Assignment 1 Question 1

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b. Investigate if $\alpha(\mathcal{P})$ is scale invariant, scale equivariant, or neither.

$$\alpha(\mathcal{P}) = \frac{\frac{1}{N}\sum_{u\epsilon\mathcal{P}}(y_u - \bar{y})^4}{\sigma^4} - 3$$

For $m > 0$, attribute $\alpha(\mathcal{P})$ is scale invariant if

$$\alpha(my_1, ..., my_N) = \alpha(y_1, ..., y_N)$$

and is scale equivariant if

$$\alpha(my_1, ..., my_N) = m\alpha(y_1, ..., y_N)$$

To begin,

$$y_u^{'} = my_u$$

and

$$\bar{y}^{'} = m\bar{y}$$

Thus,

$$\alpha(my_1, ..., my_N) = \frac{\frac{1}{N}\sum_{u\epsilon\mathcal{P}}(my_u - m\bar{y})^4}{[\frac{1}{N}\sum_{u\epsilon\mathcal{P}}(my_u - m\bar{y})^2]^2} - 3$$

$$\alpha(my_1, ..., my_N) = \frac{\frac{m^4}{N}\sum_{u\epsilon\mathcal{P}}(y_u - \bar{y})^4}{[\frac{m^2}{N}\sum_{u\epsilon\mathcal{P}}(y_u - \bar{y})^2]^2} - 3$$

$$\alpha(my_1, ..., my_N) = \frac{m^4\frac{1}{N}\sum_{u\epsilon\mathcal{P}}(y_u - \bar{y})^4}{m^4[\frac{1}{N}\sum_{u\epsilon\mathcal{P}}(y_u - \bar{y})^2]^2} - 3$$

$$\alpha(y_1 + b, ..., y_N + b) = \frac{\frac{1}{N}\sum_{u\epsilon\mathcal{P}}(y_u - \bar{y})^4}{[\frac{1}{N}\sum_{u\epsilon\mathcal{P}}(y_u - \bar{y})^2]^2} - 3$$

$$\alpha(y_1 + b, ..., y_N + b) = \frac{\frac{1}{N}\sum_{u\epsilon\mathcal{P}}(y_u - \bar{y})^4}{\sigma^4} - 3$$

$$\alpha(my_1, ..., my_N) = \alpha(y_1, ..., y_N)$$

Therefore, the excess kurtosis attribute is scale invariant.