

## BME 355 Assignment 1: Musculoskeletal Modelling

Due 11:59pm March 6, 2020

This assignment should be completed in groups of 2-3. Submit a PDF document (not a Word document) that includes the names of the group members, answers to questions, and requested plots. Also submit your Python code in a zip file.

The model equations are from the Hill-type model that was discussed in class (Millard et al., 2013). You will also take data from another paper (Winters et al., 2011). You are advised to use the partially completed code on Learn.

### References

Millard et al. (2013) Flexing computational muscle: modeling and simulation of musculotendon dynamics, *Journal of Biomechanical Engineering* 135.2: 021005.

Winters, T. M., Takahashi, M., Lieber, R. L., & Ward, S. R. (2011). Whole muscle length-tension relationships are accurately modeled as scaled sarcomeres in rabbit hindlimb muscles. *Journal of Biomechanics*, 44(1), 109-115.

### Instructions

1. Implement a simplified version of the damped muscle model from Millard et al., (2013) with damping coefficient  $\beta = 0.1$ . Assume a pennation angle of zero. Do not implement the excitation-activation dynamics. Use the following functions for normalized force-length of the SE and PE (slightly different than Millard et al.):

$$f^T(\tilde{l}^T) = \begin{cases} 0, & \tilde{l}^T < \tilde{l}_S^T \\ 10(\tilde{l}^T - \tilde{l}_S^T) + 240(\tilde{l}^T - \tilde{l}_S^T)^2, & \tilde{l}^T \geq \tilde{l}_S^T \end{cases}$$
$$f^{PE}(\tilde{l}^M) = \begin{cases} 0, & \tilde{l}^M < \tilde{l}_S^{PE} \\ 3(\tilde{l}^M - \tilde{l}_S^{PE})^2 / (.6 + \tilde{l}^M - \tilde{l}_S^{PE}), & \tilde{l}^M \geq \tilde{l}_S^{PE} \end{cases}$$

Where  $\tilde{l}_S^T = \tilde{l}_S^{PE} = 1$  are slack lengths of the SE and PE. For the CE force-velocity curve, use the implementation in the provided code, which is a regression model based on data in Millard et al. For the CE force-length curve, use your own regression model with Gaussian basis functions. Use data from Figure 2C of Winters et al. (2011) (extract the data with WebPlotDigitizer). These data points are not normalized, so you will have to normalize them in the same way Millard et al. did. You should end up with fairly similar curves to Millard et al.

Plot all four curves, using the method `plot_curves()` on the `HillTypeMuscle` class.

2. Write code to find normalized velocity  $\tilde{v}^M$  given activation  $a$ ,  $\tilde{l}^M$ , and  $\tilde{l}^T$ . Use `scipy.optimize.fsolve` to find the velocity numerically. Report the velocity for  $a = 1$ ,  $\tilde{l}^M = 1$ ,  $\tilde{l}^T = 1.01$ .

3. Create a HillTypeMuscle with maximum isometric force 100, resting muscle length .3, and resting tendon length 0.1 (units are N and m). Simulate an isometric contraction with the muscle-tendon unit at its resting length (i.e. the length it would have if  $\tilde{l}^M = \tilde{l}^T = 1$ ). Simulate for 2s, with initial state  $\tilde{l}^M = 1$  and activation  $a = 0$  for  $t < 0.5$  and  $a = 1$  thereafter. Plot the length of the contractile element and the force produced by the muscle.
4. Implement the postural stability model from class, with soleus and tibialis anterior muscles. Implement the dynamic equations from scratch (do not use PyBullet for rigid-body physics). Set non-normalized resting muscle-tendon lengths to be the muscle lengths when the body is in anatomical position. Use the following parameter values:  $f_0 M = 16000\text{N}$  for soleus and  $2000\text{N}$  for tibialis anterior; body segment  $m = 75\text{kg}$ ,  $I_{ankle} = 90\text{kgm}^2$ , and  $l_{COM} = 1\text{m}$ . Other parameters should be as discussed in class. Simulate the model with initial body angle  $\theta = \pi/2$ , constant tibialis anterior activation of 0.4, and constant soleus activation of .05. Simulate the model for 5s. Plot  $\theta$ , and torques produced by each muscle and by gravity. Explain what happened in the simulation. Is it physically realistic?
5. Devise, implement, and discuss a simple control law for the muscles that stabilizes  $\theta$ . The control law can set the activation of each muscle as a function of the system's state variables. Simulate the controlled ankle model for 10s, and plot  $\theta$  and the torques produced by each muscle and by gravity.