Image Denoising Based on Wavelet Analysis for Satellite Imagery

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Image Denoising Based on Wavelet Analysis for Satellite Imagery*

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1. Introduction

Digital images are prone to a variety of types of noise. Noise is the result of errors in the image acquisition process that result in pixel values that do not reflect the true intensities of the real scene (Gagnon & Smaili, 1996). There are several ways that noise can be introduced into an image, depending on how the image is created. For example if the image is scanned from a photograph made on film, the film grain is a source of noise. Noise can also be the result of damage to the film, or be introduced by the scanner itself. If the image is acquired directly in a digital format, the mechanism for gathering the data (such as a CCD detector) can introduce noise. Electronic transmission of image data can introduce noise. Noise is considered to be any measurement that is not part of the phenomena of interest. Noise can be categorized as Image data independent noise and image data dependent noise.

Wavelets are mathematical functions that cut up data into different frequency components, and then study each component with a resolution matched to its scale (Durand & Froment,1992). They have advantages over traditional Fourier methods in analyzing physical situations where the signal contains discontinuities and sharp spikes. Wavelets were developed independently in the fields of mathematics, quantum physics, electrical engineering, and seismic geology. Interchanges between these fields during the last ten years have led to many new wavelet applications such as image compression, turbulence, human vision, radar, and earthquake prediction.

Synthetic aperture radar is a radar technology that is used from satellite or airplane (Lee,Jukervish 1994). It produces high resolution images of earth's surface by using special signal processing techniques. Synthetic aperture radar has important role in gathering information about earth's surface because it can operate under all kinds of weather condition (whether it is cloudy, hazy or dark).However acquisition of SAR images face certain problems. SAR images contain speckle noise which is based on multiplicative noise or rayleigh noise. Speckle noise is the result of two phenomenon, first phenomenon is the coherent summation of the backscattered signals and other is the random interference of electromagnetic signals. Speckle noise degrades the appearance and quality of SAR images (Brunique,1997). Ultimately it reduces the performances of important techniques of image processing such as detection, segmentation, enhancement and classification. That is why speckle noise should be removed before applying any image processing techniques. There are three main objectives of any speckle filtering. First is to remove noise in uniform regions. Second is to preserve and enhance edges and image features and third is to provide a good visual appearance. Unfortunately 100% speckle reduction is not possible. Therefore, tradeoff has to be made among these requirements. Speckle reduction usually consists of three stages. First stage is to transform the noisy image to a new space (frequency domain). Second stage is the manipulation of coefficients. Third is to transform the resultant coefficients back to the original space (spatial domain).Currently many statistical filters are available for speckle reduction, such as, Mean, Kuan, Frost and Lee filter etc. Results show that statistical filters are good in speckle reduction but they also lose important feature details. Additionally prior knowledge about noise statistics is a prerequisite for statistical filters. In recent years, there has been active research on wavelet based speckle reduction because wavelet provides multi resolution decomposition and analysis of image (Durand & Froment,1992).. In wavelet sub bands noise is present in small coefficients and important feature details are present in large coefficients. If small coefficients are removed, we will get noise free image. Previously most of the researchers use discrete wavelet transformation for reduction of speckle. Draw back of discrete wavelet transformation is that it is not translation invariant. That means it will lose lots of important coefficients during translation from original signal to sub bands(Nabil ,2009). In order to solve this problem and to save the coefficients, derivated form of discrete wavelet transformation is used called undecimated wavelet transformation. Basic idea is that it does not lose any coefficients, all coefficients remain intact. That is why it is also called redundant wavelet transformation. It requires more storage space and need more time for computation. Whether discrete or undecimated wavelet is used, biggest problem is the selection of optimal thresholding .Some researchers use wavelet based hard or soft thresholding. Other thresholding techniques were also used such as VisuShrink, SureShrink and neighshrink.

1.1 Aim and objectives of the chapter

Image denoising has a significant role in image pre processing. As the application areas of image denoising are more, there is a big demand for efficient denoising algorithms. In this chapter, a method that involves Wavelet with shrinkage concepts is proposed and applied it to denoise images corrupted with speckle noise. The intention behind this method is to reduce the convergence time of conventional filter and thereby increase its performance. The proposed method produces excellent results and comparison is made with two wavelet shrinkage and case study is carried over for ice classification based in SAR imagery.

To this the objectives are:

1. to analysis various wavelet concepts in image processing
2. to generalize from current research thus, reaching a coherent view of the role of Wavelet in image denoising ;

3. to propose wavelet concept for describing the denoising of images using shrinkage methods
4. to produce a case study to support the denoising approach within the goal-driven concept.

Wavelet based methods are always a good choice for image denoising and has been discussed widely in literatures for the past two decades .Wavelet shrinkage permits a more efficient noise removal while preserving high frequencies based on the disbalancing of the energy of such representation. The technique denoises image in the orthogonal wavelet domain, where each coefficient is thresholded by comparing against a threshold; if the coefficient is smaller than the threshold, it is set to zero, otherwise it is kept or modified.

This chapter does not attempt to investigate in deep the theoretical properties of the proposed model in general settings. The primary goal is to demonstrate that how the performance of wavelet shrinkage based denoising methods can be applied by using the best wavelet family. In the following paragraphs, Section 2 reviews the existing works on wavelet concepts in image processing, Section 3 describes the state-of-the-art of image denoising, Section 4 explains the various shrinkage methods of wavelet, Section 5 explains about the solution for denoising in SAR imagery using wavelet Section 6 ends up with a case study and, finally, Section 7 discuss future works in the area of wavelet.

2. Review based on wavelet concepts in image processing

The main issue discussed in this section is to identify the nature of the dependence between the pixels in the original image and to estimate noise in signal. For astronomical images of sparsely distributed stars an independence assumption may be reasonable, while for many other kinds of images (including astronomical images of galaxies) such an assumption is inappropriate (Lopez & Cumplido 2004). If independence is a reasonable assumption then the CLEAN, maximum entropy, and maximally sparse methods are appropriate and the choice largely depends on the desired balance between accuracy and speed. For example, the CLEAN method is fast but can make mistakes for images containing clustered stars. For images that are expected to be relatively smooth then the Wiener filter and iterative methods are appropriate. If the images are known to satisfy some additional constraints (for example, the intensities are often known to be non-negative for physical reasons) or if the blurring function is space varying then the iterative methods such as Richardson-Lucy or constrained least squares are appropriate. Otherwise it is better to use the Wiener filter because it is fast and approximately includes the iterative methods as special cases. The wavelet methods tend to give a good compromise for images containing such a mixture of discontinuities and texture. Below are the research work in which wavelet concepts and methods are applied to image processing applications.

2.1 Motion estimation

Magarey developed a motion estimation algorithm based on a complex discrete wavelet transform. The transform used short 4-tap complex filters but did not possess the PR property. The filter shapes were very close to those used in the DT-CWT suggesting that the conclusions would also be valid for the DT-CWT. The task is to try and estimate the displacement field between successive frames of an image sequence. The fundamental property of wavelets that makes this possible is that translations of an image result in phase changes for the wavelet coefficients. By measuring the phase changes it is possible to infer the motion of the image. A major obstacle in motion estimation is that the reliability of motion estimates depends on image content. For example, it is easy to detect the motion of a single dot in an image, but it is much harder to detect the motion of a white piece of paper on a white background. Magarey developed a method for incorporating the varying degrees of confidence in the different estimates. In tests on synthetic sequences the optimised CDWT-based algorithm showed superior accuracy under simple perturbations such as additive noise and intensity scaling between frames. In addition, the efficiency of the CDWT structure minimises the usual disadvantage of phase-based schemes– their computational complexity. Detailed analysis showed that the number of floating point operations required is comparable to or even less than that of standard intensity-based hierarchical algorithms.

2.2 Classification

Efficient texture representation is important for content based retrieval of image data(Lopez &Cumplido 2004). The idea is to compute a small set of texture-describing features for each image in a database in order to allow a search of the database for images containing a certain texture. The DT-CWT has been found to be useful for classification by number of authors.Each uses the DT-CWT in different ways to compute texture features for an entire image:

1. De Rivaz and Kingsbury compute features given by the logarithm of the energy in each subband.
2. Hill, Bull, and Canagarajah compute the energies of the subbands at each scale. However, in order to produce rotationally invariant texture features, they use features based on either the Fourier transform or the auto-correlation of the 6 energies at each scale.
3. Hatipoglu, Mitra, and Kingsbury use features of the mean and standard deviations of complex wavelet subbands. However, instead of using the DT-CWT based on a fixed tree structure, they use an adaptive decomposition that continues to decompose subbands with energy greater than a given threshold (Nabil, 2009).

2.3 Denoising

In many signal or image processing applications, the input data is corrupted by some noise which need to be removed or at least reduced. Wavelet denoising techniques work by adjusting the wavelet coefficients of the signal in such a way that the noise is reduced while the signal is preserved (Sivakumar,2007). There are many different methods for adjusting the coefficients but the basic principle is to keep large coefficients while reducing small coefficients. This adjustment is known as thresholding the coefficients.One rationale for this approach is that often real signals can be represented by a few large wavelet coefficients, while (for standard orthogonal wavelet transforms) white noise signals are represented by white noise of the same variance in the wavelet coefficients. Therefore the reconstruction of the signal from just the large coefficients will tend to contain most of the signal energy but little of the noise energy. An alternative rationale comes from considering the signal as being piecewise stationary. For each piece the optimum denoising method is a Wiener filter whose frequency response depends on the local power spectrum of the signal. When the signal power is high, the power is kept mostly; when the signal power is low, the signal is attenuated. The size of each wavelet coefficient can be interpreted as an estimate of the power in some time-frequency bin and set the small ones to zero in order to approximate adaptive Wiener filtering. The first wavelet transform proposed for denoising was the standard orthogonal transform. However, orthogonal wavelet transforms (DWT) produce results that substantially vary even for small translations in the input and so a second transform was proposed, the nondecimated wavelet transform (NDWT) , which produced shift invariant results by effectively averaging the results of a DWT-based method over all possible positions for the origin. Experiments on test signals show that the NDWT is superior to the DWT. The main disadvantage of the NDWT is that even an efficient implementation takes longer to compute than the DWT, by a factor of the three times the number of levels used in the decomposition.Kingsbury has proposed the use of the DT-CWT for denoising because this transform not only reduces the amount of shift-variance but also may achieve better compaction of signal energy due to its increased directionality. In other words, at a given scale an object edge in an image may produce significant energy in 1 of the 3 standard wavelet subbands, but only 1 of the 6 complex wavelet subbands.

2.4 Compression and matching

Compression algorithms with wavelet-based transformations were selected in competition with compression using fractal transformations. FBI's standard has similarities with the JPEG2000 standard, and especially with an extension to the JPEG2000 standard. Further decomposition of the LH-, HL- and HH-bands like this may improve compression somewhat, since the effect of the filter bank application may be thought of as an "approximative orthonormalization process". The extension to the JPEG2000 standard also opens up for this type of more general subband decompositions. In FBI's standard different wavelets can be used, with the coefficients of the corresponding filter banks signalled in the code-stream. The only constraint on the filters is that there should be no more than 32 nonzero coefficients. This is much longer than lossy compression in JPEG2000 (9 nonzero coefficients).

2.5 Segmentation

Texture is an important characteristic for analyzing many types of images, including natural scenes and medical images. With the unique property of spatial-frequency localization, wavelet functions provide an ideal representation for texture analysis. Experimental evidence on human and mammalian vision support the notion of spatial-frequency analysis that maximizes a simultaneous localization of energy in both spatial and frequency domain.These psychophysical and physiological findings lead to several research works on texture-based segmentation methods based on multi-scale analysis. One important feature of wavelet transform is its ability to provide a representation of the image data in a multi-resolution fashion. Such hierarchical decomposition of the image information provides the possibility of analyzing the coarse resolution first, and then sequentially refine the segmentation result at more detailed scales. In general, such practice provides additional robustness to noise and local maxima (Mallat,1989).

3. Preliminary investigation of image denoising

Noise reduction is the process of removing noise from image. Noise reduction techniques are conceptually very similar regardless of the signal being processed, however a priori knowledge of the characteristics of an expected signal can mean the implementations of techniques vary greatly depending on the type of signal. Noise can be random or white noise with no coherence or coherent noise introduced by the device mechanism or processing algorithms. In photographic film and magnetic tape, noise (both visible and audible) is introduced due to the grain structure of the medium. In photographic film, the size of the grains in the film determines the film's sensitivity, more sensitive film having larger sized grains. In magnetic tape, the larger the grains of the magnetic particles (usually ferric oxide or magnetite), the more prone the medium is to noise. To compensate for this, larger areas of film or magnetic tape may be used to lower the noise to an acceptable level.

3.1 Noise study

Estimating the noise level from a single image seems like an impossible task: the image should be recognized whether local image variations are due to color, texture, or lighting variations from the image itself, or due to the noise. It might seem that accurate estimation of the noise level would require a very sophisticated prior model for images. Capturing a pinhole image (large depth-of-field) is important to many computer vision applications, such as 3D reconstruction, motion analysis, and video surveillance. For a dynamic scene, capturing pinhole images however is difficult: we have often to make a tradeoff between depth-of-field and motion blur. For example, if a large aperture and short exposure to avoid motion blur, the resulting images will have small depth-of-field; otherwise, if we use a small aperture and long exposure, the depth-of-field will be large, but at the expense of motion blur.

3.2 Potential disadvantages of noise filters in images

Many types of distortions limit the quality of digital images during image acquisition, formation, storage and transmission. Often, images are corrupted by impulse noise. The intensity of impulse noise has the tendency of being either relatively high or low thereby causing loss of image details. It is important to eliminate noise in images before using them for other image processing techniques like edge detection, segmentation, registration etc. Several filtering methods have been proposed in the past to address impulse noise removal(Wang &Hang 1999). One of the more famous filters for gray scale images is the standard median filter which rank orders the pixel intensities within a filtering window and replaces the center pixel with the median value. Extending the idea of a scalar median filter to color images is not straightforward due to the lack of a natural concept of ranking among the vectors. Color distortion may occur when the scalar median filter is applied separately to every single component of the color vectors. A method called Vector Median Filter (VMF) which considers all the three color components and rank orders the vectors. Various modifications of the standard VMF have been introduced like Directional Median Filter and Central Weighted Vector Median Filter. The biggest drawback of the conventional vector median approaches is that they apply median operation to each pixel, irrespective of it being corrupted or not. An intuitive solution to overcome this disadvantage is to first detect the corrupt pixels and then to apply filtering on those pixels alone (Trygve &Hakon 1999). One of the main problems with impulse noise detection is that it is difficult to differentiate between an edge and an impulse noise. In the intensity space, both these stand as peaks in their neighborhood. The difference between the center pixel with the minimum and maximum gray value in the filtering window is taken and if greater than a certain threshold, the center pixel is considered as noise. The disadvantage of this method is that the false positive rate is very high and most of the edges also get detected as noise. Coherent processing of synthetic aperture radar (SAR) data makes images susceptible to speckles (Lee,Jukervish 1994). Basically, the speckles are signal-dependent and, therefore, act like multiplicative noise. This report develops a statistical technique to define a noise model, and then successfully applies a local statistics noise filtering algorithm to a set of actual SEASAT SAR images. The smoothed images permit observers to resolve fine detail with an enhanced edge effect. Several SEASAT SAR images are used for demonstration.

The standard algorithm shows very good performance removing the additive noise. In SAR images, on the contrary, the noise is multiplicative. In particular three procedures are there to obtain the speckle noise reduction:

1. Use of a logarithmic transformation in order to translate the noise from multiplicative to additive.
2. Use of repeated applications of filtering algorithm.
3. Use of non-symmetric membership functions optimized by using a genetic algorithm.

Regarding the first point, the logarithmic transformation allowed to apply the standard fuzzy filtering algorithm obtaining a significant removal of the multiplicative noise.

The repeated application of the filtering algorithm permits to reduce the speckle noise granularity without degradation of the sharpness. This is very important for the subsequent recognition of the pattern present in the image.

3.3 Wavelet denoising in images

The fundamental objective in image enhancement is to improve or accentuate subsequent processing tasks such as detection or recognition (Chang,2000,2006). Classical image enhancement techniques consider the use of spatial-invariant operators either in the spatial or in the fourier domain. Examples of techniques in the spatial domain are related with the histogram modification by a predetermined transformation as in histogram equalization and stretching. These methods are global in the sense that the pixels are modified in the entire image. However, it is often necessary to perform the enhancement process over small patches of the image. Examples of such techniques include local histogram stretching (in overlapping or non-overlapping windows), smoothing and sharpening . In the fourier domain, most methods are based in supressing low spatial frequencies relative to high spatial frequencies as in homomorphic filtering. Local image enhancement can also be performed by means of a multiscale image representation.Fourier transform based spectral analysis is the dominant analytical tool for frequency domain analysis. However, fourier transform cannot provide information of the spectrum changes with respect to time. Fourier transform assumes the signal as stationary, but PD signal is always non-stationary. To overcome this deficiency, a modified method-short time fourier transform allows representing the signal in both time and frequency domain through time windowing functions (Akansu et al., 1992). The window length determines a constant time and frequency resolution. Thus, a shorter time windowing is used in order to capture the transient behavior of a signal by sacrificing the frequency resolution.

A continuous-time wavelet transform of f(t) is defined as:

$$CWT_v f(a,b) = W_f(b,a) = |a|^{-1/2} \int_{-\alpha}^{\alpha} f(t)\psi^*(\frac{t-b}{a})dt \quad (1)$$

Here a,bεR , $a \neq 0$ since they are dilating and translating coefficients respectively. The asterisk denotes a complex conjugate. This multiplication of $|a|^{1/2}$ is for energy normalization purposes so that the transforms signal will have the same energy at every scale. The analysis function $\psi(t)$, the so called mother wavelet, is scaled by a, so a wavelet analysis is often called a time scale analysis rather than a time frequency analysis. The wavelet transform decomposes the signal into different scales with different levels of resolution by dilating a single prototype function, the mother wavelet. Furthermore, a mother wavelet has to satisfy that it has a zero net area, which suggest that the transformation kernel of the wavelet transform is a compactly support function (localized in time), thereby offering the potential to capture the PD spikes which normally occur in a short period of time. The general wavelet denoising procedure is as follows:

- Apply wavelet transform to the noisy signal to produce the noisy wavelet coefficients to the level which it can properly distinguish the PD occurrence.
- Select appropriate threshold limit at each level and threshold method (hard or soft thresholding) to best remove the noises.
- Inverse wavelet transform of the thresholded wavelet coefficients to obtain a denoised signal.

4. Denoising using shrinkage methods

Conservative methods based on wavelet transforms have been emerged for removing Gaussian random noise from images. This local preprocessing speckle reduction technique is necessary prior to the processing of SAR images. Wavelet Shrinkage or thresholding as denoising method is the best identified method here. It is well known that increasing the redundancy of wavelet transforms can significantly improve the denoising performances. Thus a thresholding process which passes the coarsest approximation sub-band and attenuates the rest of the sub-bands should decrease the amount of residual noise in the overall signal after the denoising process (Achim et al.,2003). One dimensional dyadic discrete time wavelet transform is a transform similar to the discrete Fourier transform in that the input is a signal containing *N* numbers, say, and the output is a series of *M* numbers that describe the time-frequency content of the signal. The Fourier transform uses each output number to describe the content of the signal at one particular frequency, averaged over all time. In contrast, the outputs of the wavelet transform are localised in both time and frequency. The wavelet transform is based upon the building block shown in figure 1. This block is crucial for both understanding and implementing the wavelet transform.

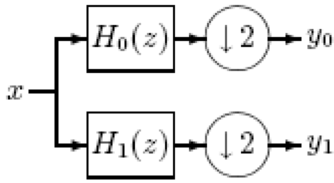

Fig. 1. Building block for wavelet transform

This diagram is to be understood as representing the following sequence of operations

1. Filter an input signal (whose value at time n is x (n)) with the filter whose Z-transform is $H_0(z)$.
2. Downsample the filter output by 2 to give output coefficients y_0 (n).
3. Filter the input signal x(n) with the filter whose Z-transform in $H_1(z)$.
4. Downsample the filter output by 2 to give output co-efficients $y_1(n)$

The two main confines in image accuracy are categorized as blur and noise. Blur is intrinsic to image acquisition systems, as digital images have a finite number of samples and must respect the sampling conditions. The second main image perturbation is noise. Image denoising is used to remove the additive noise while retaining as much as possible the important signal features. Currently a reasonable amount of research is done on wavelet thresholding and threshold selection for signal de-noising, because wavelet provides an appropriate basis for separating noisy signal from the image signal. Two shrinkage methods are used over here to calculate new pixel values in a local neighborhood. Shrinkage is a well known and appealing denoising technique. The use of shrinkage is known to be optimal for Gaussian white noise, provided that the sparsity on the signal's representation is enforced using a unitary transform. Here a new approach to image denoising, based on the image-domain minimization of an estimate of the mean squared error-Stein's unbiased risk estimate (SURE) is proposed and equation (3.1) specifies the same. Surelet method directly parameterizes the denoising process as a sum of elementary nonlinear processes with unknown weights. Unlike most existing denoising algorithms, using the SURE makes it needless to hypothesize a statistical model for the noiseless image. A key of it is, although the (nonlinear) processing is performed in a transformed domain-typically, an undecimated discrete wavelet transform, but nonorthonormal transformsis also addressed and this minimization is performed in the image domain.

$$sure(t;x) = d - 2.\#\{i : |x_i| \le t\} + \sum_{i=1}^{d} (|x_i| \Lambda t)^2 \qquad (2)$$

where d is the number of elements in the noisy data vector and x_i is the wavelet coefficients. This procedure is smoothness-adaptive, meaning that it is suitable for denoising a wide range of functions from those that have many jumps to those that are essentially smooth.

It have high characteristics as it out performs Neigh shrink method. Comparison is done over these two methods to prove the elevated need of Surelet shrinkage for the denoising the SAR images. The experimental results are projected in graph format which shows that the Surelet shrinkage minimizes the objective function the fastest, while being as cheap as neighshrink method. Measuring the amount of noise equation (3) is done by its standard deviation , $\sigma(n)$, one can define the signal to noise ratio (SNR) as

$$SNR = \frac{\sigma(\mu)}{\sigma(n)}, \tag{3}$$

Where $\sigma(\mu)$ in equation (3) denotes the empirical standard deviation of $\mu(i)$,

$$\sigma(\mu) = \left(\frac{1}{|I|} \sum_{i} (u(i) - \overline{\mu})^2 \right)^{1/2} \tag{4}$$

And $\bar{\mu} = \frac{1}{|I|} \sum_{i \varepsilon I} \mu(i)$ is the average grey level value. The standard deviation of the noise can also be obtained as an empirical measurement or formally computed when the noise model and parameters are known. This parameter measures the degree of filtering applied to the image It also demonstrates the PSNR rises faster using the proposed method than the former. Hence the resulted denoised image is conceded to the next segment for the transformation to be applied and it is also proved to improve detection process.

4.1 Wavelet shrinkage - Description and short history

Wavelet shrinkage is a quite recent denoising method compared to classical methods like the Wiener filter or convolution filters and is applied very successfully to various denoising problems (Liu &Raja, 1996).. A very interesting thing about wavelet shrinkage is that it can be motivated from very different fields of mathematics, namely partial differential equations, the calculus of variations, harmonic analysis or statistics.

A heuristic way to wavelet shrinkage goes as follows. A signal f is considered which is distributed by additive white noise: $g = f + \varepsilon$ .Since the discrete wavelet transform is linear and orthogonal, the wavelet transform of g has the form $g_\gamma = (f_\gamma) + (\varepsilon_\gamma)$ where the coefficients ε_γ of the noise are given white noise. Usually the signal f results in a few number of large wavelet coefficients and most of the coefficients are zero or nearly zero (Chen &Bui 2003). The noise on the other hand leads to a large number of small coefficients on all scales. Thus, the small coefficients g_γ mostly contain noise. Hence, it seems to be a good idea to set all the coefficients which are small to zero. But what shall happen to the large coefficients? The two most popular ones are hard and soft shrinkage. By application of hard shrinkage one leaves the large coefficients unchanged and sets the coefficients below a certain threshold to zero. Mathematically speaking one applies the function

$$S_\lambda(x) = \begin{cases} x, |x| > \lambda \\ 0, |x| \le \lambda \end{cases} \tag{5}$$

to the wavelet coefficients. Another famous way is soft shrinkage where the magnitude of all coefficients is reduced by the threshold in which one applies the function

$$S_\lambda(x) = \begin{cases} x-\lambda, x \geq \lambda \\ 0, |x| \leq \lambda \\ x+\lambda, x \leq -\lambda \end{cases} \tag{6}$$

to the coefficients. Beside these two possibilities there are many others (semi-soft shrinkage, firm shrinkage,….) and as long as the shrinkage function preserves the sign $(sign(S_\lambda(x)) = sign(x))$ and shrinks the magnitude ,one can expect a denoising effect.

The interesting thing about wavelet shrinkage is, that it appears in very different fields of mathematics in a natural way. Four places where shrinkage appears naturally are :

1. As the subgradient descent along the absolute value.
2. As the function which maps an initial value onto the minimizer of a variational functional.
3. As the function "identity minus projection onto a convex set" which is also motivated by variational analysis.
4. As the maximum a posteriori estimator for an additively disturbed signal, where the signal and the noise are distributed in a certain way.

4.2 Applying shrinkage methods

The effect of wavelet shrinkage is illustrated in this section. An image of an eye is taken which is a closeup on a man's eye. It is a suitable image for illustrative purposes because it provides very different regions: small and sharp details like the eyelashes, texture-like parts of different contrast like the eyebrows or the skin below the eye, smooth parts like the eyeball or the skin above the eye and sharp edges like the edge of the lower lid. The image has a resolution of 256 times 256 pixels, 256 gray levels For calculations the gray levels have been scaled to the interval [0, 1].

Fig. 2. The image eye.

4.3 Discrete wavelet shrinkage

This is the place where shrinkage methods have their origin and where they are used the most.

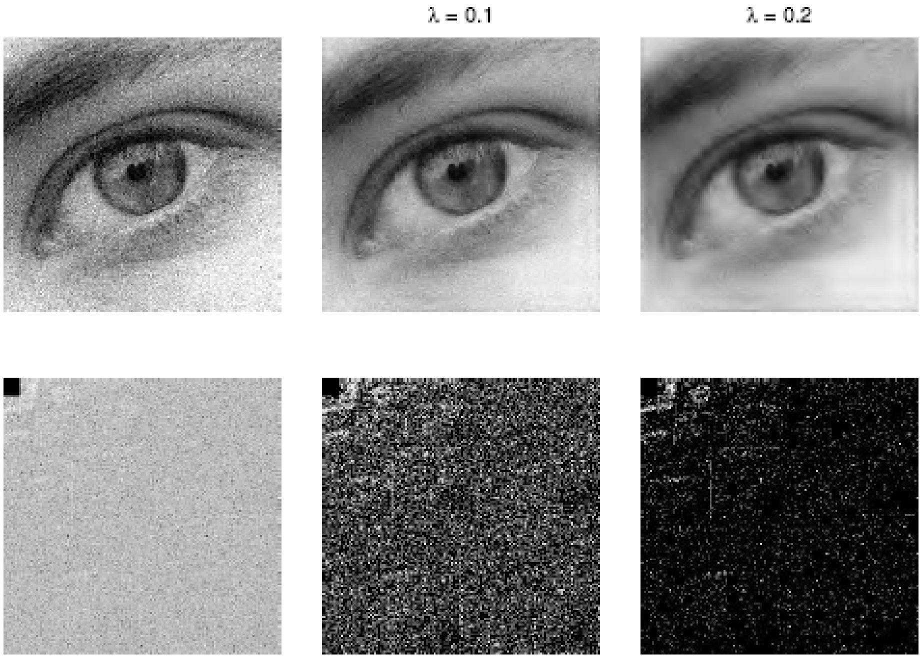

Fig. 3. Illustration of discrete wavelet shrinkage. The wavelet used here is the coiflet. Top row, from left right: the noisy image, wavelet shrinkage for different values of λ. The second row shows the discrete wavelet transform of the upper row.

Consider an orthogonal periodic wavelet base $\{\psi_\gamma)_{\gamma\varepsilon\tau}$ of $L^2(I)$

Define the orthogonal mapping as

$$W : L^2(I) \to l^2(1 \cup \Gamma) \text{ via } f \to (\langle f \mid 1 \rangle, (f_\gamma)_{\gamma\varepsilon\Gamma}) \tag{7}$$

The mapping W is invertible and isometrical.

Define $\Phi onl^2(1 \cup \Gamma)$ by $\Phi(a) = \|a\|_{l^1(1\cup\Gamma)}$ and the functional need is $\Psi : L^2(I) \to \bar{R}$ defined by

$$\Psi(f) = \Phi(Wf) = \begin{cases} |\langle f \mid 1 \rangle| + \|(f_\gamma)_{\gamma\varepsilon\Gamma}\|_{l^1(\Gamma)} \\ \infty \end{cases} \tag{8}$$

The result obtained that the discrete wavelet shrinkage

$$\mu(t) = S_t(\langle f \mid 1 \rangle) + \sum_{\gamma\varepsilon\Gamma} S_t(f_\gamma)\psi_\gamma \tag{9}$$

is a solution of the descent equation

$$\partial_t \mu + \partial\Psi(\mu) \ni 0, \mu(0) = f \tag{10}$$

4.4 Continuous wavelet shrinkage

The wavelet transform is an isometry and explained as

$$L^2(R) \to rangeL_\psi \subset L^2(R^2, dadb / a^2) \tag{11}$$

But the range of the wavelet transform is a proper subspace of the space $L^2(R^2, dadb / a^2)$. In particular range L_ψ is not invariant under shrinkage.

Unfortunately, a subgradient descent in such a subspace is in general not a subgradient descent in the original Hilbert space and vice versa.

Soft Shrinkage : The well known soft shrinkage function $S_\lambda(x) = (|x| - \lambda) + signx$ gives

$$g(|x| = (1 - \frac{(|x| - \sqrt{2\lambda})_+}{|x|} \tag{12}$$

which is a diffusivity. It is decreasing and according to the above proposition it hols g(0)=1 and g(x) $\to$ 0 for $x \to \infty$

Hard Shrinkage: The Hard shrinkage function (or hard thresholding)

$$S_\lambda(x) = x(1 - \chi_{[-\lambda\lambda]}(x)) \quad \text{leads to}$$

$$g(|x|) = \begin{cases} 1 for |x| \le \sqrt{2\lambda} \\ 0 else \end{cases} \tag{13}$$

This is a "piecewise linear diffusion" where diffusion is forbidden if the derivative has absolute value larger that $\sqrt{2\lambda}$ .With regard to wavelet shrinkage denoising, the theoretical justifications and arguments in its favor remain highly compelling. The procedure does not require any assumptions about the nature of the signal, permits discontinuities and spatial variation in the signal, and exploits the spatially adaptive multiresolution features essential to the wavelet transform. Furthermore, the procedure exploits the fact that the wavelet transform maps white noise in the signal domain to white noise in the transform domain. Thus, while signal energy becomes more concentrated into fewer coeffcients in the transform domain, noise energy does not. It is this important principle that enables the separation of signal from noise. Wavelet shrinkage denoising has been theoretically proven to be nearly optimal from the following perspectives: spatial adaptation, estimation when local smoothness is unknown, and estimation when global smoothness is unknown. This section presents an extensive study for wavelet denoising methods and shows that many of them are leading to the idea of shrinkage in a general sense.

5. Finding a solution for denoising in sar imagery using wavelet

The wavelet transform is a mathematical tool widely used in image processing. Some applications of the transform to remote sensing images have been investigated in the literature. It was found useful for texture analysis , image compression and noise reduction . The transform allows representation of a signal onto an orthonormal basis. Each term of the basis represents the signal at a given scale. In order to decompose the signal onto the basis, the algorithm developed is applied to the signal. It consists of iterations of one-dimensional high-pass and low-pass filtering steps. The algorithm creates a pyramid of low-resolution approximations as well as a wavelet pyramid in which the details a stored as wavelet coefficients. This representation is called wavelet representation(Jiang et al., 2000).. One way of image analysis, is to choose the wavelet for speckle in SAR images which is always problematic(Ali,2007). Often, it is impenitent to reduce noise-before trying to extract scene features. Many filters have been developed to improve image quality by conserving the intrinsic scene features and textures. Interpretation of SAR images by human is possible in the presence of speckle. The wavelet transform, as the mammal visual system, provides and allows for a multiscale analysis of images. This section presents how the wavelet transform can be used for extraction of linear features such as edges and thin stripes. It will also show how speckle can be relaxed by taking into account the speckle contribution to wavelet coefficients.

5.1 Noise in SAR imagery

A proportionality relation exists between Speckle noise and the wavelet coefficients . Since Speckle is approached as a multiplicative noise, this contribution will be larger for higher reflectivity regions. It is also known that speckle in SAR images is spatially correlated. That is to say the noise is colored. Therefore its behavior in the Fourier domain is such that there is a peak of a given width around the zero frequency. This is readily observed as a wavelet coefficient variance plateau when decomposing a correlated noise model on a wavelet basis. As decomposition gets to scales larger than the correlation length, the contribution from speckle decreases linearly.

5.2 Using Wavelet for SAR speckle denoising

- The main advantage of wavelet analysis is that it allows the use of long time intervals where more precise low frequency information is wanted, and shorter intervals where high frequency information is sought.
- Wavelet analysis is therefore capable of revealing aspects of data that other image analysis techniques miss, such as trends, breakdown points, and discontinuities in higher derivatives and self-similarity.
- Wavelets are also capable of compressing or de-noising a image without appreciable degradation of the original image.

5.3 Wavelet transform

The *wavelet transform* can be used to analyze time series that contain nonstationary power at many different frequencies. Assume that one has a time series, xn, with equal time spacing dt and n = 0 … N - 1. Also assume that one has a *wavelet function*, $\psi_n = (n)$, that depends on a nondimensional "time" parameter h. To be "admissible" as a wavelet, this function must have zero mean and be localized in both time and frequency space. An example is the Morlet wavelet, consisting of a plane wave modulated by a Gaussian:

$$\psi_o(\eta) = \pi^{-1/4}\ell^{i\omega_0 n}\ell^{-n^2/2} \tag{14}$$

where w0 is the nondimensional frequency, here taken to be 6 to satisfy the admissibility condition . The term "wavelet function" is used generically to refer to either orthogonal or nonorthogonal wavelets. The term "wavelet basis" refers only to an orthogonal set of functions. The use of an orthogonal basis implies the use of the *discrete wavelet transform,* while a nonorthogonal wavelet function can be used with either the discrete or the continuous wavelet transform (Grossman et al.,1989). The continuous wavelet transform of a discrete sequence *xn* is defined as the convolution of *xn* with a scaled and translated version of $\psi_n = (n)$:

$$W_n(s) = \sum_{n'=0}^{N-1} x_{n'}\psi^*\left[\frac{(n'-n)\delta t}{s}\right] \tag{15}$$

where the (*) indicates the complex conjugate. By varying the *wavelet scale s* and translating along the *localized time index n*, one can construct a picture showing both the amplitude of any features versus the scale and how this amplitude varies with time. The subscript 0 on y has been dropped to indicate that this y has also been normalized (see next section). Although it is possible to calculate the wavelet transform using (2), it is considerably faster to do the calculations in Fourier space. To approximate the continuous wavelet transform, the convolution (2) should be done *N* times for each scale, where *N* is the number of points in the time series (Kaiser 1994). (The choice of doing all *N* convolutions is arbitrary, and one could choose a smaller number, say by skipping every other point in *n*.) By choosing *N* points, the convolution theorem allows us do all *N* convolutions simultaneously in Fourier space using a discrete Fourier transform (DFT). The DFT of *xn* is

$$\hat{x}_k = \frac{1}{N}\sum_{n=0}^{N-1} x_n e^{12\pi kn/N} \tag{16}$$

where $k = 0 \ldots N - 1$ is the frequency index. In the continuous limit, the Fourier transform of a function y(t/s) is given by y$ (sw). By the convolution theorem, the wavelet transform is the inverse Fourier transform of the product:

$$W_n(s) = \sum_{k=0}^{N=1} \hat{x}_k \hat{\psi}^*(s\omega_k)e^{i\omega_k \eta\delta t} \tag{17}$$

where the angular frequency is defined as

$$\begin{aligned} \omega_k &= \left\{\frac{2\pi k}{N\delta t} : k \le \frac{N}{2}\right\} \\ \omega_k &= \left\{-\frac{2\pi k}{N\delta t} : k \le \frac{N}{2}\right\} \end{aligned} \tag{18}$$

Using (4) and a standard Fourier transform routine, one can calculate the continuous wavelet transform (for a given s) at all n simultaneously and efficiently.

5.4 Normalization

To ensure that the wavelet transforms at each scale s are directly comparable to each other and to the transforms of other time series, the wavelet function at each scale s is normalized to have unit energy:

$$\hat{\psi}(s\omega_k) = \left(\frac{2\pi s}{\delta t}\right)^{1/2} \hat{\psi}_0(s\omega_k) \tag{19}$$

Using these normalizations, at each scale s one has

$$\sum_{k=0}^{N-1} \left|\hat{\psi}(s\omega_k)\right|^2 = N \tag{20}$$

where N is the number of points. Thus, the wavelet transform is weighted only by the amplitude of the Fourier coefficients $x\$k$ and not by the wavelet function. If one is using the convolution formula (2), the normalization is

$$\psi\left[\frac{(n' - n)\delta t}{s}\right] = \left(\frac{\delta t}{s}\right)^{1/2} \psi_0\left[\frac{\left(n' - n\right)\delta t}{s}\right] \tag{21}$$

where $\psi_n = (n)$ is normalized to have unit energy.

5.5 Wavelet functions

One criticism of wavelet analysis is the arbitrary choice of the wavelet function, $\psi_n = (n)$. (It should be noted that the same arbitrary choice is made in using one of the more traditional transforms such as the Fourier, Bessel, Legendre, etc.) In choosing the wavelet function, there are several factors which should be considered

1. *Orthogonal or nonorthogonal.* In orthogonal wavelet analysis, the number of convolutions at each scale is proportional to the width of the wavelet basis at that scale. This produces a wavelet spectrum that contains discrete "blocks" of wavelet power and is useful for signal processing as it gives the most compact representation of the signal. Unfortunately for time series analysis, an a periodic shift in the time series produces a different wavelet spectrum. Conversely, a nonorthogonal analysis (such as used in this study) is highly redundant at large scales, where the wavelet spectrum at adjacent times is highly correlated. The nonorthogonal transform is useful for time series analysis, where smooth, continuous variations in wavelet amplitude are expected.
2. *Complex or real.* A complex wavelet function will return information about both amplitude and phase and is better adapted for capturing oscillatory behavior. A real wavelet function returns only a single component and can be used to isolate peaks or discontinuities.
3. *Width.* For concreteness, the width of a wavelet function is defined here as the e-folding time of the wavelet amplitude. The resolution of a wavelet function is determined by

the balance between the width in real space and the width in Fourier space. A narrow (in time) function will have good time resolution but poor frequency resolution, while a broad function will have poor time resolution, yet good frequency resolution.

4. *Shape.* The wavelet function should reflect the type of features present in the time series. For time series with sharp jumps or steps, one would choose a boxcar-like function such as the Harr, while for smoothly varying time series one would choose a smooth function such as a damped cosine. If one is primarily interested in wavelet power spectra,then the choice of wavelet function is not critical, and one function will give the same *qualitative* results as another.

5.6 Wavelet bases

Many wavelet families with various characteristics are known. Some wavelet families are especially useful for specific application. Most wavelets are based on FIR filters although research work is also done for wavelets constructed by IIR filters. The orthogonal and biorthogonal wavelets are the two basic categories of wavelets.

Type	Filter	Symmetry	Orthogonality	Fast algorithm
Haar	FIR	Symmetric	Orthogonal	Yes
Daubechies	*FIR*	*Asymmetric*	*Orthogonal*	*Yes*
Symlets	FIR	Near symmetric	Orthogonal	Yes
Coiflets	*FIR*	*Near symmetric*	*Orthogonal*	*Yes*
Spline	*FIR*	*Symmetric*	*Biorthogonal*	*Yes*
Morlet	IIR	Symmetric	No	No
Mexican Hat	IIR	Symmetric	No	No
Meyer	IIR	Symmetric	Orthogonal	No
Butterworth	*IIR*	*Asymmetric*	*Orthogonal*	*Yes*

Table 1. Main properties of wavelet families

Table 1 summarizes the main properties of most well-known wavelets. The primary consideration in the use of wavelets for surface profile analysis is the amplitude and phase transmission characteristics of the wavelet basis. A combination of good amplitude and linear phase transmission is always desired to achieve minimum distortion of surface features. Among the listed wavelets in Table 1, Haar wavelet is the oldest and simplest wavelet that is not continuous. The Symlet and Coiflet wavelet come from Daubechie wavelet, but are more symmetric (Daubechies,1989,1992). Both the scaling function and wavelet of Meyer are defined in frequency domain. Although the scaling function and wavelet of Meyer are symmetric, no fast algorithm is available for its wavelet transform.The Morlet and Mexican wavelets only have wavelet functions and the corresponding scaling functions don’t exist Four wavelets in three categories are selected for study here. They are orthogonal Daubechies wavelets and

Butterworth wavelets with nonlinear phase, orthogonal Coiflets wavelets with near linear phase, and biorthogonal Spline wavelets with linear phase (Daubechies, 1989, 1992). All of them are associated with FIR filters except Butterworth wavelets.

5.7 Wavelet families in SAR images

- Haar wavelet is the simplest of the wavelet transforms.
- This transform cross-multiplies a function against the Haar wavelet with various shifts and stretches, like the Fourier transform cross-multiplies a function against a sine wave with two phases and many stretches.
- Daubechies wavelets are a family of orthogonal wavelets defining a discrete wavelet transform and characterized by a maximal number of vanishing moments for some given support.
- Meyer's wavelet construction is fundamentally a solvent method for solving the two-scale equation.
- Symlet wavelet is only nearly symmetric, and is not exactly symmetrical.
- Coiflets are discrete wavelets designed by Ingrid Daubechies, to have scaling functions with vanishing moments.
- biorthogonal wavelet is a wavelet where the associated wavelet transform is invertible but not necessarily orthogonal.
- Reverse biorthogonal is a spline wavelet filters.

5.8 Performance evaluation

5.8.1 Subjective evaluation

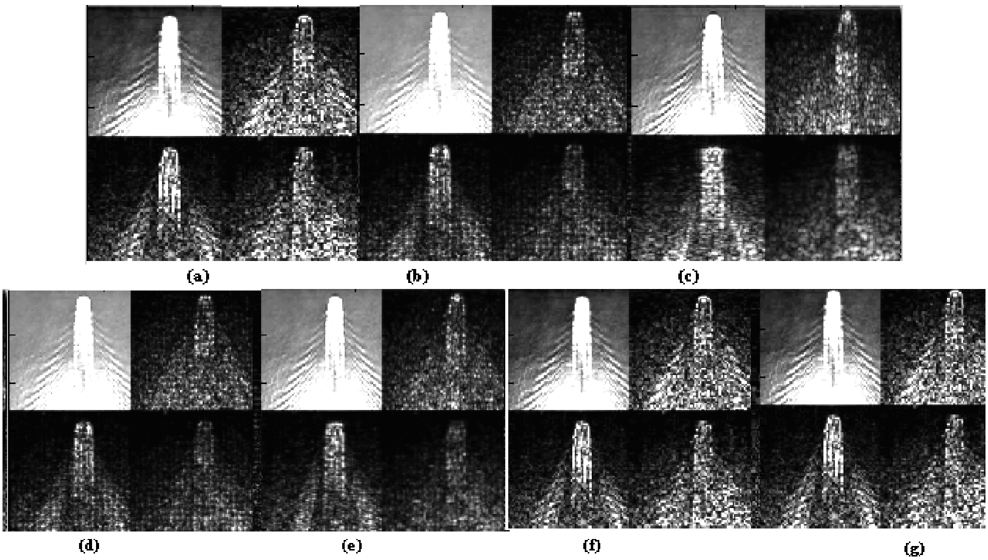

Fig. 4. (a) Haar wavelet (b) Daubechies wavelet (c) Meyer Wavelet (d) Symlet wavelet (e) Coiflets wavelet (f) Bi orthogonal wavelet (g) Reverse Bi orthogonal

5.8.2 Objective evaluation

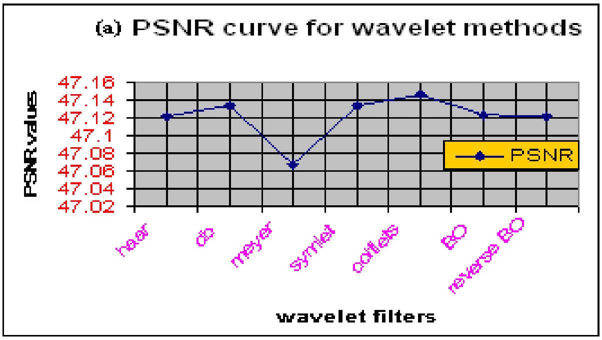

Fig. 5. Peak to Signal noise ratio for wavelet methods

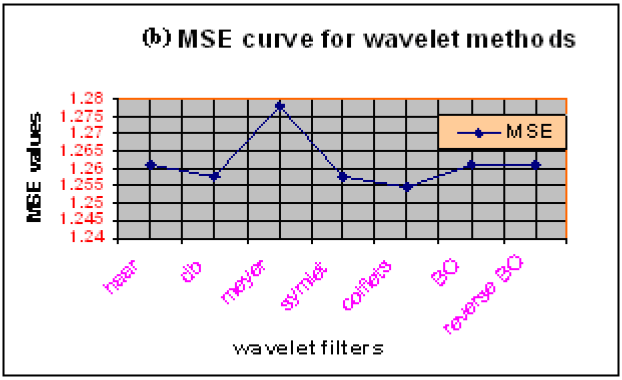

Fig. 6. Mean square error rate for wavelet methods

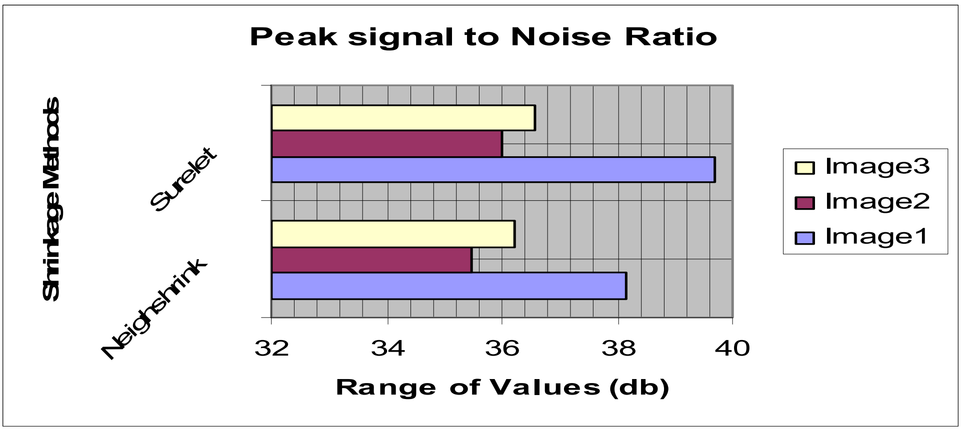

Fig. 7. PSNR values for shrinkage method based on coiflet Wavelet family

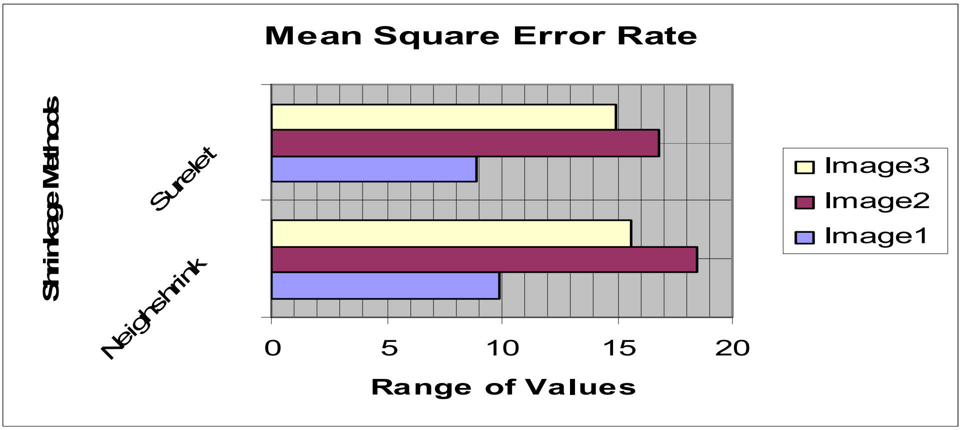

Fig. 8. MSE values for shrinkage method based on coiflet Wavelet family

The wavelet transform is computed separately for different segments of the time-domain signal at different frequencies. Discrete wavelet transform (DWT), which transforms a discrete time signal to a discrete wavelet representation. The reconstruction of image is far better in wavelet by analysis and it is implemented with the given SAR image. Some of the parameters taken for analysis of wavelet on SAR images are

Mean Square error rate - MSE is called **squared error loss**.

MSE of an estimator is one of many ways to quantify the amount by which an estimator differs from the true value of the quantity being estimated.

PSNR -This ratio is often used as a quality measurement between the original and a compressed image. The higher the PSNR, the better the quality of the compressed or reconstructed image.

The *Mean Square Error (MSE)* and the *Peak Signal to Noise Ratio (PSNR)* are the two error metrics used to compare image quality. The MSE represents the cumulative squared error between the compressed and the original image, whereas PSNR represents a measure of the peak error. The lower the value of MSE, the lower the error.

From the above got results the coiflets of wavelet denoising method out performs the rest of the wavelet families. This study presented an analysis and comparison of the wavelet families using for image denoising considering PSNR and visual quality of image as quality measure. The effects of bio orthogonal , Reverse bio orthogonal, Daubechies, coiflets and symlets wavelet families on the test images have been examined. The PSNR and visual image quality for wavelet functions of each family is also presented. The PSNR is taken as the objective measure for performance analysis of wavelets using for images denosinig. Here it is analyzed –the results for a wide range of wavelets families and found that the wavelet coifilets provides best performance for SAR images.The computational time required for the Bio orthogonal and reverse bio orthogonal wavelet families is more in comparison to other wavelet families is more in comparison to other wavelet families. The performance of wavelet function depends not only size of the image but also on the content and resolution of the images. Finally, it is concluded that the selection of wavelet for image denoising depends on size, contents and resolution of the images for desired image quality.

6. Wavelet analysis for ice classification in SAR imagery

Automation in river ice image classification assists the ice experts in extracting geophysical information from the increasing volume of images. Rivers and streams are the key elements in the terrestrial re-distribution of water. An ice cover has significant impact on rivers such as modifies ecosystem, affects microclimate, cause flooding, restrict navigation, impact hydropower generation. The regions that are affected by ice are fishing industry, coastal zone and lake water levels and navigation.

6.1 Importance of ice covers

- Impacts both global/regional energy and water cycles.
- High reflectance, thermal insulation, storage of water extent (areal coverage), depth, water equivalent (water content), wet/dry state.
- Snowfall/solid precipitation .Indicator of climate variability and change
- Input/validation of models – NWP, hydrological, climate
- Environmental monitoring/prediction – flood forecasting, severe weather(blowing snow), soil moisture/drought, forest fire risk, wildlife
- Socio-economic – hydropower production/management, agriculture, tourism

6.2 SAR basics

Satellite-based Synthetic Aperture Radar (SAR) provides a powerful vessel surveillance capability in front of time consuming traditional methods. SAR images are larger in volume. SAR images typically consist of 32 bit complex pixels with large dimensions. The entropy of SAR images is higher than optical images (Sery et al., 1996).. SAR images carry information in low frequency bands as well as high frequency bands. SAR images have larger dynamic range than optical images. SAR sea images are highly heterogeneous and this fact affects to the viability of the approach. The major advantages of SAR are (i)Sensitive to texture (ii) Good for vegetation studies (iii) Ocean waves, winds, currents(iv) Seismic Activity and Moisture content.

6.3 SAR image denoising using Wavelet

This is the first and lowest level operation to be done on images. The input and the output are both intensity images. The main idea with the preprocessing is to suppress information in the image that is not relevant for its purpose or the following analysis of the image (Subashini &Krishnaveni, 2010). The pre-processing techniques use the fact that neighboring pixels have essentially the same brightness. There are many different pre-processing methods developed for different purposes.Interesting areas of pre-processing for this work is image filtering for noise suppression. Two shrinkage methods are used over here to calculate new pixel values in a local neighborhood. Shrinkage is a well known and appealing denoising technique. On the experiment evaluation, Daubechies wavelet family of orthogonal wavelets is concluded as the appropriate family for shrinkage method as it is defined as a discrete wavelet transform and characterized by a maximal number of vanishing moments for some given support.

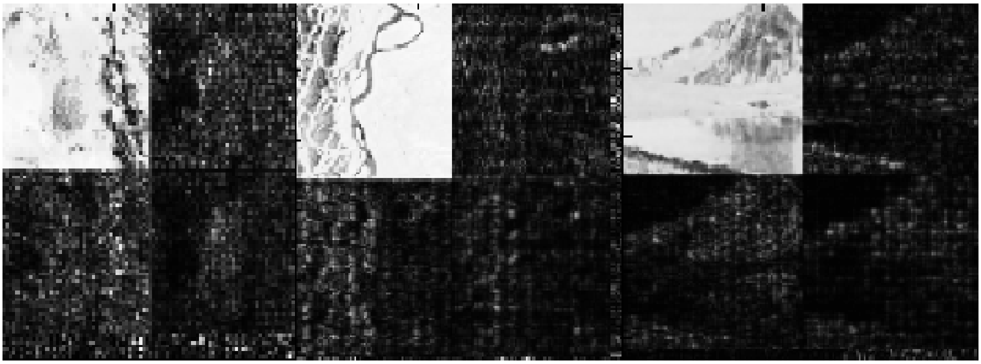

Fig. 9. Daubechies wavelet based on level 2 decomposition

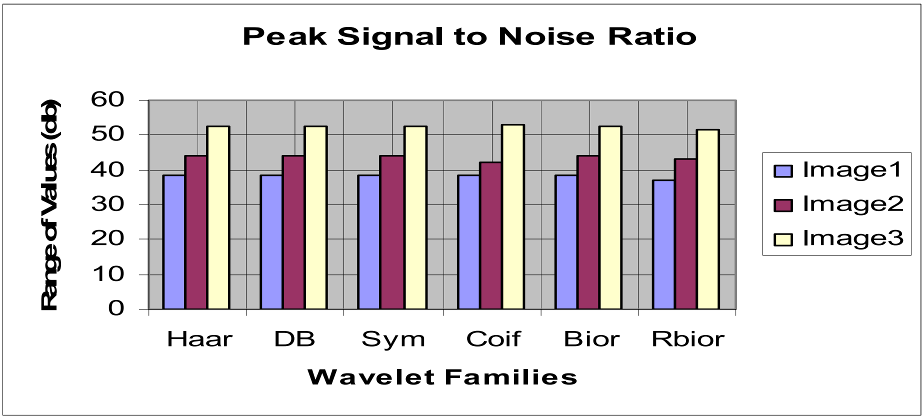

Fig. 10. Peak to Signal noise ratio for wavelet methods

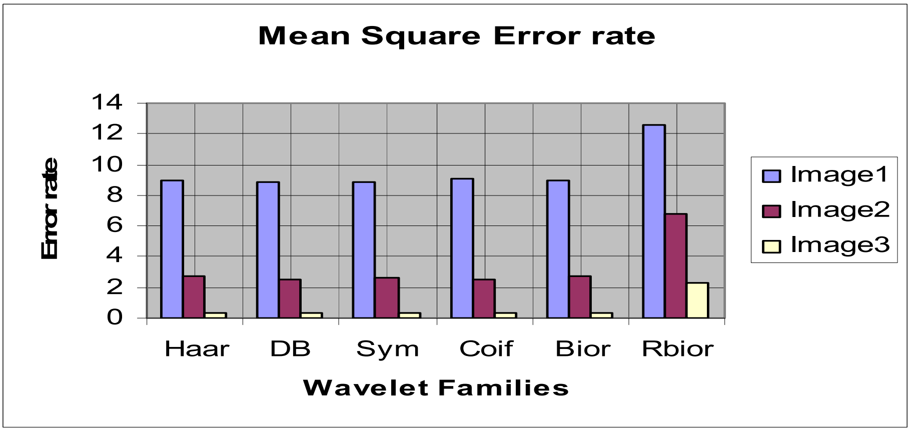

Fig. 11. Mean square error rate for wavelet methods

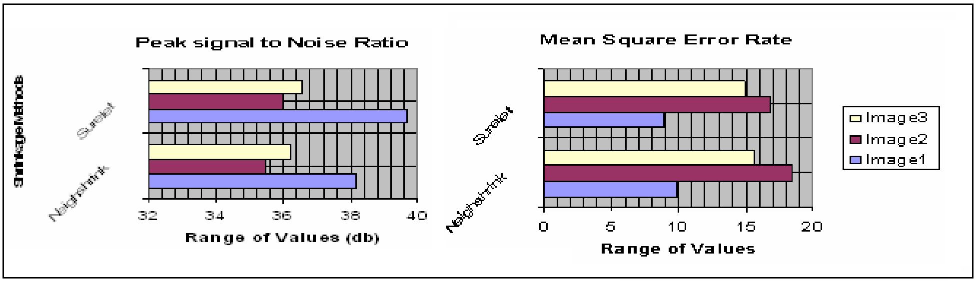

Fig. 12. PSNR and MSE values for shrinkage method based on DB Wavelet Family

Figure 10, 11 shows the evaluation of wavelet families to find the best and Daubechies wavelet is been concluded for wavelet shrinkage denoising. Figure 12 represents the objective evaluation of the shrinkage methods and finally surelet shrinkage is been concluded as the optimal method for denoising. Here an attempt is made to find a superior methodology for denoising than the conventional fixed-form neighborhoods. This approach also determines optimal results by using finest threshold instead of using the suboptimal universal threshold in all bands. It exhibits an excellent performance for ice detection and the experimental result also signifies the same by producing both higher PSNRs and enhanced visual eminence than the former and conventional methods. In future, research will be carried out to reduce the computational load of the proposed image classification algorithm and shorten the execution time of the projected approach.

7. Conclusion and further development

Recently, there has been considerable interest in using the wavelet transform as a powerful tool for recovering SAR images from noisy data. The main reason for the choice of multiscale bases of decompositions is that the statistics of many natural signals, when decomposed in such bases are significantly simplified. When multiplicative contamination is concerned, multiscale methods involve a preprocessing step consisting of a logarithmic transform to separate the noise from the original image. However, thresholding methods have two main drawbacks: i) the choice of the threshold, arguably the most important design parameter, is made in an *ad hoc* manner; and ii) the specific distributions of the signal and noise may not be well matched at different scales. To address these disadvantages, Bayesian theory can be introduced , which outperform classical linear processors and simple thresholding estimators in removing noise from visual images.

Denoising should not be confused with smoothing. Smoothing removes high frequencies and retains low frequencies whereas denoising attempts to remove whatever noise is present and retain whatever signal is present regardless of the spectral content of the noisy signal.Wavelet shrinkage denoising is denoising by shrinking (i.e., nonlinear soft thresholding) coefficients in the wavelet transform domain. It consists of three steps: 1) a linear forward wavelet transform, 2) a nonlinear shrinkage denoising, and 3) a linear inverse wavelet transform. Because of the nonlinear shrinking of coefficients in the transform domain, this procedure is distinct from those denoising methods that are entirely linear. Moreover, it is considered as a nonparametric method. Thus, it is distinct from parametric methods, including both linear and nonlinear regression, in which parameters must be estimated for a particular model that must be assumed a priori. If the common sense approach to practical problem solving is adopted, then the practitioner should exploit any and all *a priori* information available for his particular problem, and use an appropriate denoising procedure as determined by the most relevant outcome measure. Determining the most appropriate procedure necessarily involves experiments to compare the performance of a wavelet shrinkage denoising method with any other methods under consideration. In addition, issues of computational complexity must be considered. Complexity of algorithms may be measured according to CPU computing time and flops, or the number and kind of algorithm steps and their impact on firmware or hardware requirements. Here a new statistical representation for the wavelet decomposition coefficients of SAR images is introduced and it is found that shrinkage is to be more effective than traditional methods both in terms of speckle reduction and signal detail preservation. The SAR images evaluated all are coded in eight-bit. The motivation is that as wavelet transform is good at energy compaction, the small coefficients are more likely due to noise and large coefficient due to important signal features . The proposed technique is based upon the analysis of wavelet transform which uses a soft thresholding method for thresholding the small coefficients without affecting the significant features of the image. In the chapter, image denoising is studied using various wavelets for different images at various levels of decomposition and comparison are done between the families and wavelet shrinkage techniques. It is unlikely that one particular wavelet shrinkage denoising procedure will be suitable, no less optimal, for all practical problems. However, it is likely that there will be many practical problems, for which after appropriate experimentation, wavelet-based denoising with either hard or soft thresholding proves to be the most effective procedure. Estimation of the power spectrum by wavelet-based denoising of the log-periodogram may prove to be one such important application with great promise for further development.

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