Exercise 10 Sthavishtha Bhopalam Rajakumar December 29, 2017

1 Introduction

The objective of the current work is to numerically calculate the trajectory of a projectile, a fundamental problem in classical mechanics. For numerical computation, a first order accurate Euler scheme has been adopted. Two tasks have been implemented as follows in the current work:

1. To plot the trajectory of the projectile for varying magnitudes of initial velocity and angle
2. To determine the angle at which maximum range can be obtained, for varying coefficients of friction

2 Algorithm Description

The governing equations of motion of the projectile trajectory include :

$$\dot{x} \;=\; v_x \tag{1}$$

$$\dot{z} \;=\; v_z \tag{2}$$

$$\dot{v_x} \;=\; -\gamma\sqrt{v_x^2 \;+\; v_z^2}v_x \tag{3}$$

$$\dot{v_z} \;=\; -g \;-\gamma\sqrt{v_x^2 \;+\; v_z^2}v_z \tag{4}$$

where x and z refer to the horizontal and vertical directions respectively. The respective velocities along these directions include v_x and v_z. Additionally, g and γ refer to the acceleration due to gravity ($9.81m/s^2$) and friction coefficient respectively.

Using a first order accurate Euler method, the time-integration terms in the above governing equations can be discretized to yield the following equations which can be numerically solved in a step by step manner:

$$x(t_0) \;=\; x_0 \;\; z(t_0) \;=\; z_0 \tag{5}$$

$$v_x(t_0) \;=\; v_0\cos\alpha \;\; v_z(t_0) \;=\; v_0\sin\alpha \tag{6}$$

where t_0 refers to the initial time step (at $t \;=\; 0$). And α refers to the angle at which the projectile is initially thrown.

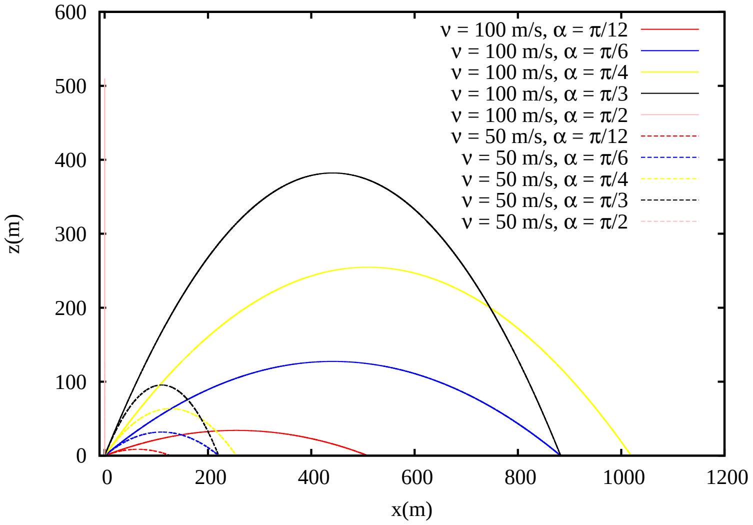
The velocity components and positions (the second order time-step term are neglected for simplicity) of the projectile at subsequent time steps (t_{n+1}) can be updated as follows:

$$v_x(t_{n+1}) \;=\; v_x(t_n) \;-\; (\Delta t\gamma\sqrt{v_x(t_n)^2 \;+\; v_z(t_n)^2})v_x(t_n) \tag{7}$$

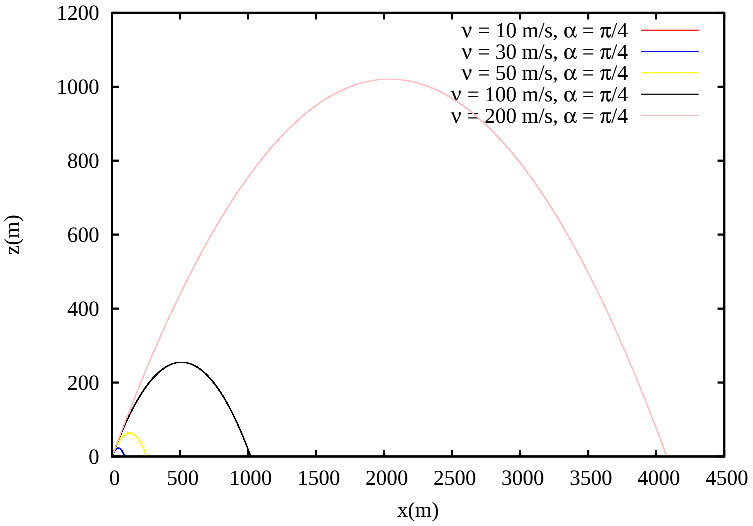
$$v_z(t_{n+1}) \;=\; v_z(t_n) \;-\; \Delta t(g \;+\; \gamma\sqrt{v_x(t_n)^2 \;+\; v_z(t_n)^2}v_x(t_n)) \tag{8}$$

$$x(t_{n+1}) \;=\; x(t_n) \;+\; \Delta t v_x(t_n) \tag{9}$$

$$z(t_{n+1}) \;=\; z(t_n) \;+\; \Delta t v_z(t_n) \tag{10}$$



(a) Projectile trajectory for constant ν (50 and 100 m/s considered) and varying α



(b) Projectile trajectory for constant α (50 and 100 m/s considered) and varying ν

Figure 1: Projectile Trajectories

3 Results and Discussions

3.1 Task 1

In this task, numerical computations of the trajectory has been made by plotting the trajectory of the projectile [1] at constant α (with varying ν - in Figure 1a) and constant ν (with varying α - in Figure 1b). In Figure 1a, it can be noticed that increasing α increases the range till $\pi/4$, later which it starts decreasing. Complementary α also lead to the same range (for $\pi/3$ and $\pi/6$). Additionally, increasing the velocity increases the range of the projectile.

3.2 Task 2

In this task, the effect of friction coefficient has been considered by plotting α for various γ which yield the maximum range (as seen in Figure 2). It can be noticed that increasing ν decreases the angle to yield the maximum range (compare ν of 10, 40 and 80 m/s for an insight). Additionally, increasing the friction coefficient requires a smaller α to yield the maximum range. Noise (or fluctuations) in

[1] For this task, the friction coefficient has been assumed to be 0

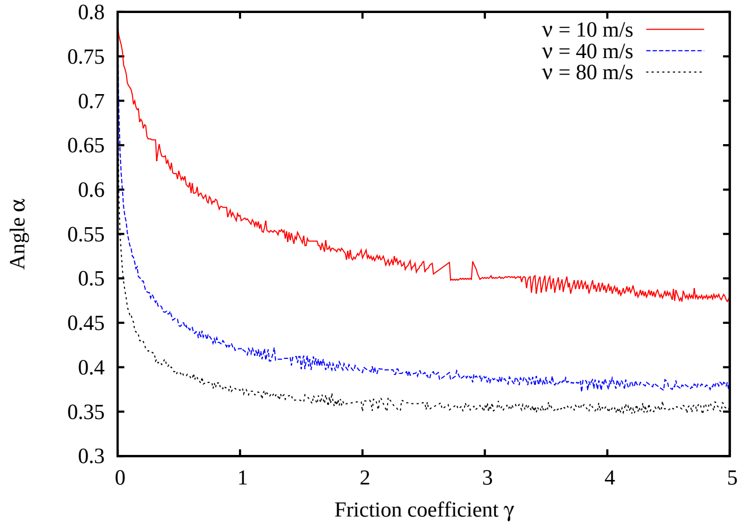


Figure 2: Graph of α at maximum range vs γ for ν of 10, 40 and 80 m/s

the plot can be observed due to the numerical errors, as the considered numerical scheme is only first order accurate.

4 Future Work

Since the order of accuracy of the considered numerical scheme is only one, it may be viable to consider a second or higher order (fourth) Runge-Kutta scheme, which is most commonly adopted in the literature. Additionally, in the current work, the friction coefficient has been assumed to be independent of the velocity, which in principle will be depending on the velocity (sometimes taken as a quadratic relationship in the literature) - this could be a recommendation for an improved understanding of the projectile trajectory.