Assignment 1 Question 1

Sheen Thusoo

c. Investigate if $\alpha\left(\mathcal{P}\right)$ is location-scale invariant, location-scale equivariant, or neither.

For $m > 0$ and $b \in \mathbb{R}$, attribute $\alpha\left(\mathcal{P}\right)$ is location-scale invariant if

$$\alpha\left(my_1 + b, ..., my_N + b\right) = \alpha\left(y_1, ..., y_N\right)$$

and is location-scale equivariant if

$$\alpha\left(my_1 + b, ..., my_N + b\right) = m\alpha\left(y_1, ..., y_N\right) + b$$

To begin,

$$y_u^{'} = my_u + b$$

and

$$\bar{y}^{'} = m\bar{y} + b$$

Thus,

$$\alpha\left(my_1 + b, ..., my_N + b\right) = \frac{\frac{1}{N}\sum_{u\epsilon\mathcal{P}}\left(my_u + b - m\bar{y} + b\right)^4}{[\frac{1}{N}\sum_{u\epsilon\mathcal{P}}\left(my_u + b - m\bar{y} + b\right)^2]^2} - 3$$

$$\alpha\left(my_1 + b, ..., my_N + b\right) = \frac{\frac{m^4}{N}\sum_{u\epsilon\mathcal{P}}\left(y_u - \bar{y}\right)^4}{[\frac{m^2}{N}\sum_{u\epsilon\mathcal{P}}\left(y_u - \bar{y}\right)^2]^2} - 3$$

$$\alpha\left(my_1 + b, ..., my_N + b\right) = \frac{m^4\frac{1}{N}\sum_{u\epsilon\mathcal{P}}\left(y_u - \bar{y}\right)^4}{m^4[\frac{1}{N}\sum_{u\epsilon\mathcal{P}}\left(y_u - \bar{y}\right)^2]^2} - 3$$

$$\alpha\left(my_1 + b, ..., my_N + b\right) = \frac{\frac{1}{N}\sum_{u\epsilon\mathcal{P}}\left(y_u - \bar{y}\right)^4}{[\frac{1}{N}\sum_{u\epsilon\mathcal{P}}\left(y_u - \bar{y}\right)^2]^2} - 3$$

$$\alpha\left(my_1 + b, ..., my_N + b\right) = \frac{\frac{1}{N}\sum_{u\epsilon\mathcal{P}}\left(y_u - \bar{y}\right)^4}{\sigma^4} - 3$$

$$\alpha\left(my_1 + b, ..., my_N + b\right) = \alpha\left(y_1, ..., y_N\right)$$

Therefore, the excess kurtosis attribute is location-scale invariant.