Hilbert Spaces as Generalized Conceptual Spaces
in Information Dynamics of Thinking Theory

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1 Conceptual Spaces

2 Hilbert Spaces

Hilbert spaces are the means by which the ordinary experience of Euclidian concepts can be extended meaningfully into the idealized constructions of more complex math.

2.1 Complete Inner Product Space

- **Vector Space:** Dimensions and rules for combining vectors
- **Norm:** Measure of the size of a vector
- **Inner Product:** Defines orthogonality, projections, and angles of vectors
- **Completeness:** Space is big enough to include norm of converging sequences

3 Abstraction Process

1. Define $\langle \cdot, \cdot \rangle$ on $\mathcal{H}$
2. Generate $\{\varphi_n\}_{n=1}^{\infty}$ through Orthogonalization Process
3. Perform Linear Regression to determine trajectory function
4. Transform to $L^2(\Omega)$ with $\sum_{n=1}^{\infty} \langle f, \varphi_n \rangle \widetilde{\varphi_n}$ with φ_n from $\mathcal{H}$ and $\widetilde{\varphi_n}$ from $L^2(\Omega)$
5. Perform Fourier Transform
6. Repeat

4 Trajectories

5 Implementation Specification

$V_i^{\alpha,\delta} = \langle S^{\alpha,\delta}, \langle \cdot, \cdot \rangle_i \rangle$

$V_i^{\alpha,\delta}$: Representation of applying inner product i to vector set S

$S_i^{\alpha,\delta}$: Set of vectors in this dimension at this level of abstraction

$f \in S$: Function vector representing the spectrum of one segment. The spectrum of one segment refers to the fourier transform of a trajectory in the inferior abstraction layer.

Fourier Tranform: $\hat{f}(\xi) = \int_{-\infty}^{\infty} f(x) e^{-2\pi i x \xi} dx$

where $f(x)$ is the trajectory function of the inferior segment.

$\langle \cdot, \cdot \rangle_i$: inner product i to apply to set of vectors