NAME : AISHWARYA.L
NUMBER : 6770324

EXERCISE - 12

1. Maxwell equation in 1-D (z direction) for Ey and Hx

$$\frac{\partial}{\partial z} E_y(t,z) = \mu_0 \frac{\partial H_x(t,z)}{\partial t} \quad \text{—— (*)}$$

$$\frac{\partial}{\partial z} H_x(t,z) = \varepsilon_0 \varepsilon_r(z) \frac{\partial}{\partial t} E_y(t,z) + J_y(t,z) \quad \text{—— (**)}$$

Define $E_i^j = E_y(t_j, z_i)$ and $H_i^j = H_x(t_j^\circ, z_i)$

using central difference scheme:

$$f'(x) \approx \frac{f(x+h/2) - f(x-h/2)}{h} + O(h^3)$$

$$\frac{\partial}{\partial z} E_i^j \approx \frac{E_{i+\frac{1}{2}}^j - E_{i-\frac{1}{2}}^j}{\Delta z}, \quad \frac{\partial}{\partial t} E_i^j \approx \frac{E_i^{j+\frac{1}{2}} - E_i^{j-\frac{1}{2}}}{\Delta t}$$

$$\frac{\partial}{\partial z} H_i^j \approx \frac{H_{i+\frac{1}{2}}^j - H_{i-\frac{1}{2}}^j}{\Delta z}, \quad \frac{\partial}{\partial t} H_i^j \approx \frac{H_i^{j+\frac{1}{2}} - H_i^{j-\frac{1}{2}}}{\Delta t}$$

Combining the equations,

$$\frac{E_{i+\frac{1}{2}}^j - E_{i-\frac{1}{2}}^j}{\Delta z} = \mu_0 \left[\frac{H_i^{j+\frac{1}{2}} - H_i^{j-\frac{1}{2}}}{\Delta t} \right]$$

$$H_i^{j+\frac{1}{2}} = H_i^{j-\frac{1}{2}} + \frac{\Delta t}{\Delta z \mu_0} \left[E_{i+\frac{1}{2}}^j - E_{i-\frac{1}{2}}^j \right]$$

$$\varepsilon_0 \varepsilon_r(z_i) \left[\frac{E_i^{j+\frac{1}{2}} - E_i^{j-\frac{1}{2}}}{\Delta t} \right] = J_y(t_j, z_i) - \frac{H_{i+\frac{1}{2}}^j - H_{i-\frac{1}{2}}^j}{\Delta z}$$

$$E_i^{j+\frac{1}{2}} = E_i^{j-\frac{1}{2}} + \frac{\Delta t}{\varepsilon_0 \varepsilon_r(z_i)} \left[\frac{1}{\Delta z} \left(H_{i+\frac{1}{2}}^j - H_{i-\frac{1}{2}}^j \right) - J_y(t_j, z_i) \right]$$

PTO →

Move the grid in time by $+1/2$ units and ~~Del~~ it remain the same in space.

$$H_i^{j+1} = H_i^j + \frac{\Delta t}{\Delta z\, \mu_0}\left(E_{i+1/2}^{j+1/2} - E_{i-1/2}^{j-1/2}\right)$$

Now, move the grid in time by $+1$ units $+\frac{1}{2}$ units in space

$$E_{i+\frac{1}{2}}^{j+1/2} = E_{i+1/2}^{j+1/2} + \frac{\Delta t}{\varepsilon_0 \varepsilon_R (z\,i)}\left[\frac{1}{\Delta z}\left(H_{i+1}^{j+1} - H_{i-1}^{j+1}\right) - J_y\left(t_{j+1}, z\, i+1/2\right)\right]$$

No displacements. Electrical and magnetic fields are perpendicular to eachother. So, all point in different directions in 3-D.

(2)(a) Because we use small distances. ~~there is~~ with propagation in 1-direction only.

(e) VERIFICATION:

In the graph plotted with dielectric slab, we see that there are other waves besides the source waves, which are produced by interference & reflections (due to dielectric slab + vacuum).

In the graph plotted without dielectric slab, we see ~~the~~ ~~only~~ only the propagation of source.