PROBLEM SHEET - 10

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EXERCISE 1 :

① $\frac{d^2y}{dt^2} = 1 - 2y^2$

ⓐ $\frac{dy}{dt} = z$ $\quad y(0)=0 \quad ; y(1)=0$

ⓑ $\frac{dz}{dt} = 1 - 2y^2$ [PROGRAM IN MATLAB FILE]

EXERCISE 2 :

$y'' = 4xy' - y$ —①

$y'' \simeq \frac{y_{i+1} - 2y_i - y_{i-1}}{h^2}$ —②

$y' \simeq \frac{y_{i+1} - y_i}{h}$ —③

SUB ②, ③ in ①

$$\frac{y_{i+1} - 2y_i + y_{i-1}}{h^2} = 4x\left(\frac{y_{i+1} - y_i}{h}\right) - y_i$$

$$y_{i+1} - 2y_i + y_{i-1} = 4xh(y_{i+1} - y_i) - h^2 y_i$$

$$y_{i+1} - 4xhy_{i+1} = 2y_i - 4xhy_i - h^2 y_i - y_{i-1}$$

SUB $h=1$

$$\Rightarrow \boxed{y_{i+1}(1-4x) = (1-4x)y_i - y_{i-1}}$$

$i=1 ; x=1$

$$y_2(1-4(1)) = (1-4(1))y_1 - y_0$$

$$y_0 + 3y_1 - 3y_2 = 0$$

$$\Rightarrow 3y_1 - 3y_2 = 0$$

$i=2, x=2$

$$y_3(1-4(2)) = (1-4(2))y_2 - y_1$$

$$y_1 + 7y_2 - 7y_3 = 0$$

$i=3, x=3$

$$y_4(1-4(3)) = y_3(1-4(3)) - y_2$$

$$y_2 + 11y_3 - 11y_4 = 0$$

$i=4, x=4$

$$y_5(1-4(4)) = y_4(1-4(4)) - y_3$$

$$y_3 + 15y_4 - 15y_5 = 0$$

$i=5, x=5$

$$y_6(1-4(5)) = y_5(1-4(5)) - y_4$$

$$y_4 + 19y_5 - 19y_6 = 0$$

$$y''(x) = 4xy'(x) - y(x)$$

$$y''(5) = (4)(5)\, y'(5) - y(5)$$

$$y(6) = 60 + y(5) - y(4)$$

$$\therefore \quad 3y(1) - 3y(2) = 0$$

$$y(1) + 7y(2) - 7y(3) = 0$$

$$y(2) + 11y(3) - 11y(4) = 0$$

$$y(3) + 15y(4) - 15y(5) = 0$$

$$y(4) + 19y(5) - 19y(6) = 0$$

But $y(6) = 60 + y(5) - y(4)$

$$\therefore \quad y(4) + 19y(5) - (19 \times 60) - 19y(5) + 19y(4)$$

$$\Rightarrow -18y(4) - 1140 = 0$$

$$\begin{bmatrix} 3 & -3 & 0 & 0 & 0 \\ 1 & 7 & -7 & 0 & 0 \\ 0 & 1 & 11 & -11 & 0 \\ 0 & 1 & 15 & -15 & 0 \\ 0 & 0 & 20 & 0 & 0 \end{bmatrix} \quad A$$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \end{bmatrix} \quad Y$$

$$= \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1140 \end{bmatrix} \quad B$$

$$Y = A^{-1}B$$