

TDT4171-assignment3-Stian-Mogen

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Exercise 1 - Umbrella World

- What is the set of unobserved variable(s) for a given time-slice t (denoted X_t in the book)?

The unobserved set X_t contains R_t . which true or false depending on if it rains on timeslice t .

- What is the set of observable variable(s) for a given time-slice t (denoted E_t in the book)?

The observable variables for E_t is U_t which is true or false depending on if the director brings an umbrella on time-slice t .

- Present the dynamic model $P(X_t | X_{t-1})$ and the observation model $P(E_t | X_t)$ as matrices.

$$\text{Dynamic / transition model} = T = P(X_t | X_{t-1}) = \begin{bmatrix} 0.7 & 0.3 \\ 0.3 & 0.7 \end{bmatrix}$$

$$\text{Observation / Sensor model} = O = P(E_t | X_t) = \begin{bmatrix} 0.9 & 0.1 \\ 0.8 & 0.2 \end{bmatrix}$$

- Which assumptions are encoded in this model? Are the assumptions reasonable for this particular domain? (See 14.1 Time and Uncertainty on Page 479).

The Markov assumption, which states that the current state depends on only a finite fixed number of previous states, with the simplest Markov Process being the First-order Markov Process where the current state depends only on the previous state. Time-homogenous process for changes in world state is assumed, meaning that the rules themselves do not change over time. This implies that the conditional probability $P(R_t | R_{t-1})$ is the same for all t . The Sensor Markov Assumption says that any state should suffice for generating the sensor observed value.

The Markov Assumption is in my estimation a reasonable simplification given

a large enough conditional dependence. Time-homogenous appears to be too simple. The probabilities would change over time, for example one might be more likely to bring an umbrella in the autumn, rather than in the summer, anticipating a potential change in weather. The Sensor Markov Assumption is a necessary assumption for the situation to hold any merit. Failure to observe the umbrella may very well happen, but calculating for uncertainty in this regard to the observation is unreasonable.

Exercise 2 - Implementing Forward Operations

Results from evidence [True, True]

$$P(X_2 | E_{1:2}) = \begin{bmatrix} 0.883 \\ 0.117 \end{bmatrix}$$

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Running exercise with evidence: [True, True]
f(1:1) = [[0.81818182]
          [0.18181818]]
f(1:2) = [[0.88335704]
          [0.11664296]]
P(X_1 | e(1:2)) = [[0.88335704]
                   [0.11664296]]
P(X_2 | e(1:2)) = [[0.88335704]
                   [0.11664296]]
```

Results from evidence [True, True, False, True, True]

$$P(X_5 | E_{1:5}) = \begin{bmatrix} 0.867 \\ 0.133 \end{bmatrix}$$

```
Running exercise with evidence: [True, True, False, True, True]
f(1:1) = [[0.81818182]
          [0.18181818]]
f(1:2) = [[0.88335704]
          [0.11664296]]
f(1:3) = [[0.19066794]
          [0.80933206]]
f(1:4) = [[0.730794]
          [0.269206]]
f(1:5) = [[0.86733889]
          [0.13266111]]
P(X_1 | e(1:5)) = [[0.86733889]
                   [0.13266111]]
```

$$\begin{aligned}
P(X_2 | e(1:5)) &= \begin{bmatrix} 0.82041905 \\ 0.17958095 \end{bmatrix} \\
P(X_3 | e(1:5)) &= \begin{bmatrix} 0.30748358 \\ 0.69251642 \end{bmatrix} \\
P(X_4 | e(1:5)) &= \begin{bmatrix} 0.82041905 \\ 0.17958095 \end{bmatrix} \\
P(X_5 | e(1:5)) &= \begin{bmatrix} 0.86733889 \\ 0.13266111 \end{bmatrix}
\end{aligned}$$