# Bivariate Spatial Regression Model for Mixed-Response Lattice Data

MS Creative Component with Dr. Emily Berg

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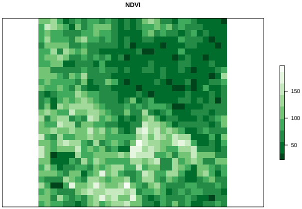
# Overview

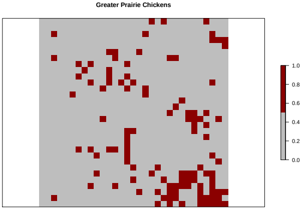
# Data Description

- 2+ response variables, possibly correlated, distributions of mixed types
- Spatial data collected on a grid (discrete-indexed)
  - Model dependence in terms of adjacency rather than distance
- Study interested in both covariates and dependence relationships
  - Need interpretable dependence parameters

Examples:

- ecology (presence/absence + environmental variables)
- agriculture (yield + categorical plant characteristics)
- weather/meteorology
- image pixel analysis



# Notation

- $\{s_i : i = 1, ..., n\}$ : locations
- $Y(s_i)$ : Binary variable (0-1) at location $s_i$
- $Z(s_i)$ : Gaussian variable at location $s_i$
- $\boldsymbol{z} : \{z(s_i) : i = 1, ..., n\}$ and $\boldsymbol{y} : \{y(s_i) : i = 1, ..., n\}$: all obs. of the same type
- $\boldsymbol{z}(\bar{s}_i) : \{z(s_j) : i \neq j\}$ and $\boldsymbol{y}(\bar{s}_i) : \{y(s_j) : i \neq j\}$: all obs. of the same type except for location $s_i$
- $N_i$ : Neighborhood of location $s_i$
  - $j \in N_i$ if $s_i$ and $s_j$ are neighboring locations
- $M_i$ : Number of neighbors ($|N_i|$) of location $s_i$
- $\delta_i$ , $\mu_i$ : Global/marginal mean parameters (node potentials)
- $\eta_y$ , $\eta_z$ : Spatial dependence parameters for variables of the same type
- $\rho$ : Dependence parameter for variables of different types

# Background - Markov Random Field Models

Markov Random fields (auto-models) for lattice data from Besag (1974):

- One response variable from a one-parameter exponential family
- Specify expectation at each location conditionally on neighboring values: $\mathrm{E}[\mathrm{x}(\mathrm{s}_i)|x(\bar{s}_i)] = \mu_i + \sum_{j \neq i} \eta_{ij} x(s_j)$

- Sain, Furrer, and Cressie (2011): Extension to multivariate auto-normal model by "stacking" the univariate lattices
  - Direct extension of Besag's univariate conditional model rather than specifying MVN covariance of response variables
- Caragea and Berg (2014): Model two tree species' presence in bivariate autologistic model
  - Both response variables are binary

# Background - Mixed Graphical Models

Graphical models for learning the edge structure given *n* realizations of a set of mixed nodes (variables):

- Chen, et al. (2014): Edge potential parameter spaces for mixed one-parameter exponential family nodes corresponding to valid joint
- Lee and Hastie (2015): Specific case where any continuous variable is conditionally Gaussian, any discrete variable is conditionally logistic regression
- Gaussian nodes:

$$p(z(s_i)|\boldsymbol{z}(\bar{s}_i),\boldsymbol{y})=\frac{1}{\sqrt{2\pi\sigma^2}}\exp\left\{\frac{-\sigma^{-2}}{2}\left[\frac{\mu_i+\sum_{i=1}^{n}\rho_{ij}y(s_j)+\sum_{i=1}^{n}\eta_{z,ij}z(s_j)}{\sigma^{-2}}-z(s_i)\right]^2\right\}$$

- Discrete nodes:

$$p(y(s_i)|\boldsymbol{y}(\bar{s}_i),\boldsymbol{z})=\frac{\exp\left\{y(s_i)\left(\delta_i+\sum_{i=1}^{n}\rho_{ij}z(s_j)+\sum_{i=1}^{n}\eta_{y,ij}y(s_j)\right)\right\}}{1+\exp\{y(s_i)\left(\delta_i+\sum_{i=1}^{n}\rho_{ij}z(s_j)+\sum_{i=1}^{n}\eta_{y,ij}y(s_j)\right)\}}$$

# Bivariate Mixed MRF Model

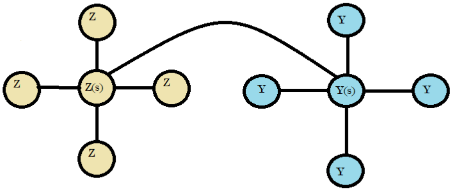
## Mixed Bivariate Markov Random Fields

Goal: Combine the interpretive properties of auto-models with compatible node-conditional specifications from mixed graphical models

- View each response variable at each location as a node, combine with (fixed/known) set of edges into an undirected graph with $2n$ total nodes
- Use this graph structure in the context of auto-models

# Assumptions

- Simplifying assumptions for the models in this work:
  - Four-nearest neighbor structure (pairwise-only dependence): $M = 4$
  - Binary and Gaussian nodes are only connected at each location
  - Isotropy: $\eta_{y,ij} = \eta_y$ and $\eta_{z,ij} = \eta_z$
  - Interior locations only: each node has same number of neighbors (no edge effects)
  - Common conditional variance for Gaussian nodes (variance $= \sigma^2 \forall z(s_i)$)



# Conditional Distributions

- Gaussian node at location $s_i$ has conditional distribution

$$p(z(s_i)|\boldsymbol{z}(\bar{s}_i), y(s_i)) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{ \frac{-1}{2\sigma^2} \left[z(s_i) - \alpha(s_i)\right]^2 \right\}$$

where

$$\alpha(s_i) = \mu_i + \frac{\eta_z}{M} \sum_{j \in N_i} (z(s_j) - \mu_j) + \rho \left( y(s_i) - \frac{\exp(\delta_i)}{1 + \exp(\delta_i)} \right)$$

# Conditional Distributions

- Binary node at location $s_i$ has conditional distribution

$$p(y(s_i)|\boldsymbol{y}(\bar{s}_i), z(s_i)) = \frac{\exp\{y(s_i)\,(\phi(s_i))\}}{1 + \exp\{y(s_i)\,(\phi(s_i))\}}$$

  where

$$\phi(s_i) = \delta_i + \frac{\eta_y}{M}\sum_{j \in N_i}\left[y(s_j) - \frac{\exp(\delta_j)}{1 + \exp(\delta_j)}\right] + \frac{\rho}{\sigma^2}(z(s_i) - \mu_i)$$

- "Centering" in the auto-regressive terms allow separation of global and local effects for a more sensible interpretation of $\eta_y$, $\eta_z$, and $\rho$

# Properties of the Conditional Specification

- $\mu_i$ and $\frac{\exp(\delta_j)}{1+\exp(\delta_i)}$ are independence expectations, i.e., marginal means
- Incorporate covariates $\boldsymbol{X_y}, \boldsymbol{X_z}$ by setting $\mu_i = \boldsymbol{X_{z,i}\beta_z}$ and $\delta_i = \boldsymbol{X_{y,i}\beta_y}$
- The remaining terms in $\alpha(s_i)$ and $\phi(s_i)$ govern the clustering or "local" behavior due to dependence/auto-correlation
- Full set of node-conditionals combine into a negpotential function

$$\boldsymbol{Q(y,z)} = \sum_{i=1}^{n}\left[\frac{\mu_i}{\sigma^2}z(s_i) - \frac{1}{2\sigma^2}z(s_i)^2 + \delta_i y(s_i)\right] + \frac{\rho}{\sigma^2}\sum_{i=1}^{n}\left[(z(s_i)-\mu_i)\left(y(s_i) - \frac{\exp(\delta_i)}{1+\exp(\delta_i)}\right)\right]$$
$$- \frac{1}{2\sigma^2}\sum_{i=1}^{n}\sum_{j\in N_i}\left[\frac{\eta_z}{M}(z(s_i)-\mu_i)(z(s_j)-\mu_j)\right]$$
$$+ \frac{1}{2}\sum_{i=1}^{n}\sum_{j\in N_i}\left[\frac{\eta_y}{M}\left(y(s_i) - \frac{\exp(\delta_i)}{1+\exp(\delta_i)}\right)\left(y(s_j) - \frac{\exp(\delta_j)}{1+\exp(\delta_j)}\right)\right]$$

- $p(\boldsymbol{y}, \boldsymbol{z}) = \frac{\exp(\boldsymbol{Q(y,z)})}{\int_\Omega \exp(\boldsymbol{Q(y,z)})d\boldsymbol{y}d\boldsymbol{z}}$ is a valid joint distribution if $\int_\Omega \exp(\boldsymbol{Q(y,z)})d\boldsymbol{y}d\boldsymbol{z} < \infty$, which holds if $\eta_z < 1$ and $\sigma^2 > 0$

# Model Breakdown

- When dependence becomes too strong, no longer have clearly separated marginal (global) and conditional (local) effects
  - Large $\eta_z$ or $\eta_y$ cause local dependence to overwhelm global properties
  - Data are heavily clustered/chaotic
- Kaiser (2007) quantifies boundaries for "nicely behaved" models:
  - Auto-logistic models: $\eta_y < 4$
  - Auto-normal models: Breakdown as $\eta_z \to 1$, especially $\eta_z \geq 0.99$
- In bivariate case, we have 3 different dependence parameters ($\eta_y$,$\eta_z$, $\rho$) and want all parameters to reflect appropriate properties of the data
- My rule of thumb: $\rho > \sqrt{\sigma^2(1-\eta_z)(4-\eta_y)}$ implies <u>strong dependence</u> for bivariate model

# Estimation and Inference

# Maximum Pseudo-likelihood Estimation

- For most practical applications, maximizing full likelihood requires repeated evaluation of high-dimensional integral in full joint distribution
- Instead, minimize the negative of the log pseudo-likelihood $\tilde{\ell}$, which only uses lower-dimensional node-conditionals:

$$\tilde{\ell} = \sum_{i=1}^{n} \log\left(p(z(s_i)|\boldsymbol{z}(\bar{s}_i), y(s_i))\right) + \sum_{i=1}^{n} \log\left(p(y(s_i)|\boldsymbol{y}(\bar{s}_i), z(s_i))\right)$$

- Hughes, et al. (2011) show loss in efficiency for MPLE is negligible when the lattice is reasonably large ($\sim$ 400 points)
- Exact or asymptotic distributions for parameter estimates are not straightforward from pseudo-likelihood methods

# Simulation and Inference

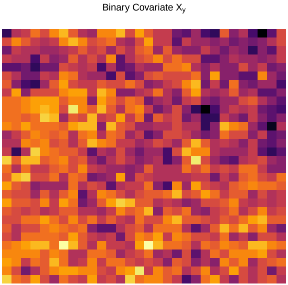
- Use Gibbs sampling to simulate from full set of specified node-conditional distributions
  - Simulate sequentially node-by-node from each conditional distribution, update, iterate until convergence
- Relatively simple simulation algorithm enables ready implementation of parametric bootstrap for inference:
  1. For a set of parameters $\hat{\boldsymbol{\Theta}}$ estimated from the original data, use Gibbs sampling to simulate bootstrap replicates from $\hat{\boldsymbol{\Theta}}$
  2. Obtain MPLE estimates for each bootstrapped dataset to get an empirical distribution of parameters, $\theta^* \in \boldsymbol{\Theta}^*$
  3. Lower and upper bounds for a 100(1-$\alpha$)% confidence interval for a parameter $\theta \in \boldsymbol{\Theta}$ are:
     $L_\alpha = 2\hat{\theta} - \theta^*_{1-\alpha/2}$ and $U_\alpha = 2\hat{\theta} - \theta^*_{\alpha/2}$

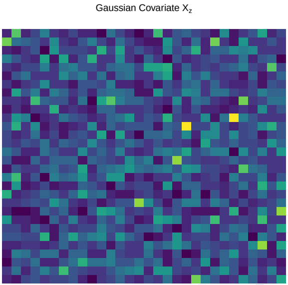
# Simulation Studies

# Study 1: MPLE Estimation Set-up

- Goal: Compare performance for full model vs. previous reduced models
- 3 different dependence regimes (weak, moderate, strong)
- For each dependence regime, obtain MPLE estimates (8 parameters for full model) for 1,000 simulated datasets on a 30 x 30 lattice

Figure: Covariates used in simulation: Normal with spatial correlation $\eta_z = 0.9$, $\mu = 1$, $\sigma^2 = 1$ (left) and $Gamma(3, 4)$ (right)



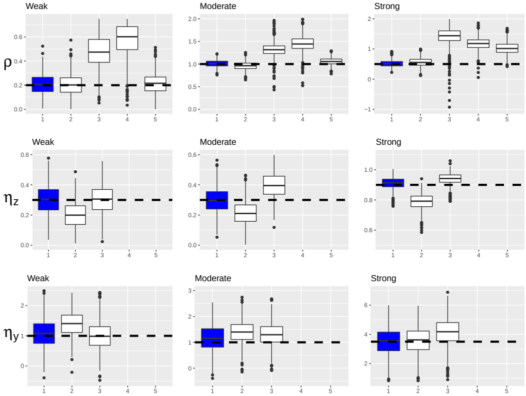



| Parameter Regimes Used to Simulate Data | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| **Dependence** | $\rho$ | $\eta_z$ | $\eta_y$ | $\beta_{0y}$ | $\beta_{1y}$ | $\beta_{0z}$ | $\beta_{1z}$ | $\sigma^2$ |
| Weak | 0.2 | 0.3 | 1 | -1 | 0.5 | 1 | 0.5 | 1 |
| Moderate | 1 | 0.3 | 1 | -1 | 0.5 | 1 | 0.5 | 1 |
| Strong | 0.5 | 0.9 | 3.5 | -1 | 0.5 | 1 | 0.5 | 1 |

| **Model** | **Gaussian Cond. Mean Parameter** $\alpha(s_i)$ |
|---|---|
| Full (1) | $\beta_0+\beta_1 X_{z,i}+\frac{\eta_z}{4}\sum_{j\in N_i}\left(z(s_j)-\mu_j\right)+\rho\left(y(s_i)-\frac{\exp(\delta_i)}{1+\exp(\delta_i)}\right)$ |
| Constant Mean (2) | $\beta_0+\frac{\eta_z}{4}\sum_{j\in N_i}\left(z(s_j)-\mu_j\right)+\rho\left(y(s_i)-\frac{\exp(\delta_i)}{1+\exp(\delta_i)}\right)$ |
| Univariate Spatial (3) | $\beta_0+\beta_1 X_{z,i}+\beta_\rho Y(s_i)+\frac{\eta_z}{4}\sum_{j\in N_i}\left(z(s_j)-\mu_j\right)$ |
| Univariate Non-Spatial (4) | $\beta_0+\beta_1 X_{z,i}+\beta_\rho Y(s_i)$ |
| Bivariate Non-Spatial (5) | $\beta_0+\beta_1 X_{z,i}+\rho\left(y(s_i)-\frac{\exp(\delta_i)}{1+\exp(\delta_i)}\right)$ |

# MPLE Study Results - Dependence

- Only bivariate spatial models (1 and 2) capture the dependence structure adequately
- Strong dependence (far right column): only full bivariate spatial model obtains unbiased estimates for all three dependence parameters



# MPLE Study Results - Covariates

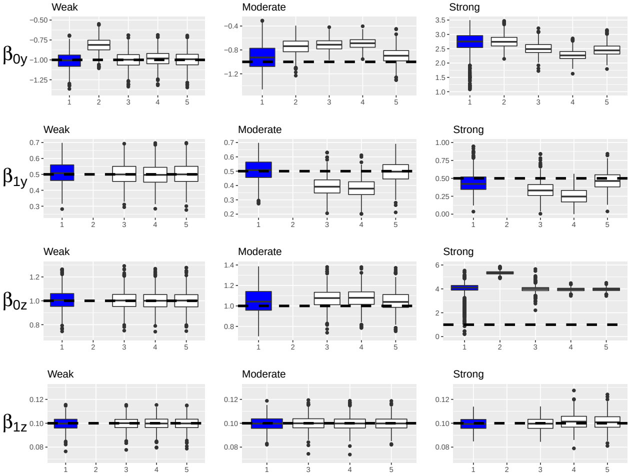



Figure: Mean Parameter Estimates for Simulated Data

# MPLE Study Summary

Summary

- Weak/Moderate dependence: all models provide unbiased estimates of the regression coefficients $\beta$
- Strong dependence: only full model is remotely close to estimating the true parameter values
- The strong dependence regime "breaks" the data-generating mechanism:
  - Expected marginal behavior is not observed due to local dependence effects

    $\Longrightarrow$ Caution for parametric bootstrap if we encounter strong dependence

# Study 2: Bootstrap C.I. Coverage

Goal: Investigate empirical vs. nominal confidence interval coverage under different dependence

Set-up:

- No covariates
- 2 dependence regimes: weak/small variance and strong/large variance
- 30 x 30 lattice
- 95% confidence intervals via parametric bootstrap using 500 bootstrap replicates for 1,000 simulated datasets

# Study 2: Bootstrap C.I. Coverage

- Generate $m = 1,000$ datasets from the true parameter values, $\Theta = \{\rho, \eta_z, \eta_y, \delta, \mu, \sigma^2\}$
- For each of the $i = 1, ..., 1000$ simulated datasets:
  - Estimate the parameters using MPLE to get $\hat{\Theta}_i$
  - Use $\hat{\Theta}_i$ to generate 500 bootstrap replicates for each dataset
  - For each of the bootstrap replicates, obtain MPLE estimates of the parameters and collect into empirical distribution $\Theta_i^*$ of $\Theta_i$ for data set $i$
  - Obtain $C = 95\%$ confidence intervals for $\theta \in \Theta$ by taking $U_i = 2 \cdot \hat{\theta}_i - \theta^*_{i,.025}$ and $L_i = 2 \cdot \hat{\theta}_i - \theta^*_{i,.975}$ as the bounds.
- For a Monte Carlo sample of size $m = 1000$, the empirical coverage is $C^* = (\frac{1}{1000}) \sum_{i=1}^{1000} \mathbb{1}\{\theta \in (L_i, U_i)\}$

# Study 2: Bootstrap C.I. Coverage

- Under weak or moderate dependence conditions, empirical coverage is close to 95%
- Slight undercoverage matches the results for the bivariate autologistic model from Caragea and Berg (2014)

| Weak Dependence | | | | |
|---|---|---|---|---|
| **Parameter** | **True Value** | **MC Estimate** | **MC Std. Error** | **95% CI Coverage** |
| $\rho$ | 0.2 | 0.202 | 0.073 | 92.9% |
| $\eta_z$ | 0.5 | 0.493 | 0.083 | 93.2% |
| $\eta_y$ | 1 | 0.959 | 0.419 | 94.5% |
| $\delta$ | 0.5 | 0.508 | 0.090 | 94.1% |
| $\mu$ | 1 | 1.001 | 0.052 | 93.6% |
| $\sigma^2$ | 1 | 0.996 | 0.053 | 92.9% |

Table: Bootstrap confidence interval coverage for weak dependence regime

# Study 2: Bootstrap C.I. Coverage

- Strong dependence: empirical coverage is further from the nominal 95% value for the unbiased dependence parameters
- Unable to obtain unbiased estimates or reliable confidence intervals for mean parameters $\delta$ and $\mu$

| **Strong Dependence** | | | | |
|---|---|---|---|---|
| **Parameter** | **True Value** | **MC Estimate** | **MC Std. Error** | **95% CI Coverage** |
| $\rho$ | 5 | 5.024 | 0.911 | 96.9% |
| $\eta_z$ | 0.9 | 0.901 | 0.042 | 86.5% |
| $\eta_y$ | 3 | 3.031 | 0.710 | 98.0% |
| $\delta$ | 0.5 | -1.683 | 1.103 | 49.4% |
| $\mu$ | 1 | -17.845 | 10.419 | 50.0% |
| $\sigma^2$ | 100 | 99.622 | 5.582 | 92.6% |

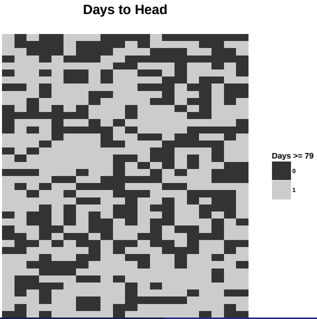
Table: Bootstrap confidence interval coverage for strong dependence regime

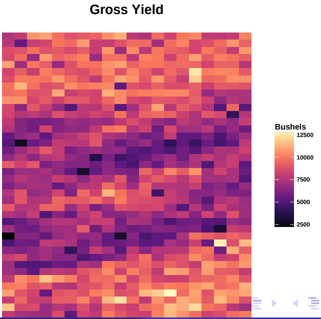
# Illustrative Examples

# Illustrative Example 1: Model Estimation and Selection

# Agricultural Field Trials - Set-Up

- Lado, et al. (2013) used basic spatial adjustments to aid in prediction of wheat yields
- Re-purpose yield data to implement bivariate mixed MRF model
- Goal: Evaluate model performance for full model vs. reduced models
    - Gaussian response $Z(s_i)$ : Gross Yield (1000s of bushels)
    - Binary response $Y(s_i)$: Slow/Fast Growth (1 if $>$ 78 days to head)
    - Thousand-Kernel Weight (g) used a covariate for both responses
    - Fields arranged on a 40 x 20 lattice



# Agricultural Field Trials - Estimation Results

Regression Coefficient Estimates (and 95% Confidence Intervals):

- Estimates are similar across models
- In 3 out of 4 models, Thousand-Kernel Weight has significant relationship with both response variables

| **Model** | **Binary (Y) Intercept ($\beta_{0,y}$)** | **1000-Kernel Wt $\beta_{1,y}$** | **Gaussian (Z) Intercept ($\beta_{0,z}$)** | **1000-Kernel Wt ($\beta_{1,z}$)** |
|---|---|---|---|---|
| Full | 6.591<br>(5.109 , 7.794) | -0.137<br>(-0.162, -0.106) | 6.842<br>(5.915, 7.606) | 0.022<br>(0.008 , 0.036) |
| Bivariate<br>No Covariates | 0.336<br>(0.130 , 0.534) | - | 8.014<br>(7.496 , 8.384) | - |
| Univariate | 6.600<br>(4.319 , 7.104) | -0.135<br>(-0.145 , -0.087) | 7.065<br>(6.969 , 8.370) | 0.019<br>(-0.011 , 0.019) |
| Univariate<br>Non-Spatial | 6.561<br>(5.177 , 7.818) | -0.134<br>(-0.161 , -0.105) | 5.330<br>(4.435 , 6.250) | 0.057<br>(0.038 , 0.077) |
| Bivariate<br>Non-Spatial | 6.873<br>(5.328 , 8.266) | -0.141<br>(-0.171 , -0.110) | 5.442<br>(4.485 , 6.507) | 0.055<br>(0.032 , 0.076) |

# Agricultural Field Trials - Estimation Results

Dependence Parameters:

- Estimates are quite different between bivariate and univariate models
- Moderate within-type (spatial) dependence
- Significant cross-dependence for 2/3 models

| **Model** | $\rho$ | $\eta_z$ | $\eta_y$ |
|---|---|---|---|
| Full Bivariate | 0.299 (0.127 , 0.474) | 0.888 (0.795 , 1.008) | 0.846 (0.000 , 1.947) |
| Bivariate No Covariates | 0.126 (-0.026 , 0.264) | 0.900 (0.809, 0.996) | 1.001 (0.135 , 2.001) |
| Univariate Spatial | - | 0.618 (0.634 , 0.921) | 1.232 (0.622 , 2.710) |
| Univariate Non-Spatial | - | - | - |
| Bivariate Non-Spatial | 0.233 (0.282 , 0.487) | - | - |

# Agricultural Field Trials - Model Selection

- We use continuous-ranked probability score to compare in-sample predictive capability of each model:

$$CRPS(F(x(s_i)|\boldsymbol{x}(\bar{s}_i), \hat{\boldsymbol{\Theta}}), x_i) = \int_{-\infty}^{\infty} (F(x(s_i)) - \mathbb{1}_{[x(s_i) \geq x]})^2 dx(s_i)$$

  where $F(x(s_i)|\boldsymbol{x}(\bar{s}_i), \hat{\boldsymbol{\Theta}})$ is the conditional CDF at each node
- Sum CRPS over all points to get CRPS(M) for the model
- Corresponding skill score for model $M$ vs. constant mean model $c$ is

$$Skill = 100 \cdot \left[1 - \frac{CRPS(M)}{CRPS(c)}\right]$$

- Minimize CRPS or maximize Skill to select best model

# Agricultural Field Trials - Model Selection

| **Model** | $CRPS_y$ | $Skill_y$ | $CRPS_z$ | $Skill_z$ |
|---|---|---|---|---|
| Full | 0.20 | 16.74 | 0.66 | 34.23 |
| Intercept-Only Spatial | 0.24 | 1.62 | 0.66 | 33.46 |
| Univariate | 0.21 | 14.82 | 0.70 | 29.57 |
| Univariate Non-Spatial | 0.22 | 8.11 | 0.97 | 2.59 |
| Bivariate Non-Spatial | 0.22 | 10.44 | 0.96 | 3.37 |

- Full bivariate spatial model with covariate has best in-sample prediction for both response variables
- Spatial models outperform non-spatial models for Gaussian response (Gross Yield)
- Covariate (Thousand-Kernel Weight) important for fitting the binary data

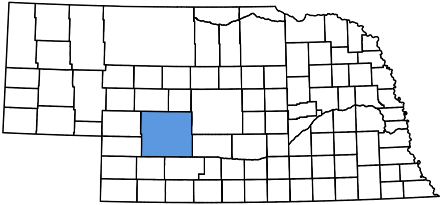
# Illustrative Example 2: Strong Spatial Dependence

# Greater Prairie Chickens - Data

Setting: Lincoln County (Western Nebraska)

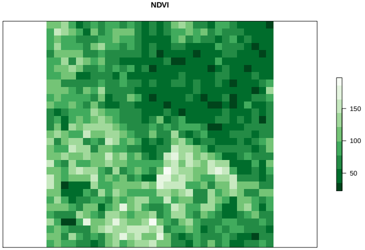
Data from three sources:

- Greater Prairie Chicken observations (March-May, since 1990) from Cornell Lab of Ornithology
- Vegetative Cover: April 2018 NDVI (Normalized Difference Vegetation Index) collected via satellite, from USDA National Agricultural Statistics Survey's VegScape API
- Soil properties (covariates) from NRCS Web Soil Survey



# Greater Prairie Chickens - Data

- Data sources overlaid/combined/aggregated onto 31 x 31 lattice (each point ~2 square miles)
- NDVI (continuous response) has very strong spatial autocorrelation
- Prairie Chicken nesting sites (binary response) has visible spatial autocorrelation, but somewhat sparse



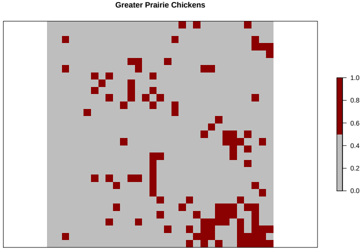




Figure: NDVI (left) and Greater Prairie Chicken observations (right)

# Greater Prairie Chickens - Methodology

Methodology:

- NDVI as continuous response $Z(s_i)$
- Greater Prairie Chicken observations (1 if observed since 1990, 0 if not) as binary response $Y(s_i)$
- Exploratory analysis to select possible soil covariates
- Estimate the full bivariate MRF model use 90% confidence intervals for inference
- Can we find any evidence for:
    - Either response being related to soil properties?
    - Spatial dependence or cross-dependence among NDVI and nesting sites?

# Greater Prairie Chickens - Estimation Results

| Parameter | Estimate | Std. Err. | 90% Bootstrap C.I. |
|---|---|---|---|
| Y-Intercept($\beta_{0,y}$) | -0.417 | 1.962 | (-2.343 , 4.183) |
| Wind Erodibility Index ($\beta_{1,y}$) | -0.005 | 0.006 | (-0.017 , 0.002) |
| % Sand ($\beta_{2,y}$) | 0.024 | 0.018 | ( 0.003 , 0.063) |
| Water Supply ($\beta_{3,y}$) | 0.019 | 0.045 | (-0.061 , 0.083) |
| Z-Intercept($\beta_{0,z}$) | 204.543 | 15.970 | (-43.172 , 66.560) |

Covariates:

- Soil variables not significantly related with either response, even with reduced confidence level
- Standard errors quite large for all variables except % Sand composition as covariate for Greater Prairie Chicken presence

# Greater Prairie Chickens - Estimation Results

| Parameter | Estimate | Std. Err. | 90% Bootstrap C.I. |
|---|---|---|---|
| $\eta_y$ (Presence-Absence) | 3.613 | 0.697 | (2.356 , 4.614) |
| $\eta_z$ (NDVI) | 0.991 | 0.017 | (0.955 , 1.012) |
| $\rho$ (Cross-dependence) | 0.446 | 0.260 | (0.000 , 0.844) |
| $\sigma^2$ | 16.765 | 0.949 | (15.246 , 18.245) |

Dependence Parameters:

- Moderate/Strong spatial correlation (near 4) for Greater Prairie Chicken presence
- NDVI spatial correlation is near the boundary of the parameter space (very strong dependence)
- Some cross-dependence among response variables
  - Weak evidence supporting a positive dependence between vegetative cover and probability of observing a Greater Prairie Chicken nest
- In-sample predictive capability was poor for Prairie Chicken presence-absence (Skill $< 0$)

# Conclusions and Future Work

# Summary

The proposed bivariate Gaussian-Binary Markov random field model:

- Provides interpretable dependence parameters in terms of conditional means by modeling dependence directly
- Allows for flexible specification of covariates related to each response
- Has easy-to-implement estimation and inference procedures from conditional specification
- Outperforms previous univariate auto-models in simulations and application
- Introduces mixed-response multivariate auto-models that can be extended to more complex applications

# Conclusions and Future Work

Areas for future work:

- Formally quantifying/defining/interpreting/remedying strong dependence settings
- Expanded/irregular neighborhood structures
- Extension to other distributions and higher-dimensional responses
- Faster/more exact inference methods

QUESTIONS?

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