Modeling & Simulation
Problem Sheet 8

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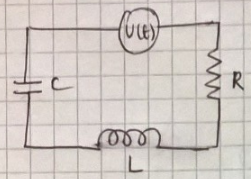
1. (a)



$$U(t) = V_R + V_L + V_C$$

$$U(t) = RI(t) + LI'(t) + \frac{q}{C}$$

$$V_C = \frac{q}{C}$$

$$I = \frac{dq}{dt}$$

$$V_C' = \frac{1}{C}\frac{dq}{dt} = \frac{1}{C} I(t)$$

$$\Rightarrow LI'(t) = U(t) - RI(t) - V_C(t)$$

$$\boxed{I'(t) = \frac{1}{L}\left[U(t) - RI(t) - V_C(t)\right] \quad V_C'(t) = \frac{1}{C} I(t)} \quad \left\{ U(t) = U_0 \cos(\omega t) \right\}$$

1. (d) RK-2 and ODE-45 give the same results. ODE-45 uses RK-4 which is more accurate.

2(a) $y' = \lambda y$

$$y_{i+1} = y_i + hf\left(t_{i+\frac{1}{2}}, y_{i+\frac{1}{2}}\right)$$
$$= y_i + h\lambda y_{i+\frac{1}{2}}$$
$$y_{i+\frac{1}{2}} = y_i + \frac{h}{2} f(t_i, y_i)$$
$$= y_i + \frac{h}{2}\lambda y_i$$

$$y_{i+1} = y_i + h\lambda\left(y_i + \frac{h}{2}\lambda y_i\right)$$
$$= y_i + h\lambda y_i + \frac{h^2}{2}\lambda^2 y_i$$

$$\therefore y_{i+1} = y_i\left[1 + h\lambda + \frac{h^2\lambda^2}{2}\right]$$

For A-Stable

$$\left|1 + h\lambda + \frac{h^2\lambda^2}{2}\right|^2 \leq 1 \quad (\text{for RK2})$$