Assignment 1 Question 1

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d. Investigate if $\alpha\left(\mathcal{P}\right)$ is replication invariant, replication equivariant, or neither.

$$\alpha\left(\mathcal{P}\right) = \frac{\frac{1}{N}\sum_{u\epsilon\mathcal{P}}\left(y_u - \bar{y}\right)^4}{\sigma^4} - 3$$

The attribute $\alpha\left(\mathcal{P}\right)$ is replication invariant if

$$\alpha\left(\mathcal{P}^k\right) = \alpha\left(\mathcal{P}\right)$$

and is replication equivariant if

$$\alpha\left(\mathcal{P}^k\right) = k \cdot \alpha\left(\mathcal{P}\right)$$

To begin,

$$\alpha\left(\mathcal{P}^k\right) = \frac{\frac{1}{N}\sum_{u\epsilon\mathcal{P}^k}\left(y_u - \bar{y}\right)^4}{[\frac{1}{N}\sum_{u\epsilon\mathcal{P}^k}\left(y_u - \bar{y}\right)^2]^2} - 3$$

$$\alpha\left(\mathcal{P}^k\right) = \frac{\frac{1}{kN}\sum_{u\epsilon\mathcal{P}}k\left(y_u - \bar{y}\right)^4}{[\frac{1}{kN}\sum_{u\epsilon\mathcal{P}}k\left(y_u - \bar{y}\right)^2]^2} - 3$$

$$\alpha\left(\mathcal{P}^k\right) = \frac{\frac{k}{kN}\sum_{u\epsilon\mathcal{P}}\left(y_u - \bar{y}\right)^4}{[\frac{k}{kN}\sum_{u\epsilon\mathcal{P}}\left(y_u - \bar{y}\right)^2]^2} - 3$$

$$\alpha\left(\mathcal{P}^k\right) = \frac{\frac{1}{N}\sum_{u\epsilon\mathcal{P}}\left(y_u - \bar{y}\right)^4}{[\frac{1}{N}\sum_{u\epsilon\mathcal{P}}\left(y_u - \bar{y}\right)^2]^2} - 3$$

$$\alpha\left(\mathcal{P}^k\right) = \alpha\left(\mathcal{P}\right)$$

Therefore, the excess kurtosis attribute is replication invariant.