Assignment 1 Question 1

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e. [3 marks] If we replace σ^4 with $\sigma_*^4 = [\frac{1}{N-1}\sum_{u\epsilon\mathcal{P}^k}(y_u - \bar{y})^2]^2$ in the definition of $\alpha(\mathcal{P})$, does your answer to parts (b) and (d) change? Provide the details of your response.

After changing σ^4 to $\sigma_*^4 = [\frac{1}{N-1}\sum_{u\epsilon\mathcal{P}^k}(y_u - \bar{y})^2]^2$ in the definition of $\alpha(\mathcal{P})$, the attribute becomes

$$\alpha(y_1, ..., y_N) = \frac{\frac{1}{N}\sum_{u\epsilon\mathcal{P}}(y_u - \bar{y})^4}{[\frac{1}{N-1}\sum_{u\epsilon\mathcal{P}}(y_u - \bar{y})^2]^2} - 3$$

First, we investigate if this new attribute is scale invariant, scale equivariant, or neither.

$$\alpha(my_1, ..., my_N) = \frac{\frac{1}{N}\sum_{u\epsilon\mathcal{P}}(my_u - m\bar{y})^4}{[\frac{1}{N-1}\sum_{u\epsilon\mathcal{P}}(my_u - m\bar{y})^2]^2} - 3$$

$$\alpha(my_1, ..., my_N) = \frac{\frac{1}{N}\sum_{u\epsilon\mathcal{P}}(my_u - m\bar{y})^4}{[\frac{1}{N-1}\sum_{u\epsilon\mathcal{P}}(my_u - m\bar{y})^2]^2} - 3$$

$$\alpha(my_1, ..., my_N) = \frac{\frac{m^4}{N}\sum_{u\epsilon\mathcal{P}}(y_u - \bar{y})^4}{[\frac{m^2}{N-1}\sum_{u\epsilon\mathcal{P}}(y_u - \bar{y})^2]^2} - 3$$

$$\alpha(my_1, ..., my_N) = \frac{m^4\frac{1}{N}\sum_{u\epsilon\mathcal{P}}(y_u - \bar{y})^4}{m^4[\frac{1}{N-1}\sum_{u\epsilon\mathcal{P}}(y_u - \bar{y})^2]^2} - 3$$

$$\alpha(my_1, ..., my_N) = \frac{\frac{1}{N}\sum_{u\epsilon\mathcal{P}}(y_u - \bar{y})^4}{[\frac{1}{N-1}\sum_{u\epsilon\mathcal{P}}(y_u - \bar{y})^2]^2} - 3$$

$$\alpha(my_1, ..., my_N) = \alpha(y_1, ..., y_N)$$

Therefore, this new attribute with σ_*^4 is still scale invariant and hasn't changed from the answer in part b). This is because the m constant still cancels out in the numerator and denominator.

Next, we investigate if this new attribute is replication invariant, scale equivariant, or neither.

$$\alpha(\mathcal{P}^k) = \frac{\frac{1}{N}\sum_{u\epsilon\mathcal{P}^k}(y_u - \bar{y})^4}{[\frac{1}{N-1}\sum_{u\epsilon\mathcal{P}^k}(y_u - \bar{y})^2]^2} - 3$$

$$\alpha(\mathcal{P}^k) = \frac{\frac{1}{kN}\sum_{u\epsilon\mathcal{P}}k(y_u - \bar{y})^4}{[\frac{1}{k(N-1)}\sum_{u\epsilon\mathcal{P}}k(y_u - \bar{y})^2]^2} - 3$$

$$\alpha\left(\mathcal{P}^k\right) = \frac{\frac{1}{kN}\sum_{u\epsilon\mathcal{P}} k\left(y_u - \bar{y}\right)^4}{[\frac{1}{kN-1}\sum_{u\epsilon\mathcal{P}} k\left(y_u - \bar{y}\right)^2]^2} - 3$$

$$\alpha\left(\mathcal{P}^k\right) = \frac{\frac{k}{kN}\sum_{u\epsilon\mathcal{P}} \left(y_u - \bar{y}\right)^4}{[\frac{k}{kN-1}\sum_{u\epsilon\mathcal{P}} \left(y_u - \bar{y}\right)^2]^2} - 3$$

$$\alpha\left(\mathcal{P}^k\right) = \frac{\frac{1}{N}\sum_{u\epsilon\mathcal{P}} \left(y_u - \bar{y}\right)^4}{[\frac{k}{kN-1}\sum_{u\epsilon\mathcal{P}} \left(y_u - \bar{y}\right)^2]^2} - 3$$

This equation does not simplify further. Therefore, this new attribute with σ_*^4 is neither replication invariant nor equivariant and has changed from the answer in part d) which was replication invariant. This is because we have an extra k coefficient on the N term in the denominator and a k constant in the numerator. These cannot be cancelled out.