

# Joint Super-Resolution and Optical Flow Estimation

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## Outline

- 1 Introduction
- 2 Energy Minimization Approach
- 3 Results
- 4 Future Work



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## Introduction

- Super-Resolution
  - Enhance resolution of images
  - Gain Additional Details

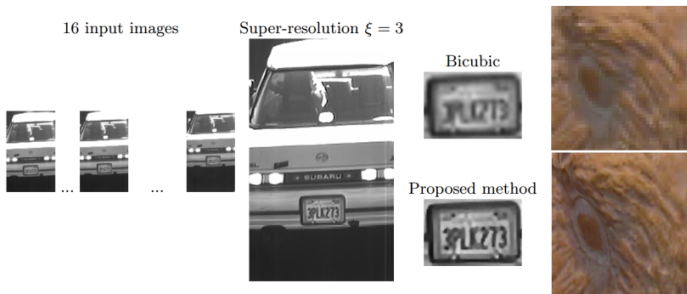
# Introduction

- Super-Resolution
  - Naive Upsampling



# Introduction

- Super-Resolution
  - Using multiple images



First image: Unger, Pock, Werlberger, Bischof 2010.

Second image: Goldlücke, Aubry, Kolev, Cremers 2014.

# Introduction

- Optical Flow Estimation
  - Estimate the movement in the images





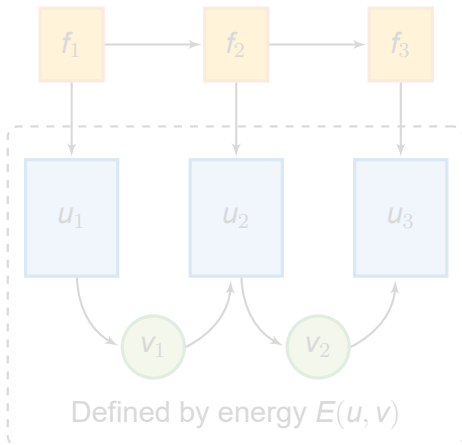
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# Energy Minimization Approach

Input Images:



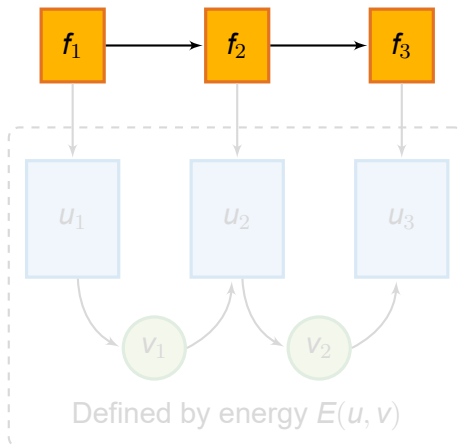
Alternating Optimization:

$$v^{k+1} \leftarrow \underset{v}{\operatorname{argmin}} E(u^k, v)$$

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# Energy Minimization Approach

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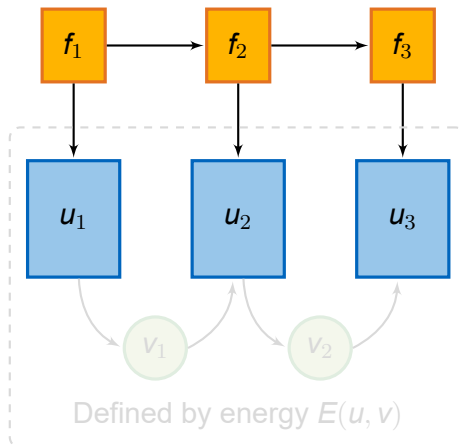
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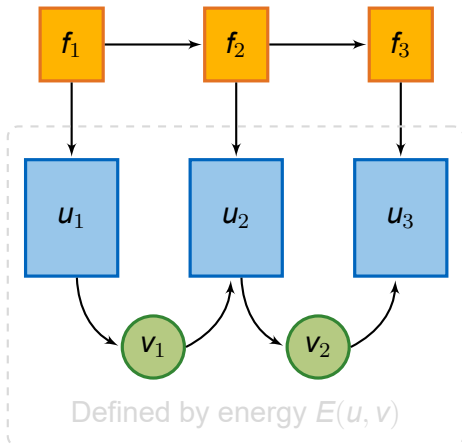
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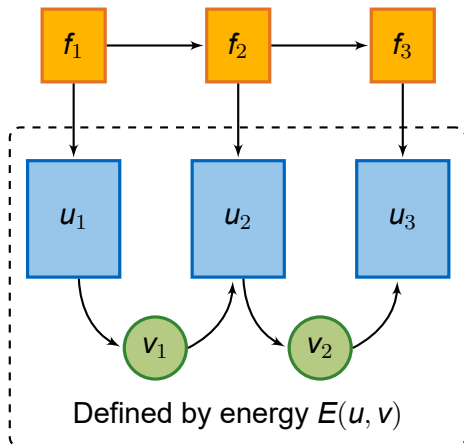
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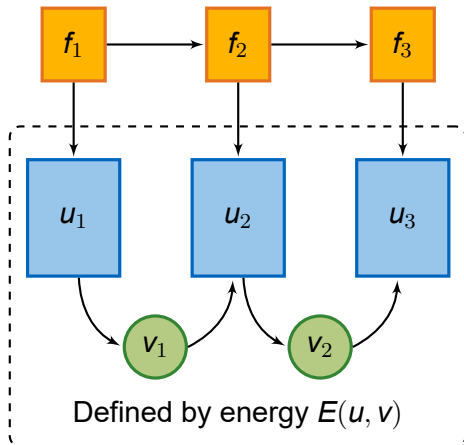
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## Flow Field Energy

- Optical Flow Constraint:  $u_i(x) \stackrel{!}{=} u_{i+1}(x + v_i(x))$   
→ minimize  $\|u_t - \nabla u^T \cdot v\|_1$
- Total Variation:  $TV(v)$

$$E_{flow}(v) = \gamma \|u_t - \nabla u^T \cdot v\|_1 + TV(v)$$

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# Super-Resolution Energy

- Super-Resolution

→ minimize:  $\|Au - f\|_1$

- Optical Flow Constraint

→ minimize:  $\|u_t - \nabla u^T \cdot v\|_1$

- Total Variation:  $TV(u)$

$$E_{super}(u) = \alpha \|Au - f\|_1 + \beta TV(u) + \gamma \|u_t - \nabla u^T \cdot v\|_1$$



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## Total Energy

$$E_{\text{flow}}(v) = \gamma \|u_t - \nabla u^T \cdot v\|_1 + TV(v)$$

+

$$E_{\text{super}}(u) = \alpha \|Au - f\|_1 + \beta TV(u) + \gamma \|u_t - \nabla u^T \cdot v\|_1$$

↓

$$E(u, v) = \underbrace{\alpha \|Au - f\|_1 + \beta TV(u)}_{\text{Super-Resolution}} + \underbrace{TV(v)}_{\text{Flow}} + \underbrace{\gamma \|u_t - \nabla u^T \cdot v\|_1}_{\text{Coupling}}$$

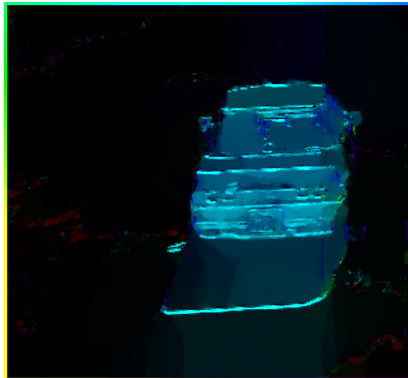


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## Results

- Optical Flow Estimation
  - Live Demo ...



# Results

## ■ Super-Resolution with Optical Flow Estimation

### ■ Input:



Input 1



Input 2

# Results

- Super-Resolution with Optical Flow Estimation
  - Result:



Super-Resolution 1



Input 1

## Results

- Super-Resolution with Optical Flow Estimation
  - Result:



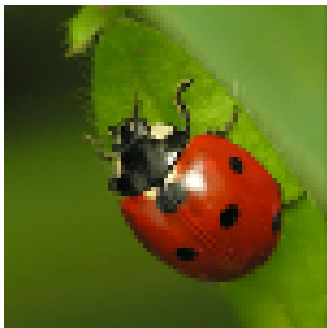
Super-Resolution 1



Input 1

# Results

- Super-Resolution with Optical Flow Estimation
  - Input:



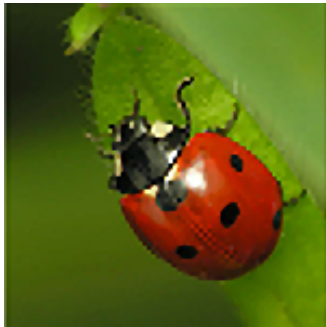
Downscaled



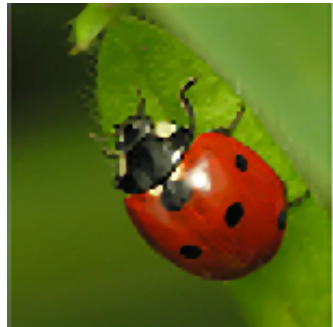
Original

# Results

- Super-Resolution with Optical Flow Estimation
  - Result:



2 Input Images



4 Input Images





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## Conclusion and Future Work

- More than two input images
- Arbitrary scaling
- Optical flow estimation for movements  $> 1$  Pixel



End

Thank you for your  
attention!