

Modeling & Simulation
PROBLEM SHEET 7

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$$\begin{aligned} (1) (a) (i) \quad E(t) &= x^2(t) + y^2(t) \\ E'(t) &= 2x(t)x'(t) + 2y(t)y'(t) \\ &= 2x(t)y(t) + 2y(t)[-x(t)] \\ &= 2x(t)y(t) - 2x(t)y(t) \\ &= 0 \end{aligned}$$

\Rightarrow ENERGY IS CONSERVED

$$(ii) \quad x^2 + y^2 = \text{CONSTANT}$$

This is the equation of a circle

(d) (i) Forward Euler method is unstable as the results are outside the unit circle. Whereas, in the other methods (Backward, semi-implicit, Runge-Kutta), the results are inside the unit circle. Hence, they are stable.

(ii) It can be inferred that the forward method does not conserve energy compared to the other methods. This can be seen with the increase in 'N'. As 'N' increases, results of forward Euler method increases. While in Backward, semi-implicit & Runge Kutta schemes, the results stay within the unit circle.