Assignment 1 Question 1

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a. Investigate if $\alpha\left(\mathcal{P}\right)$ is location invariant, location equivariant, or neither.

$$\alpha\left(\mathcal{P}\right) = \frac{\frac{1}{N}\sum_{u\epsilon\mathcal{P}}\left(y_u - \bar{y}\right)^4}{\sigma^4} - 3$$

For $b \in \mathbb{R}$, attribute $\alpha\left(\mathcal{P}\right)$ is location invariant if

$$\alpha\left(y_1 + b, ..., y_N + b\right) = \alpha\left(y_1, ..., y_N\right)$$

and is location equivariant if

$$\alpha\left(y_1 + b, ..., y_N + b\right) = \alpha\left(y_1, ..., y_N\right) + b$$

To begin,

$$y_u^{'} = y_u + b$$

and

$$\bar{y}^{'} = \bar{y} + b$$

Thus,

$$\alpha\left(y_1 + b, ..., y_N + b\right) = \frac{\frac{1}{N}\sum_{u\epsilon\mathcal{P}}\left(y_u + b - \bar{y} - b\right)^4}{[\frac{1}{N}\sum_{u\epsilon\mathcal{P}}\left(y_u + b - \bar{y} - b\right)^2]^2} - 3$$

$$\alpha\left(y_1 + b, ..., y_N + b\right) = \frac{\frac{1}{N}\sum_{u\epsilon\mathcal{P}}\left(y_u - \bar{y}\right)^4}{[\frac{1}{N}\sum_{u\epsilon\mathcal{P}}\left(y_u - \bar{y}\right)^2]^2} - 3$$

$$\alpha\left(y_1 + b, ..., y_N + b\right) = \frac{\frac{1}{N}\sum_{u\epsilon\mathcal{P}}\left(y_u - \bar{y}\right)^4}{\sigma^4} - 3$$

$$\alpha\left(y_1 + b, ..., y_N + b\right) = \alpha\left(y_1, ..., y_N\right)$$

Therefore, the excess kurtosis attribute is location invariant.