A **logistic partially linear regression model** is a hybrid between two types of models:

1. **Logistic regression** for modeling binary outcomes.
2. **Partially linear regression** to model part of the relationship as linear while allowing flexibility for another part to be nonlinear.

The general idea is to split the predictor variables X into two sets:

- One part that has a linear relationship with the outcome (e.g., Z).
- Another part that can have a nonlinear relationship with the outcome (e.g., X).

General form of the logistic partially linear regression model:

$$\log\left(\frac{P(y=1|X,Z)}{1-P(y=1|X,Z)}\right) = Z^T\beta + g(X)$$

where:

- $P(y=1|X,Z)$ is the probability of the binary outcome $y=1$ given the predictor variables X and Z.
- Z is a vector of predictors with a **linear** relationship with the log-odds of y.
- β is a vector of coefficients for the linear part Z.
- $g(X)$ is an unspecified, potentially nonlinear function of X, which can be estimated using nonparametric methods (e.g., splines, kernel methods, or generalized additive models).

Key features:

- **Linear term** ($Z^T\beta$): The model assumes a linear relationship between the covariates Z and the log-odds of the outcome.
- **Nonlinear term** ($g(X)$): The function $g(X)$ allows the model to capture complex or unknown relationships between the covariates X and the log-odds of the outcome, which are not restricted to being linear.

Estimation:

- The linear part is typically estimated using maximum likelihood methods.
- The nonlinear part $g(X)$ is often estimated using nonparametric or semi-parametric techniques such as splines or local smoothing.

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This model is particularly useful when you believe part of the relationship between predictors and the outcome can be linear, but another part is more complex and requires a flexible, nonparametric approach.