Exercise 04 Sthavishtha Bhopalam Rajakumar December 29, 2017

1 Introduction

The objective of the current work is to compute the fractal dimension of a percolating cluster at critical occupation probability for a square lattice. For obvious reasons (less computing power and efficient algorithm), the Hoshen Kopelman Algorithm has been adopted for finding the percolating (or the largest) cluster.

2 Algorithm Description

The Hoshen-Kopelman algorithm was used (implemented in the code `hoshen_kopelman.cpp`) for finding out the largest cluster in the square lattice as follows :

- As an extension of the previously developed code for implementing Hoshen-Kopelman Algorithm, additional user-defined functions were added in the code.
- Initially, the largest cluster (`largest_cluster(int)`) was found by identifying the cluster with maximum number of sites.
- Additionally, the sites referencing this largest cluster number were identified and flagged with 1, whereas the remaining sites were flagged as 0 (`flag_find(int, int*, int)`).

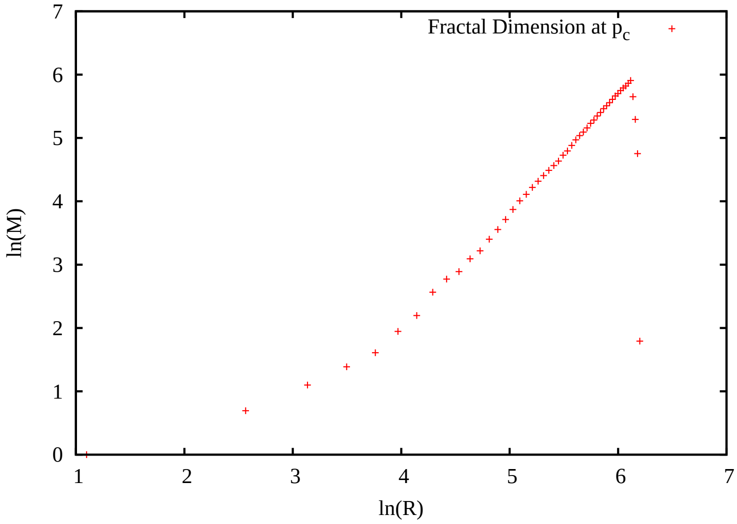
Once the largest cluster along with its flagged cluster sites were identified, the sandbox algorithm was implemented (in the code `sandbox.cpp`) to compute the fractal dimension at critical occupation probability.

- According to this algorithm, a square box lying close to the center of the system with an occupied site was identified (`center_occ_pos(int[], int&, int&, int)`).
- Then, the mass (or the number of sites) belonging to the largest cluster (which were flagged as 1) were counted (`mass_radius(int, int, int, int[], int[], int])`), such that they lie within a square box of increasing size (or radius R).
- The size of the square box was varied till it enclosed all lattice sites in the square box (that is $R = L$).
- The mass and corresponding radius of the box were plotted in a graph, and the slope of the curve was determined to obtain the fractal dimension of the cluster.

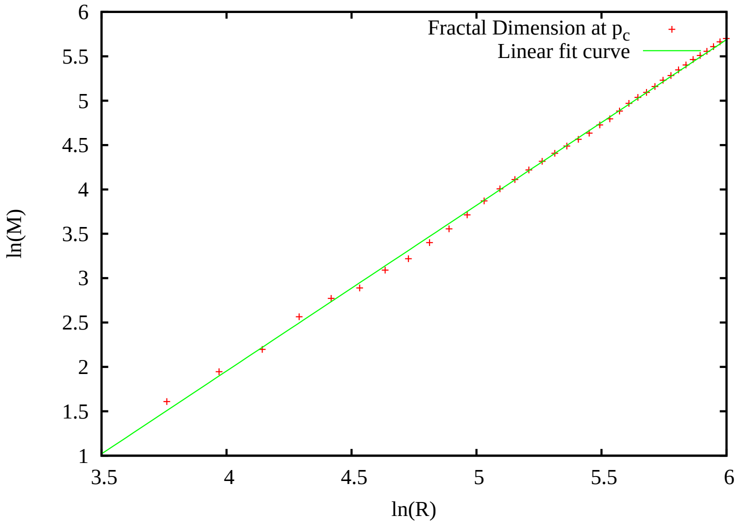
For this purpose, a square lattice of 500×500 sites with 100 repetitive samples were considered for averaging the number of sites, in order to determine the fractal dimension of the cluster.

3 Results and Discussions

Since the fractal dimension can be computed from the slope of the graph of number of occupied sites (of the largest cluster) vs the radius (or size) of the sand box, a graph has been plotted for the same (as seen in Figure 1a). Due to finite size effects, the deviation from linearity can be observed at large radii of the square box. Hence, only the linear portion of the graph has been considered for obtaining the slope, which has been shown in Figure 1b. Fitting a linear line determines a fractal dimension of 1.87, quite close to the actual one of 1.8958 [1].



(a) Graph of Number of occupied sites (of the largest cluster) vs the size of the sand box enclosing the lattice



(b) Fitting a linear curve to determine the fractal dimension of the percolating cluster

Figure 1: Computing the fractal dimension at p_c

References

[1] Hermann, Hans J., *Lecture notes on introduction to computational physics*, Institute for Building Materials, ETH Zürich, 2017.