CSCI 360 – Project #1

Stephen Tsung-Han Sher (7555500940)

Theoretical Part: A*

Let $A*_1$ break ties by smallest g values first

Let $A*_2$ break ties by the largest g values first

Given an instance of a graph G we want to run $A*$ on, the goal state $s(goal)$ will have a heuristic of 0 and a g -value equaling the distance from the start state $s(start)$ to $s(goal)$. Therefore:

$$\begin{aligned} f(goal) &= h(goal) + g(goal) \\ &= 0 + g(goal) \\ &= g(goal) \end{aligned}$$

Because in general $A*$, we will be expanding smallest f -values first, every node with an f -value less than the f -value of the goal state will always be expanded. Similarly, every node with an f -value greater than the f -value of the goal state will never be expanded (since we will reach the goal state and terminate).

Therefore, for both $A*_1$ and $A*_2$:

$$\begin{aligned} Always \quad Expand &: \{\exists s \in G \mid f(s) < g(goal)\} \\ Never \quad Expand &: \{\exists s \in G \mid f(s) > g(goal)\} \end{aligned}$$

Now consider the following 4×4 grid with the start cell at the upper-left, the goal cell at the lower right, and each cell's heuristic being the true distance from the goal cell:

	A	B	C	D
1	S:6	5	4	3
2	5	4	3	2
3	4	3	2	1
4	3	2	1	G:0

$A*_1$ will produce the following expansions:

$$< Column >< Row >: \{h(s), g(s), f(s)\}$$
$$A1:\{6,0,6\} \to B1:\{5,1,6\} \to A2:\{5,1,6\} \to C1:\{4,2,6\} \to B2:\{4,2,6\} \to A3:\{4,2,6\} \to$$
$$D1:\{3,3,6\} \to C2:\{3,3,6\} \to B3:\{3,3,6\} \to A4:\{3,3,6\} \to D2:\{2,4,6\} \to C3:\{2,4,6\} \to$$
$$B4:\{2,4,6\} \to D3:\{1,5,6\} \to C4:\{1,5,6\} -> D4:\{0,6,6\}$$

$A*_2$ will produce the following expansions:

$$< Column >< Row >: \{h(s), g(s), f(s)\}$$
$$A1:\{6,0,6\} \rightarrow B1:\{5,1,6\} \rightarrow C1:\{4,2,6\} \rightarrow D1:\{3,3,6\} \rightarrow D2:\{2,4,6\} \rightarrow D3:\{1,5,6\} \rightarrow D4:\{0,6,6\}$$

For this specific search problem, $A*_2$ expands fewer nodes than $A*_1$. This is because when breaking ties, selecting the node with the greater g -value has the effect of expanding the node that has progressed the furthest from $s(start)$. In this specific problem since all the f -values of every single cell is 6, we want to make progress in terms of moving towards the goal state, and hence selecting the larger g -value achieves this result.

For general search problems, both $A*_1$ and $A*_2$ ensures smallest f-value first, which ensures progress towards our goal state from the aggregation of both the h and g -values. However, when faced with ties, $A*_2$ will expands towards the goal state in terms of making progress in distance travelled, whereas $A*_1$ will expand the node furthest from the goal state, making less progress towards the goal state. This results in $A*_2$ always expanding the same node as $A*_1$ or expanding nodes that are closer to the goal state compared to the nodes $A*_1$ expands due to the heuristic being consistent.