Assignment 2 Question 2

Sheen Thusoo

Part a)

$$L(\boldsymbol{\theta}; y_1, ..., y_n) = f_{\boldsymbol{\theta}}(y_1) \times ... \times f_{\boldsymbol{\theta}}(y_n)$$

$$\ell(\boldsymbol{\theta}; y_1, ..., y_n) = \log\left(L(\boldsymbol{\theta}; y_1, ..., y_n)\right)$$
$$\ell(\boldsymbol{\theta}; y_1, ..., y_n) = \log(f_{\boldsymbol{\theta}}(y_1) \times ... \times f_{\boldsymbol{\theta}}(y_n))$$

$$\ell(\boldsymbol{\theta}; y_1, ..., y_n) = \log(f_{\boldsymbol{\theta}}(y_1)) + ... + log(f_{\boldsymbol{\theta}}(y_n))$$

$$\ell(\boldsymbol{\theta}; y_1, ..., y_n) = \sum_{i \in P} \log(f_{\boldsymbol{\theta}}(y_i))$$

$$f(y) = \frac{\beta^\alpha y^{\alpha-1}}{\Gamma(\alpha)} \exp(-y\beta)$$

$$\ell(\boldsymbol{\theta}; y_1, ..., y_n) = \sum_{i \in P} \log\left(\frac{\beta^\alpha y_i^{\alpha-1}}{\Gamma(\alpha)} \exp(-y_i\beta)\right)$$

Therefore the log-likelihood function of the parameter vector $\theta = (\alpha, \beta)$ is,

$$\ell(\boldsymbol{\theta}; y_1, ..., y_n) = \sum_{i \in P} \alpha \log(\beta) + (\alpha - 1)\log(y_i) - \log(\Gamma(\alpha)) - y_i\beta$$