On the uniform, achromatic rotation of CMB polarization

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Abstract

Polarization angles are sharp probes of spacetime geometry. On a metric-compatible Riemann–Cartan background, a single variational structure yields a uniform, frequency-independent rotation β of CMB linear polarization without post hoc fitting. A Fujikawa computation fixes the abelian vertex $g_{a\gamma\gamma} = -1/(2\pi)$; geometric optics gives $d\psi/d\eta = \frac{1}{2}\, g_{a\gamma\gamma}\, da/d\eta$; a conformal trace stationarity at fixed connection, normalized in the torsionless GR limit, fixes $|\Delta a| = \varphi_0 = 0.0533 \pm 0.0007$. Fit-free with respect to CMB data: after anomaly normalization and the GR renormalization condition, no parameters are tuned to observations; the resulting prediction is $\beta_{\rm th} = \varphi_0/(4\pi) = 0.243^\circ \pm 0.003^\circ$ (degrees). Planck PR4 and ACT DR6 are consistent with this level and show no ν^{-2} slope. Angles are quoted in our convention $\beta \equiv -\beta_{\rm exp}$. Index-level steps, robustness to non-axial torsion and minimal deformations, and calibration-aware likelihood details are provided in the Supplementary material (online).

Keywords: cosmic birefringence, CMB polarization, trace anomaly, axial anomaly, Riemann–Cartan torsion, geometric optics

1. Setup and conventions

Heaviside–Lorentz units with $c = \hbar = 1$, signature $(+,-,-,-)$, and Levi–Civita symbol $\varepsilon_{0123} = +1$ are used; the dual is $\tilde{F}^{\mu\nu} = \frac{1}{2}\,\varepsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}$. Hats denote torsionless curvature, e.g. $\hat{R}_{\mu\nu\rho\sigma}$; ∇ is the full Riemann–Cartan covariant derivative. Angles in equations are in radians. The polarization-angle convention matches TB/EB analyses via $\beta \equiv -\beta_{\rm exp}$. The field a is a *dimensionless* axial phase (not the cosmological scale factor). To avoid dimensional ambiguities in the trace equation, the gravitational coupling is understood in the standard renormalized form $\bar{\kappa} := \mu\,\kappa_{\rm phys}$; the bar is dropped below. Observables derived here are RG-stationary at one loop once the GR renormalization condition is imposed.

Result (no fit).. In the setting above, the chiral Jacobian in Fujikawa regularization fixes $g_{a\gamma\gamma} = -1/(2\pi)$ with the adopted dual and interaction conventions. The modified Maxwell equations imply $d\psi/d\eta = \frac{1}{2}\, g_{a\gamma\gamma}\, da/d\eta$, so $\beta = \frac{1}{2}\, g_{a\gamma\gamma}\,\Delta a$ along the light path. The trace channel of the single action at fixed connection yields a monotone condition $I(\kappa, L) = \kappa^3 - 2c_3^2\kappa^2 - 8b_1c_3^6 L = 0$ with $L := \ln(1/\varphi_0)$, $\partial I/\partial L < 0$. Imposing in the torsionless limit the renormalization condition $I(\kappa_{\rm P}, L^*) = 0$ at the GR point fixes L^* uniquely and therefore $\varphi_0 = e^{-L^*}$; no CMB data enter this fixation. With the physical orientation $\Delta a = -\varphi_0$, the prediction is $\beta_{\rm th} = \varphi_0/(4\pi) = 0.243^\circ \pm 0.003^\circ$ (degrees).

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2. Single action, field equations, and trace stationarity

A metric-compatible Riemann–Cartan geometry with torsion $T^{\rho}{}_{\mu\nu}$ and contorsion $K^{\rho}{}_{\mu\nu}$ is considered; axial torsion is $S_\mu = \frac{1}{6}\varepsilon_{\mu\nu\rho\sigma}T^{\nu\rho\sigma}$. The bulk action reads

$$S = \int d^4x\,\sqrt{-g}\Big[\frac{1}{2\kappa^2}(\hat{R} - \nabla_\lambda K^{\lambda}{}_{\mu}{}^{\mu} + K_{\mu\nu\lambda}K^{\mu\nu\lambda}) - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} - \tfrac{1}{4}g_{a\gamma\gamma}\, a\, F_{\mu\nu}\tilde{F}^{\mu\nu} + \tfrac{\lambda}{2}\,\partial_\mu a\, S^\mu + \tfrac{1}{3}\,U(\kappa, L)\Big] + S_{\partial\mathcal{M}}, \quad (1)$$

with $L = \ln(1/\varphi_0)$. Variation in S^μ gives

$$\partial_\mu a = \lambda\, S_\mu, \quad (2)$$

and for purely axial torsion

$$K_{\mu\nu\rho} = \tfrac{1}{2}\,\varepsilon_{\mu\nu\rho\sigma}\, S^\sigma = \frac{1}{2\lambda}\,\varepsilon_{\mu\nu\rho\sigma}\,\partial^\sigma a, \quad (3)$$

which makes the leading eikonal transport achromatic. Metric variation at fixed connection yields

$$Q_{\mu\nu} - \tfrac{1}{2}g_{\mu\nu}\, Q = \kappa^2\, T^{(a)}_{\mu\nu} + \kappa^2\, T^{(\rm EM)}_{\mu\nu} + \tfrac{1}{3}\, g_{\mu\nu}\, U, \quad (4)$$

with

$$Q_{\mu\nu} = \hat{R}_{\mu\nu} - \nabla_\lambda K^{\lambda}{}_{\mu\nu} + K_{\mu\alpha\beta}K_{\nu}{}^{\alpha\beta}, \qquad Q = g^{\mu\nu}Q_{\mu\nu}. \quad (5)$$

A conformal variation $g_{\mu\nu} \to e^{2\sigma}g_{\mu\nu}$ at fixed connection isolates the trace channel. On the geometric-optics vacuum, matter enters only through the renormalization of U; normalizing

$$\frac{\partial U}{\partial L} = \frac{8}{3}\, b_1\, c_3^6. \quad (6)$$

where $c_3 = 1/(8\pi)$ by the abelian anomaly (Sec. 3).

Stationarity $\delta S/\delta\sigma = 0$ gives

$$I(\kappa, L) := \kappa^3 - 2\,c_3^2\kappa^2 - 8\,b_1\,c_3^6\,L = 0. \tag{7}$$

Since $\partial I/\partial L = -8\,b_1\,c_3^6 < 0$, for fixed κ there is a unique root L^*, hence $\varphi_0 = e^{-L^*} \in (0, 1)$.

$$\mathcal{V}_{\mu\nu} := Q^{\mathrm{TF}}_{\mu\nu} + g_{\mu\nu}\,I(\kappa, L). \tag{8}$$

The full variation is equivalent to $\mathcal{V}_{\mu\nu} = 0$.

Fixed-connection conformal stationarity.. "Metric variation at fixed connection" means that in the conformal variation $g_{\mu\nu} \to e^{2\sigma}g_{\mu\nu}$ the affine structure Γ is held fixed (Palatini logic with metric-compatibility imposed), so the trace channel is isolated without altering the transport sector. This is a renormalization prescription, not a gauge choice; the Einstein–Hilbert boundary term ensures a well-posed variational principle for Dirichlet (g, Γ) data.

3. Anomaly normalization and achromatic transport

A local chiral rotation $\psi \to e^{\frac{i}{2}a(x)\gamma_5}\psi$ produces the Fujikawa Jacobian

$$\Delta\mathcal{L}_{\mathrm{anom}} = +\frac{a(x)}{8\pi}\,\partial_\mu K^\mu, \qquad K^\mu = \varepsilon^{\mu\nu\rho\sigma}A_\nu\partial_\rho A_\sigma, \tag{9}$$

and since $F_{\mu\nu}\tilde{F}^{\mu\nu} = 2\,\partial_\mu K^\mu$, matching to $-\frac{1}{4}g_{a\gamma\gamma}aF_{\mu\nu}\tilde{F}^{\mu\nu}$ fixes

$$g_{a\gamma\gamma} = -\frac{1}{2\pi}. \tag{10}$$

Equivalently, matching $-\frac{1}{4}\,g_{a\gamma\gamma}\,aF\tilde{F} = +c_3\,aF\tilde{F}$ gives $g_{a\gamma\gamma} = -4c_3$ and therefore $c_3 = 1/(8\pi)$ in our conventions. The modified Maxwell equations read

$$\nabla_\mu F^{\mu\nu} + \tfrac{1}{2}\,g_{a\gamma\gamma}\,\partial_\mu a\,\tilde{F}^{\mu\nu} = 0, \tag{11}$$

and in geometric optics

$$\frac{d\psi}{d\eta} = \frac{1}{2}\,g_{a\gamma\gamma}\,\frac{da}{d\eta} \quad\Rightarrow\quad \beta = \frac{1}{2}\,g_{a\gamma\gamma}\,\Delta a. \tag{12}$$

Sign check.. With $\varepsilon_{0123} = +1$ and the above dual, Eq. (9) fixes the overall sign in (10). Since $d\psi/d\eta = \frac{1}{2}g_{a\gamma\gamma}da/d\eta$ and $\Delta a = -\varphi_0 < 0$, one has $\beta_{\mathrm{th}} = \varphi_0/(4\pi) > 0$, identical in sign to the angles reported by experiments after the $\beta \equiv -\beta_{\mathrm{exp}}$ conversion.

4. Trace anomaly and fixation of b_1

On metric-compatible Riemann–Cartan backgrounds with minimal couplings and purely axial torsion, the Maxwell operator is unchanged at one loop. The dilatation anomaly takes the form

$$\langle T^\mu{}_\mu\rangle = \frac{b_1}{16\pi^2}\,F_{\mu\nu}F^{\mu\nu} + \cdots, \tag{13}$$

and b_1 equals the abelian trace coefficient of the Standard Model, $b_1 = 41/10$ in GUT normalization. Axial torsion redistributes purely geometric terms in the DeWitt–Seeley coefficient a_2, while the $F_{\mu\nu}F^{\mu\nu}$ coefficient retains its torsionless value at one loop under minimal couplings; non-axial torsion and non-minimal operators do not mix into F^2 at this order [6, 7].

EWSB mapping (explicit).. With $g_1^2 = \frac{5}{3}g'^2$ and $A_\mu = \sin\theta_W B_\mu + \cos\theta_W W^3_\mu$, kinetic diagonalization removes $F_{\mu\nu}Z^{\mu\nu}$ at one loop under minimal couplings; the abelian trace piece projects onto the canonically normalized photon operator $F_{\mu\nu}F^{\mu\nu}$ with unit weight. Hence the coefficient in front of F^2 remains $b_1 = 41/10$ after EWSB at this order.

5. Renormalization condition at the GR point and prediction

The torsionless limit must reproduce the Einstein–Hilbert sector $(2\kappa_{\mathrm{P}}^2)^{-1}R$ at the chosen renormalization scale; this is a renormalization condition, not a fit. Imposing

$$I(\kappa_{\mathrm{P}}, L^*) = 0 \tag{14}$$

fixes L^* uniquely because $\partial I/\partial L = -8b_1c_3^6 < 0$, hence $\varphi_0 = e^{-L^*}$ is determined without reference to CMB data. Using $c_3 = 1/(8\pi)$ from (10) and the Standard Model value $b_1 = 41/10$, one finds numerically

$$\varphi_0 = 0.0533 \pm 0.0007, \qquad \beta_{\mathrm{th}} = \frac{\varphi_0}{4\pi} = 0.243^\circ \pm 0.003^\circ.$$

The quoted uncertainty is the direct propagation of $\delta\varphi_0$ through $\beta_{\mathrm{th}} = \varphi_0/(4\pi)$ (angles converted to degrees); scheme shifts in U cancel against the GR renormalization condition, and higher-order geometric corrections are $\ll 10^{-2}$ deg across CMB bands and are not included.

RG stationarity.. Differentiating $I(\kappa_{\mathrm{P}}, L^*) = 0$ with respect to $\ln\mu$ shows that the change $U \to U + \frac{8}{3}b_1c_3^6\delta\ell$ under $\mu \to \mu e^{\delta\ell}$ shifts I by a constant that is removed by (14), so $dL^*/d\ln\mu = 0$ and $d\beta_{\mathrm{th}}/d\ln\mu = 0$ at one loop.

6. Consistency with current measurements

All angles below are quoted in our convention $\beta \equiv -\beta_{\mathrm{exp}}$. Planck PR4 reports $\beta = 0.300^\circ \pm 0.110^\circ$ (68% C.L.) and a dedicated ACT DR6 analysis with absolute-angle priors finds $\beta = 0.215^\circ \pm 0.074^\circ$ (68% C.L.), both compatible with our prediction $\beta_{\mathrm{th}} = \varphi_0/(4\pi)$. Because Planck and ACT rely on independent absolute-angle calibrations with different priors and systematics, a common positive β in our sign convention cannot be produced by miscalibration alone; the joint ν^{-2} slope fit consistent with zero further disfavors Faraday-like leakage. A joint multi-band fit of a Faraday-like slope $\beta(\nu) = \beta_0 + m\,(\nu/\mathrm{GHz})^{-2}$ yields m consistent with zero, as expected for achromatic transport.

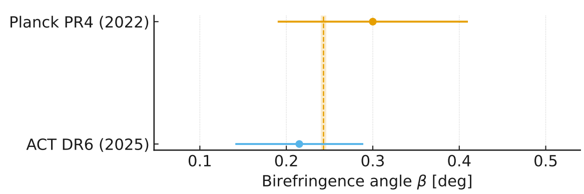

Figure 1: Uniform birefringence angle. Horizontal band: $\beta_{\mathrm{th}} \pm \sigma_{\mathrm{th}}$. Points: Planck PR4 and ACT DR6 (68% C.L.), in our convention $\beta = -\beta_{\mathrm{exp}}$.

Table 1: Prediction vs. measurements (degrees; 68% C.L.).

Entry	Value	Note
Prediction $\beta_{\rm th}$	0.243 ± 0.003	from $\varphi_0/(4\pi)$
Planck PR4	0.300 ± 0.110	nearly full-sky
ACT DR6	0.215 ± 0.074	absolute-angle priors

7. Robustness

Non-axial torsion modifies $K^{\rho}{}_{\mu\nu}$ but leaves $g_{a\gamma\gamma}$ and the F^2 anomaly coefficient unchanged in the minimal theory [6, 7]. The uniform rotation depends on $\partial_\mu a$ via (2) and is achromatic at leading eikonal order [1, 8]. Small non-minimal operators organize as higher-derivative corrections to the constitutive tensor and would induce suppressed frequency dependence; the joint multi-band slope consistent with zero constrains such effects at the $\lesssim 10^{-2\,\circ}$ level across CMB bands. Being a total derivative, the Nieh–Yan density renormalizes L-independent geometric counterterms and does not renormalize the $aF\tilde{F}$ vertex or the F^2 coefficient in the minimal, metric-compatible setup.

8. Falsifiability

A single-experiment 5σ rejection of $\beta_{\rm th}$ against $\beta = 0$ requires a total polarization-angle uncertainty $\sigma_{\rm tot} \lesssim 0.049°$; for 3σ, $\sigma_{\rm tot} \lesssim 0.081°$. These thresholds set the target for absolute polarization-angle calibration; once reached, the prediction $\beta_{\rm th} = \varphi_0/(4\pi)$ is immediately testable.

9. Summary

A single variational structure on metric-compatible Riemann–Cartan backgrounds predicts a uniform, achromatic rotation of CMB polarization with no tunable parameters. The anomaly fixes $g_{a\gamma\gamma}$, geometric optics fixes the transport law, and the trace equation with the GR renormalization condition fixes $|\Delta a| = \varphi_0$, yielding $\beta_{\rm th} = \varphi_0/(4\pi) = 0.243° \pm 0.003°$. Present data are consistent with a constant rotation at this level and show no ν^{-2} slope. Algebraic steps, RG details, and calibration-aware likelihoods are provided in the Supplementary material (online).

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