MULTIVARIATE ANALYSIS OF VARIANCE

Seth Howells
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OVERVIEW

Multivariate analysis of variance (MANOVA) is an extension of analysis of variance (ANOVA), where MANOVA accommodates more than one dependent metric variable. In contrast to many multivariate techniques, MANOVA incorporates nonmetric (or, categorical) data in the independent variables. In many multivariate techniques, the independent variables consist of metric data that impacts the metric or nonmetric dependent variable. For example, multiple regression requires two or more metric independent and a metric dependent; multiple discriminant analysis requires two or more metric independent variables and a single nonmetric dependent variable.

	metric	*metric*
Multiple Regression:	$\hat{y} = b_0 + b_1x_1 + b_2x_2 +_{\ldots} b_px_p + \varepsilon$	
	nonmetric	*metric*
Multiple Discriminant Analysis	$F = \beta_0 + \beta_1X_1 + \beta_2X_2 + \ldots + \beta_pX_p + \varepsilon$	
	metric	*nonmetric*
MANOVA	$Y_1 + Y_2 + \ldots Y_N = X_1 + X_2 + \ldots X_N$	

In contradistinction at two critical levels, MANOVA stands out from the rest. First, the multivariate aspect is not concerned with the number of independent variables, but rather the number of dependent variables. Second, nonmetric independent variables impact metric dependent variables. Because of those two criteria, MANOVA computationally stands out among other multivariate dependence methods.

ANOVA vs MANOVA

While MANOVA stands out from all the other multivariate techniques due to the computational structure, it also has the same structure as its univariate predecessor, ANOVA (analysis of variance). Both analyses are concerned with differences between groups, which is apparent in their structure. That is, differences in the nonmetric independent variables directly impact the metric values in the dependent variables.

One example of groups that MANOVA would assess would be: customer satisfaction, likely to recommend, and likely to purchase. These groups are particularly suited for marketing and purchasing decision for a company. A key difference between ANOVA and MANOVA is in the tests: ANOVA uses a t-test while MANOVA uses **Hotelling's T^2**, assesses the statistical significance of the difference on the means of two or more variables between groups.

MANOVA DECISION PROCESS

MULTIVARIATE ANALYSIS OF VARIANCE

1. Objectives of MANOVA
    a. Analyze a dependent relationship represented as the differences in a set of dependent measure across a series of groups formed by one or more categorical independent measures.
2. Research Design of MANOVA
    a. Researchers must consider the following in MANOVA design:
    b. Sample size requirements
    c. Factorial designs
        - Selecting Treatments: types and number, interaction effects
    d. Covariates – ANCOVA and MANCOVA
3. Assumptions in MANOVA
    a. Assumptions are to be met in order to properly conduct multivariate analysis of variance. Assumptions for MANOVA are below:
    b. Observations must be independent
    c. Variance-covariance matrices must be equal for all treatment groups
    d. Dependent variables must have a multivariate normal distribution
        - Multivariate normal distribution is often hard to assess
        - Univariate normality does not guarantee multivariate normality
        - Univariate normality must is required, and thus multivariate normality is inconsequential.
4. Estimating the MANOVA model and Assessing Overall Fit
    a. Statistical tests for MANOVA:
    b. **Roy's greatest characteristic root** – testing the null hypothesis (group means are equal, no statistically significant difference)
    c. **Wilk's Lambda** – testing the null hypothesis, also referred to as the maximum likelihood criterion or U statistic.
    d. **Pillai's criterion** – similar to Wilk's Lambda, test for multivariate differences
    e. **Hotelling's trace** – see *ANOVA vs MANOVA*
5. Interpreting the MANOVA variate
    a. Effects of covariates
    b. Assessment regarding dependent variables and their differences across the groups of each treatment
        - Statistical significance
        - Ordinal interactions
        - Disordinal interactions
    c. Identifying whether the groups differ on a single dependent variable or the entire dependent variate.
        - Post Hoc methods: Scheffe, Tukey's (HSD), Duncan's multiple range, Newman-Kuels test
6. Validation of the results
    a. Replication
    b. Covariates
    c. Assessing causation