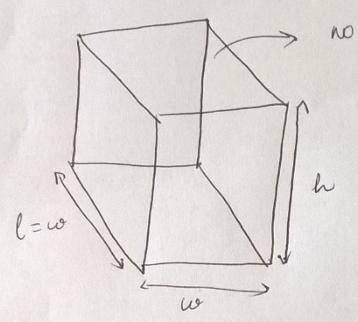
Name : Aishwarya Lakshminarayanan
Number: 6770324

PROBLEM_SHEET 11

1. Consider a rectangular box with square base as shown below :



no lid => no top surface are

~~Tot~~ Total surface area: $2wh + w^2 + 2wh$ (since no lid)
volume : $lwh = w^2h$

$\therefore$ Minimize : $2wh + w^2 + 2wh$ —— (1)
constraint = $w^2 h = 1000$ ml

$\Rightarrow h = \frac{1000}{w^2}$ —— (2)

sub (2) in (1)

$A(x) = 4 \times w \times \frac{1000}{w^2} + w^2$

$A(x) = \frac{-4000}{w} + w^2$

Since we want to minimize the material required, we ~~dif~~ take the first derivative of $A(x)$

$A'(x) = \frac{4000}{w^2} + 2w = 0 \quad = -4000 + 2w^3 = 0$

$w^3 = \sqrt[3]{2000} \simeq 12.599$

$\therefore h = 6.299$
$l = 12.599$

[Matlab program => Gradient descent method attached.]

2)

$$R(t) = R_0(1+\alpha Y(t))$$

$$Y(t) = T - T_0$$

$$Y'(t) = -\frac{1}{\tau} Y(t) + B I^2(t) \quad \text{— } 1$$

$$u(t) = I(t) R(t)$$

$$u(t) = I(t) R_0 [1+\alpha Y(t)]$$

$$I(t) = \frac{u(t)}{R_0[1+\alpha Y(t)]} \quad \text{— } 2$$

Put 2 in 1

$$Y'(t) = -\frac{1}{\tau} Y(t) + B \frac{u^2(t)}{R_0^2 [1+\alpha Y(t)]^2}$$

for sinusoidal voltage source

$$u(t) = \sin\left(\frac{2\pi t}{20}\right) \quad \text{for } t = 0 \to 101$$

$$f = \frac{1}{T} = \frac{1}{20}$$

for square voltage source

$$u(t) = \begin{cases} 0 & 0 \le t \le 1 \\ 1 & 1 \le t \le 11 \\ 0 & 11 \le t \le 21 \\ \vdots & \\ 0 & 91 \le t \le 101 \end{cases}$$

$$y'(t) = f(u(t), y(t))$$

FE scheme: $y_{n+1} = y_n + h y_n'$

$$I_n = \frac{u_n}{R_0(1+\alpha y_n)}$$

MATLAB PROGRAM:

```
clc
clear all
close all
To = 20;
Y1(1) = 0;
Y2(1) = 0;
Ro = 50;
alpha = 0.5;
beta = 200;
h = 1/100;
timeconstant = 5.6778324;

% define voltage sources
fnc = @(y,v) -(1/timeconstant)*y + beta*((v^2)/Ro*(1+alpha*y))^2);
                                            % Functions for passing
                                              values in loop
t = 0:1/100:101;
n1 = length(t);
 for i = 1:n1;
   v1(i) = sin(pi/10)*t(i));
   v2(i) = (1+ square(2*pi*0.05*(t(i)-1)))/2;
   if (i<n1)
        Y1(i+1) = Y1(i)+h* fnc(Y1(i), v1(i));
        Y2(i+1) = Y2(i) +h* fnc(Y2(i), V2(i));
   end
   I1(i) = v1(i)/(Ro * (1+alpha * Y1(i)));
   I2(i) = V2(i) /(Ro * (1+alpha* Y2(i)));

end
```

```
subplot (3,1,1)
plot (t, v1)
ylabel('voltage v(t)')
xlabel ('time in seconds')

subplot(3,1,2)
plot (t, I1)
ylabel('current I(t)')
xlabel ('time in sec')

subplot (3,1,3)
plot (t, y1)
ylabel ('y(t)')
xlabel ('time in sec')

figure
subplot (3,1,1)
plot(t,V2)
ylabel ('voltage v(t)')
xlabel ('time in sec')

subplot (3,1,2)
plot (t, I2)

subplot (3,1,3)
plot (t, y2)
ylabel ('y(t)')
xlabel ('time in sec')
```