\left(Yd^\dagger Yd\right)$$

$$- 48\lambda^2\mathrm{Tr}\left(Ye^\dagger Ye\right) - 48\lambda^2\mathrm{Tr}\left(yN^\dagger yN\right) + \frac{85}{6}g_1^2\lambda\mathrm{Tr}\left(Yu^\dagger Yu\right) + \frac{25}{6}g_1^2\lambda\mathrm{Tr}\left(Yd^\dagger Yd\right)$$

$$+ \frac{25}{2}g_1^2\lambda\mathrm{Tr}\left(Ye^\dagger Ye\right) + \frac{5}{2}g_1^2\lambda\mathrm{Tr}\left(yN^\dagger yN\right) + \frac{45}{2}g_2^2\lambda\mathrm{Tr}\left(Yu^\dagger Yu\right) + \frac{45}{2}g_2^2\lambda\mathrm{Tr}\left(Yd^\dagger Yd\right)$$

$$+ \frac{15}{2}g_2^2\lambda\mathrm{Tr}\left(Ye^\dagger Ye\right) + \frac{15}{2}g_2^2\lambda\mathrm{Tr}\left(yN^\dagger yN\right) + 80g_3^2\lambda\mathrm{Tr}\left(Yu^\dagger Yu\right) + 80g_3^2\lambda\mathrm{Tr}\left(Yd^\dagger Yd\right)$$

$$- \frac{19}{4}g_1^4\mathrm{Tr}\left(Yu^\dagger Yu\right) + \frac{5}{4}g_1^4\mathrm{Tr}\left(Yd^\dagger Yd\right) - \frac{25}{4}g_1^4\mathrm{Tr}\left(Ye^\dagger Ye\right) - \frac{1}{4}g_1^4\mathrm{Tr}\left(yN^\dagger yN\right)$$

$$+ \frac{21}{2}g_1^2g_2^2\mathrm{Tr}\left(Yu^\dagger Yu\right) + \frac{9}{2}g_1^2g_2^2\mathrm{Tr}\left(Yd^\dagger Yd\right) + \frac{11}{2}g_1^2g_2^2\mathrm{Tr}\left(Ye^\dagger Ye\right) - \frac{1}{2}g_1^2g_2^2\mathrm{Tr}\left(yN^\dagger yN\right)$$

$$- \frac{9}{4}g_2^4\mathrm{Tr}\left(Yu^\dagger Yu\right) - \frac{9}{4}g_2^4\mathrm{Tr}\left(Yd^\dagger Yd\right) - \frac{3}{4}g_2^4\mathrm{Tr}\left(Ye^\dagger Ye\right) - \frac{3}{4}g_2^4\mathrm{Tr}\left(yN^\dagger yN\right)$$

$$- 3\lambda\mathrm{Tr}\left(Yu^\dagger YuYu^\dagger Yu\right) - 42\lambda\mathrm{Tr}\left(Yu^\dagger YdYd^\dagger Yu\right) - 3\lambda\mathrm{Tr}\left(Yd^\dagger YdYd^\dagger Yd\right)$$

$$- \lambda\mathrm{Tr}\left(Ye^\dagger YeYe^\dagger Ye\right) - 14\lambda\mathrm{Tr}\left(Ye^\dagger yNyN^\dagger Ye\right) - \lambda\mathrm{Tr}\left(yN^\dagger yNyN^\dagger yN\right)$$

$$- \frac{8}{3}g_1^2\mathrm{Tr}\left(Yu^\dagger YuYu^\dagger Yu\right) + \frac{4}{3}g_1^2\mathrm{Tr}\left(Yd^\dagger YdYd^\dagger Yd\right) - 4g_1^2\mathrm{Tr}\left(Ye^\dagger YeYe^\dagger Ye\right)$$

$$- 32g_3^2\mathrm{Tr}\left(Yu^\dagger YuYu^\dagger Yu\right) - 32g_3^2\mathrm{Tr}\left(Yd^\dagger YdYd^\dagger Yd\right) + 30\mathrm{Tr}\left(Yu^\dagger YuYu^\dagger YuYu^\dagger Yu\right)$$

$$- 6\mathrm{Tr}\left(Yu^\dagger YuYu^\dagger YdYd^\dagger Yu\right) - 6\mathrm{Tr}\left(Yu^\dagger YdYd^\dagger YdYd^\dagger Yu\right)$$

$$+ 30\mathrm{Tr}\left(Yd^\dagger YdYd^\dagger YdYd^\dagger Yd\right) + 10\mathrm{Tr}\left(Ye^\dagger YeYe^\dagger YeYe^\dagger Ye\right)$$

$$- 2\mathrm{Tr}\left(Ye^\dagger YeYe^\dagger yNyN^\dagger Ye\right) - 2\mathrm{Tr}\left(Ye^\dagger yNyN^\dagger yNyN^\dagger Ye\right)$$

$$+ 10\mathrm{Tr}\left(yN^\dagger yNyN^\dagger yNyN^\dagger yN\right)$$

$$\beta^{(1)}(lPhi) = +80lPhi^2 + 2lHphi^2$$

$$\beta^{(2)}(lPhi) = -3840lPhi^3 - 80lHphi^2lPhi - 16lHphi^3 + 4g_1^2lHphi^2 + 12g_2^2lHphi^2$$

$$- 12lHphi^2\mathrm{Tr}\left(Yu^\dagger Yu\right) - 12lHphi^2\mathrm{Tr}\left(Yd^\dagger Yd\right) - 4lHphi^2\mathrm{Tr}\left(Ye^\dagger Ye\right)$$

$$- 4lHphi^2\mathrm{Tr}\left(yN^\dagger yN\right)$$

$$\beta^{(1)}(lHphi) = +12lHphi\lambda + 32lHphilPhi + 8lHphi^2 - \frac{3}{2}g_1^2lHphi - \frac{9}{2}g_2^2lHphi$$

$$+ 6lHphi\mathrm{Tr}\left(Yu^\dagger Yu\right) + 6lHphi\mathrm{Tr}\left(Yd^\dagger Yd\right) + 2lHphi\mathrm{Tr}\left(Ye^\dagger Ye\right)$$

$$+ 2lHphi\mathrm{Tr}\left(yN^\dagger yN\right)$$

$$\beta^{(2)}(lHphi) = -144lHphi^2\lambda - 384lHphi^2lPhi - 60lHphi\lambda^2 - 640lHphilPhi^2 - 44lHphi^3$$

$$+ 24g_1^2 lHphi\lambda + 72g_2^2 lHphi\lambda + 2g_1^2 lHphi^2 + 6g_2^2 lHphi^2 + \frac{557}{48}g_1^4 lHphi$$

$$+ \frac{15}{8}g_1^2 g_2^2 lHphi - \frac{65}{16}g_2^4 lHphi - 72 lHphi\lambda \text{Tr}\left(Yu^\dagger Yu\right) - 72 lHphi\lambda \text{Tr}\left(Yd^\dagger Yd\right)$$

$$- 24 lHphi\lambda \text{Tr}\left(Ye^\dagger Ye\right) - 24 lHphi\lambda \text{Tr}\left(yN^\dagger yN\right) - 24 lHphi^2 \text{Tr}\left(Yu^\dagger Yu\right)$$

$$- 24 lHphi^2 \text{Tr}\left(Yd^\dagger Yd\right) - 8 lHphi^2 \text{Tr}\left(Ye^\dagger Ye\right) - 8 lHphi^2 \text{Tr}\left(yN^\dagger yN\right)$$

$$+ \frac{85}{12}g_1^2 lHphi\text{Tr}\left(Yu^\dagger Yu\right) + \frac{25}{12}g_1^2 lHphi\text{Tr}\left(Yd^\dagger Yd\right) + \frac{25}{4}g_1^2 lHphi\text{Tr}\left(Ye^\dagger Ye\right)$$

$$+ \frac{5}{4}g_1^2 lHphi\text{Tr}\left(yN^\dagger yN\right) + \frac{45}{4}g_2^2 lHphi\text{Tr}\left(Yu^\dagger Yu\right) + \frac{45}{4}g_2^2 lHphi\text{Tr}\left(Yd^\dagger Yd\right)$$

$$+ \frac{15}{4}g_2^2 lHphi\text{Tr}\left(Ye^\dagger Ye\right) + \frac{15}{4}g_2^2 lHphi\text{Tr}\left(yN^\dagger yN\right) + 40g_3^2 lHphi\text{Tr}\left(Yu^\dagger Yu\right)$$

$$+ 40g_3^2 lHphi\text{Tr}\left(Yd^\dagger Yd\right) - \frac{27}{2}lHphi\text{Tr}\left(Yu^\dagger YuYu^\dagger Yu\right)$$

$$- 21 lHphi\text{Tr}\left(Yu^\dagger YdYd^\dagger Yu\right) - \frac{27}{2}lHphi\text{Tr}\left(Yd^\dagger YdYd^\dagger Yd\right)$$

$$- \frac{9}{2}lHphi\text{Tr}\left(Ye^\dagger YeYe^\dagger Ye\right) - 7 lHphi\text{Tr}\left(Ye^\dagger yNyN^\dagger Ye\right)$$

$$- \frac{9}{2}lHphi\text{Tr}\left(yN^\dagger yNyN^\dagger yN\right)$$

3.5 Trilinear couplings

$$\beta^{(1)}(cR_3) = -\frac{3}{2}cR_3 g_1^2 - \frac{9}{2}cR_3 g_2^2 + 12 cR_3\lambda + 6 cR_3\text{Tr}\left(Yu^\dagger Yu\right) + 6 cR_3\text{Tr}\left(Yd^\dagger Yd\right)$$
$$+ 2 cR_3\text{Tr}\left(Ye^\dagger Ye\right) + 2 cR_3\text{Tr}\left(yN^\dagger yN\right)$$

$$\beta^{(2)}(cR_3) = + \frac{557}{48}cR_3 g_1^4 + \frac{15}{8}cR_3 g_1^2 g_2^2 - \frac{65}{16}cR_3 g_2^4 + 24 cR_3 g_1^2\lambda + 72 cR_3 g_2^2\lambda - 60 cR_3\lambda^2$$

$$- 4 cR_3 lHphi^2 + \frac{85}{12}cR_3 g_1^2\text{Tr}\left(Yu^\dagger Yu\right) + \frac{25}{12}cR_3 g_1^2\text{Tr}\left(Yd^\dagger Yd\right)$$

$$+ \frac{25}{4}cR_3 g_1^2\text{Tr}\left(Ye^\dagger Ye\right) + \frac{5}{4}cR_3 g_1^2\text{Tr}\left(yN^\dagger yN\right) + \frac{45}{4}cR_3 g_2^2\text{Tr}\left(Yu^\dagger Yu\right)$$

$$+ \frac{45}{4}cR_3 g_2^2\text{Tr}\left(Yd^\dagger Yd\right) + \frac{15}{4}cR_3 g_2^2\text{Tr}\left(Ye^\dagger Ye\right) + \frac{15}{4}cR_3 g_2^2\text{Tr}\left(yN^\dagger yN\right)$$

$$+ 40 cR_3 g_3^2\text{Tr}\left(Yu^\dagger Yu\right) + 40 cR_3 g_3^2\text{Tr}\left(Yd^\dagger Yd\right) - 72 cR_3\lambda\text{Tr}\left(Yu^\dagger Yu\right)$$

$$- 72 cR_3\lambda\text{Tr}\left(Yd^\dagger Yd\right) - 24 cR_3\lambda\text{Tr}\left(Ye^\dagger Ye\right) - 24 cR_3\lambda\text{Tr}\left(yN^\dagger yN\right)$$

$$- \frac{27}{2}cR_3\text{Tr}\left(Yu^\dagger YuYu^\dagger Yu\right) - 21 cR_3\text{Tr}\left(Yu^\dagger YdYd^\dagger Yu\right)$$

$$- \frac{27}{2}cR_3\text{Tr}\left(Yd^\dagger YdYd^\dagger Yd\right) - \frac{9}{2}cR_3\text{Tr}\left(Ye^\dagger YeYe^\dagger Ye\right) - 7 cR_3\text{Tr}\left(Ye^\dagger yNyN^\dagger Ye\right)$$

$$- \frac{9}{2}cR_3\text{Tr}\left(yN^\dagger yNyN^\dagger yN\right)$$

3.6 Scalar mass couplings

$$\beta^{(1)}(\mu_2) = -\frac{3}{2}g_1^2\mu_2 - \frac{9}{2}g_2^2\mu_2 - 2cR_3^2 + 12\lambda\mu_2 - 8MPhilHphi + 6\mu_2\text{Tr}\left(Yu^\dagger Yu\right)$$
$$+ 6\mu_2\text{Tr}\left(Yd^\dagger Yd\right) + 2\mu_2\text{Tr}\left(Ye^\dagger Ye\right) + 2\mu_2\text{Tr}\left(yN^\dagger yN\right)$$

$$\beta^{(2)}(\mu_2) = +\frac{557}{48}g_1^4\mu_2 + \frac{15}{8}g_1^2g_2^2\mu_2 - \frac{65}{16}g_2^4\mu_2 - \frac{1}{2}cR_3^2g_1^2 - \frac{3}{2}cR_3^2g_2^2 + 24g_1^2\lambda\mu_2 + 72g_2^2\lambda\mu_2$$
$$+ 36cR_3^2\lambda - 60\lambda^2\mu_2 - 4lHphi^2\mu_2 + 32MPhilHphi^2 + \frac{85}{12}g_1^2\mu_2\text{Tr}\left(Yu^\dagger Yu\right)$$
$$+ \frac{25}{12}g_1^2\mu_2\text{Tr}\left(Yd^\dagger Yd\right) + \frac{25}{4}g_1^2\mu_2\text{Tr}\left(Ye^\dagger Ye\right) + \frac{5}{4}g_1^2\mu_2\text{Tr}\left(yN^\dagger yN\right)$$
$$+ \frac{45}{4}g_2^2\mu_2\text{Tr}\left(Yu^\dagger Yu\right) + \frac{45}{4}g_2^2\mu_2\text{Tr}\left(Yd^\dagger Yd\right) + \frac{15}{4}g_2^2\mu_2\text{Tr}\left(Ye^\dagger Ye\right)$$
$$+ \frac{15}{4}g_2^2\mu_2\text{Tr}\left(yN^\dagger yN\right) + 40g_3^2\mu_2\text{Tr}\left(Yu^\dagger Yu\right) + 40g_3^2\mu_2\text{Tr}\left(Yd^\dagger Yd\right)$$
$$+ 6cR_3^2\text{Tr}\left(Yu^\dagger Yu\right) + 6cR_3^2\text{Tr}\left(Yd^\dagger Yd\right) + 2cR_3^2\text{Tr}\left(Ye^\dagger Ye\right) + 2cR_3^2\text{Tr}\left(yN^\dagger yN\right)$$
$$- 72\lambda\mu_2\text{Tr}\left(Yu^\dagger Yu\right) - 72\lambda\mu_2\text{Tr}\left(Yd^\dagger Yd\right) - 24\lambda\mu_2\text{Tr}\left(Ye^\dagger Ye\right) - 24\lambda\mu_2\text{Tr}\left(yN^\dagger yN\right)$$
$$- \frac{27}{2}\mu_2\text{Tr}\left(Yu^\dagger YuYu^\dagger Yu\right) - 21\mu_2\text{Tr}\left(Yu^\dagger YdYd^\dagger Yu\right) - \frac{27}{2}\mu_2\text{Tr}\left(Yd^\dagger YdYd^\dagger Yd\right)$$
$$- \frac{9}{2}\mu_2\text{Tr}\left(Ye^\dagger YeYe^\dagger Ye\right) - 7\mu_2\text{Tr}\left(Ye^\dagger yNyN^\dagger Ye\right) - \frac{9}{2}\mu_2\text{Tr}\left(yN^\dagger yNyN^\dagger yN\right)$$

$$\beta^{(1)}(MPhi) = -4lHphi\mu_2 + 32MPhilPhi$$

$$\beta^{(2)}(MPhi) = -8g_1^2lHphi\mu_2 - 24g_2^2lHphi\mu_2 - 4cR_3^2lHphi + 16lHphi^2\mu_2 - 640MPhilPhi^2$$
$$- 8MPhilHphi^2 + 24lHphi\mu_2\text{Tr}\left(Yu^\dagger Yu\right) + 24lHphi\mu_2\text{Tr}\left(Yd^\dagger Yd\right)$$
$$+ 8lHphi\mu_2\text{Tr}\left(Ye^\dagger Ye\right) + 8lHphi\mu_2\text{Tr}\left(yN^\dagger yN\right)$$

3.7 Vacuum-expectation values

Definitions:

$$H \;:\; \frac{1}{\sqrt{2}}\Pi_2 \rightarrow \frac{1}{\sqrt{2}}\left(\Pi_2 + vSM\right)$$
$$phiR \;:\; phiR \rightarrow phiR + vPQ$$

RGEs:

$$\beta^{(1)}(vSM) = +\frac{3}{4}g_1^2vSM + \frac{1}{4}\xi g_1^2vSM + \frac{9}{4}g_2^2vSM + \frac{3}{4}\xi g_2^2vSM - 3vSM\text{Tr}\left(Yu^\dagger Yu\right)$$
$$- 3vSM\text{Tr}\left(Yd^\dagger Yd\right) - vSM\text{Tr}\left(Ye^\dagger Ye\right) - vSM\text{Tr}\left(yN^\dagger yN\right)$$

$$\beta^{(2)}(vSM) = -\frac{431}{96}g_1^4 vSM + \frac{1}{8}\xi g_1^4 vSM + \frac{1}{8}\xi^2 g_1^4 vSM - \frac{9}{16}g_1^2 g_2^2 vSM + \frac{3}{4}\xi g_1^2 g_2^2 vSM$$

$$+ \frac{3}{4}\xi^2 g_1^2 g_2^2 vSM + \frac{191}{32}g_2^4 vSM + \frac{27}{8}\xi g_2^4 vSM - \frac{85}{24}g_1^2 vSM\mathrm{Tr}\left(Yu^\dagger Yu\right)$$

$$- \frac{3}{2}\xi g_1^2 vSM\mathrm{Tr}\left(Yu^\dagger Yu\right) - \frac{25}{24}g_1^2 vSM\mathrm{Tr}\left(Yd^\dagger Yd\right) - \frac{3}{2}\xi g_1^2 vSM\mathrm{Tr}\left(Yd^\dagger Yd\right)$$

$$- \frac{25}{8}g_1^2 vSM\mathrm{Tr}\left(Ye^\dagger Ye\right) - \frac{1}{2}\xi g_1^2 vSM\mathrm{Tr}\left(Ye^\dagger Ye\right) - \frac{5}{8}g_1^2 vSM\mathrm{Tr}\left(yN^\dagger yN\right)$$

$$- \frac{1}{2}\xi g_1^2 vSM\mathrm{Tr}\left(yN^\dagger yN\right) - \frac{45}{8}g_2^2 vSM\mathrm{Tr}\left(Yu^\dagger Yu\right) - \frac{9}{2}\xi g_2^2 vSM\mathrm{Tr}\left(Yu^\dagger Yu\right)$$

$$- \frac{45}{8}g_2^2 vSM\mathrm{Tr}\left(Yd^\dagger Yd\right) - \frac{9}{2}\xi g_2^2 vSM\mathrm{Tr}\left(Yd^\dagger Yd\right) - \frac{15}{8}g_2^2 vSM\mathrm{Tr}\left(Ye^\dagger Ye\right)$$

$$- \frac{3}{2}\xi g_2^2 vSM\mathrm{Tr}\left(Ye^\dagger Ye\right) - \frac{15}{8}g_2^2 vSM\mathrm{Tr}\left(yN^\dagger yN\right) - \frac{3}{2}\xi g_2^2 vSM\mathrm{Tr}\left(yN^\dagger yN\right)$$

$$- 20g_3^2 vSM\mathrm{Tr}\left(Yu^\dagger Yu\right) - 20g_3^2 vSM\mathrm{Tr}\left(Yd^\dagger Yd\right) + \frac{27}{4}vSM\mathrm{Tr}\left(Yu^\dagger Yu Yu^\dagger Yu\right)$$

$$- \frac{3}{2}vSM\mathrm{Tr}\left(Yu^\dagger Yd Yd^\dagger Yu\right) + \frac{27}{4}vSM\mathrm{Tr}\left(Yd^\dagger Yd Yd^\dagger Yd\right)$$

$$+ \frac{9}{4}vSM\mathrm{Tr}\left(Ye^\dagger Ye Ye^\dagger Ye\right) - \frac{1}{2}vSM\mathrm{Tr}\left(Ye^\dagger yN yN^\dagger Ye\right)$$

$$+ \frac{9}{4}vSM\mathrm{Tr}\left(yN^\dagger yN yN^\dagger yN\right) - 6\lambda^2 vSM - 2lHphi^2 vSM$$

$$\beta^{(1)}(vPQ) = 0$$

$$\beta^{(2)}(vPQ) = -64lPhi^2 vPQ - 4lHphi^2 vPQ$$