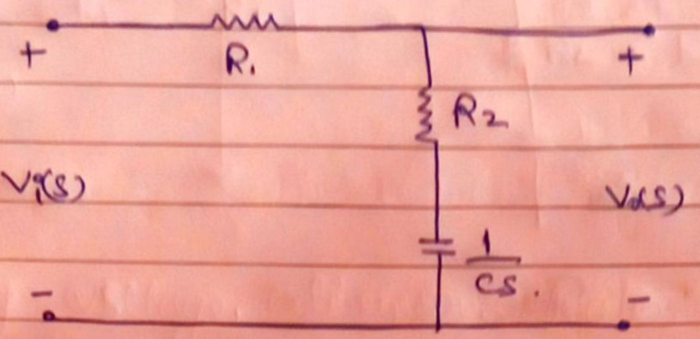
Compute the Transfer function & Pole-Zero plot of the following :-

a) Lag compensator.



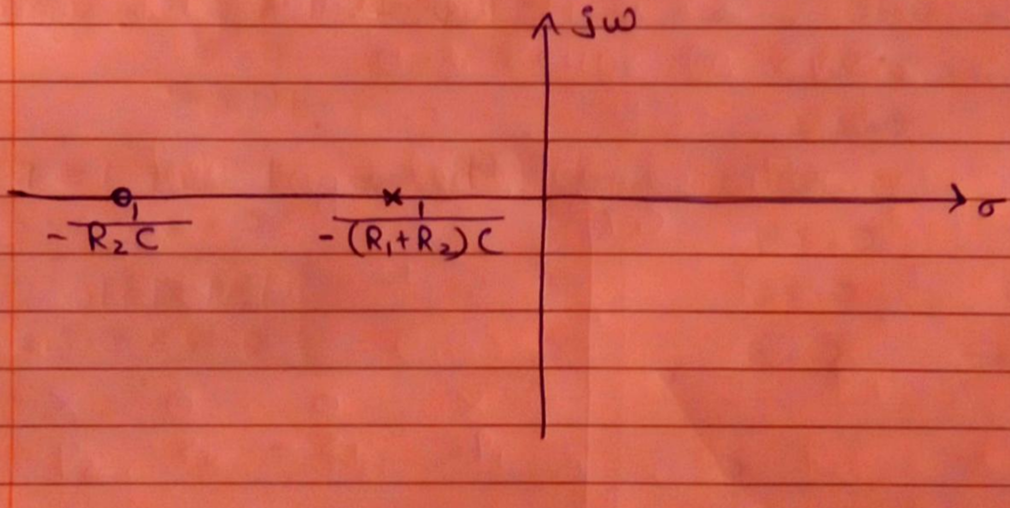
By voltage division rule :-

$$V_o(S) = V_i(S)\left[\frac{R_2 + \frac{1}{CS}}{R_2 + \frac{1}{CS} + R_1}\right]$$

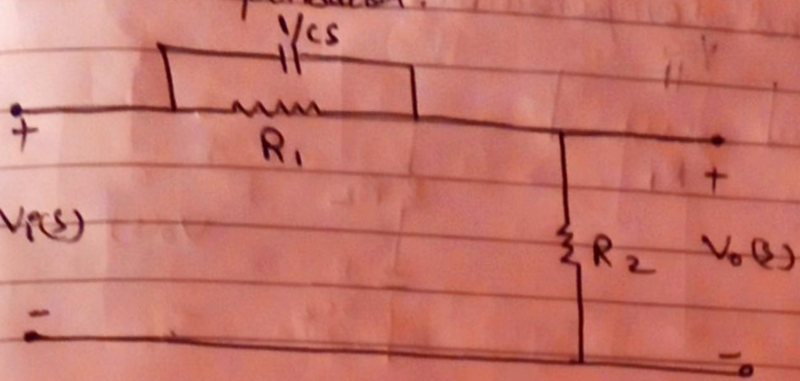
$$\frac{V_o(S)}{V_i(S)} = \frac{R_2 C S + 1}{(R_1+R_2)CS + 1}$$

Lag compensator :- $\tan^{-1} R_2 C - \tan^{-1}(R_1+R_2)C < 0$

$$R_2 C < (R_1+R_2) C$$



b) Lead Compensator.



By voltage division Rule.

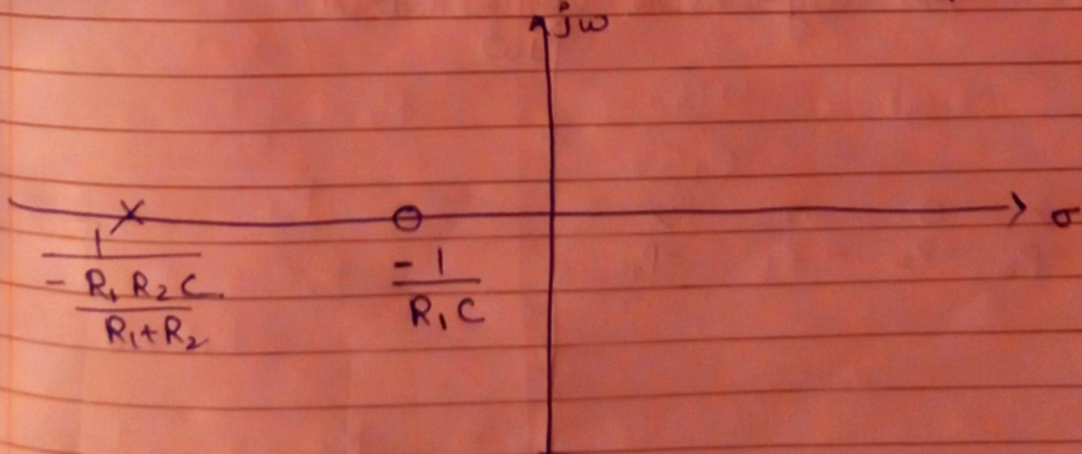
$$\frac{V_o(s)}{\cancel{V}} = V_i(s)\left[\frac{R_2}{R_2 + \frac{R_1}{1+R_1Cs}}\right]$$

$$\frac{V_o(s)}{V_i(s)} = \frac{R_2(1+R_1Cs)}{R_1 + R_1R_2Cs + R_2}$$

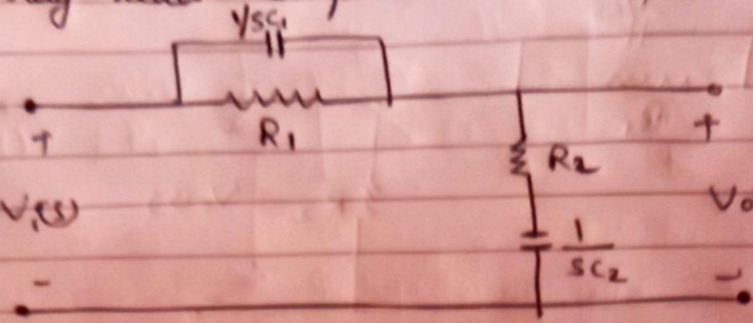
$$= \frac{R_2(R_1Cs+1)}{R_1+R_2\left(1+\frac{R_1R_2Cs}{R_1+R_2}\right)}$$

Lead compensator :- $\tan^{-1} R_1C - \tan^{-1}\frac{R_1R_2C}{R_1+R_2} > 0$

$$R_1C > \frac{R_1R_2C}{R_1+R_2}$$



c) Lag-Lead Compensator.



By voltage division Rule,

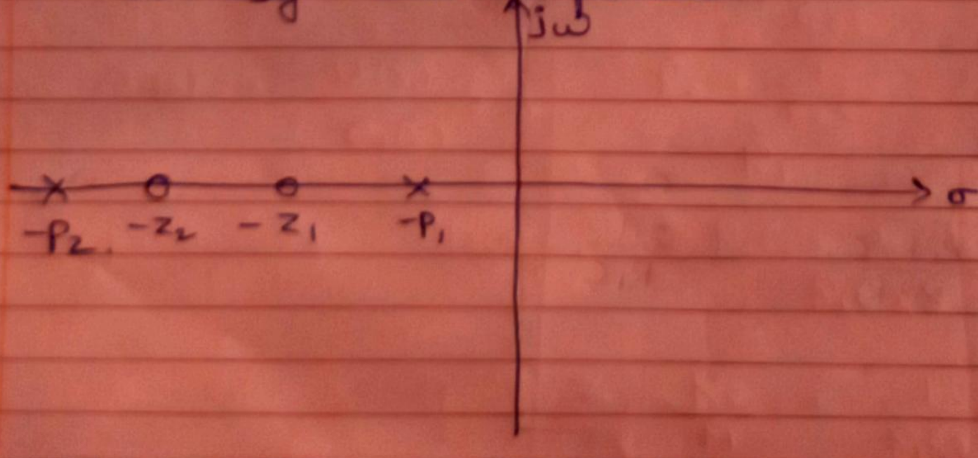
$$V_o(s) = V_i(s)\left[\frac{R_2 + \frac{1}{sC_2}}{R_2 + \frac{1}{C_2 S} + \frac{R_1}{1 + R_1C_1S}}\right]$$

$$\frac{V_o(s)}{V_i(s)} = \frac{(C_2R_2S + 1)(1 + R_1C_1S)}{(R_2C_2S + R_2C_2R_1C_1S^2 + 1 + R_1C_1S + R_1C_2S)}$$

$$= \frac{(R_1C_1S + 1)(R_2C_2 + 1)}{R_2C_2R_1C_1S^2 + (R_1C_1 + R_1C_2 + R_2C_2)S +}$$

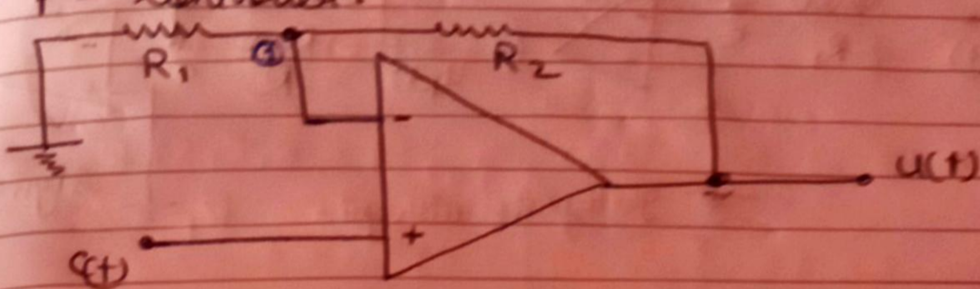
Let Poles be P_1 & P_2
and Zeros be Z_1 & Z_2

For lag-lead compensator



Here x_1 =

4) P - Controller.



~~By~~ Let Voltage at node ① be V(t).

$$\frac{V(t)}{R_1} + \frac{V(t) - u(t)}{R_2} = 0$$

$$V(t)\left[\frac{1}{R_1} + \frac{1}{R_2}\right] = \frac{+u(t)}{R_2}$$

$$V(t) = \frac{R_1}{R_1 + R_2} u(t)$$

Also by virtual ground concept, $v(t) = c(t)$.

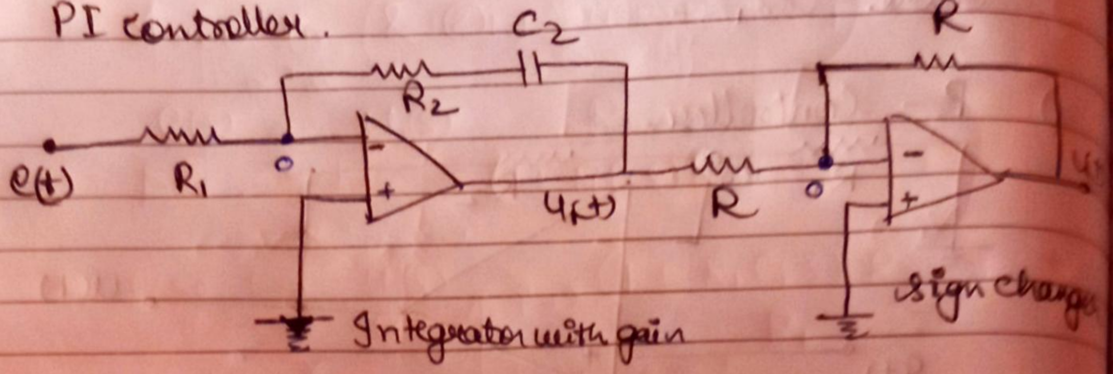
$$c(t) = \frac{R_1}{R_1 + R_2} u(t)$$

$$\frac{C(s)}{U(s)} = \frac{R_1}{R_1 + R_2}$$

$$T(s) = \frac{U(s)}{C(s)} = \frac{R_1 + R_2}{R_1}$$

There are ~~both~~ no poles & zeros in the system

e) PI controller.



$$\frac{0-e(s)}{R_1} + \frac{0-u_1(s)}{R_2+\frac{1}{C_2 s}} = 0$$

$$\frac{e_1(s)}{R_1} = -\,u_1(s)\,\frac{1}{R_2+\frac{1}{C_2 s}} \quad \text{——— ①}$$

$$\frac{0-u_1(s)}{R} + \frac{0-u(s)}{R} = 0$$

$$u_1(s) = -\,u(s). \quad \text{——— ②}$$

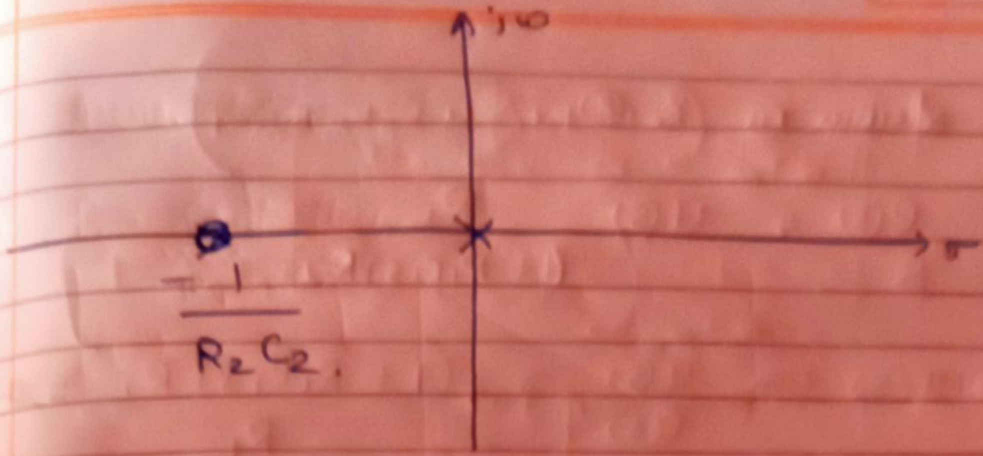
from ① & ②.

$$\frac{e_1(s)}{R_1} = u(s)\,\frac{1}{R_2+\frac{1}{C_2 s}}$$

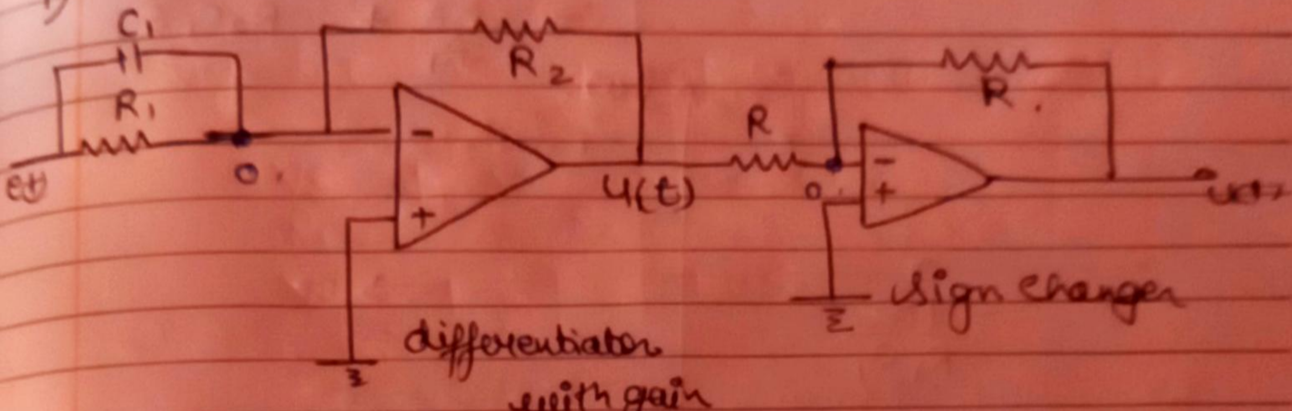
$$T(s) = \frac{u(s)}{e_1(s)} = \frac{R_2+\frac{1}{C_2 s}}{R_1}$$

$$T(s) = \frac{R_2 C_2 s + 1}{R_1 C_2 s}$$

$$= \frac{R_2}{R_1} + \frac{1}{R_1 C_2 s}.$$



f) PD - Controller



For differentiator with gain part

$$\frac{0 - e(s)}{R_1/1+R_1C_1s} + \frac{0 - u_1(s)}{R_2} = 0$$

$$e(s) = -u_1(s)\frac{R_1}{(1+R_1C_1s)R_2} \quad \text{—— (1)}$$

For sign-changer part :

$$\frac{0 - u_1(s)}{R} + \frac{0 - u(s)}{R} = 0$$

$$u_1(s) = -u(s) \quad \text{—— (2)}$$

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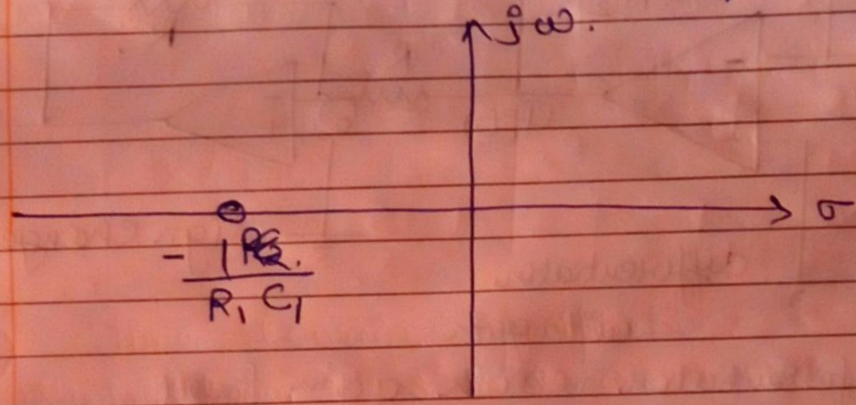
from ① & ②,

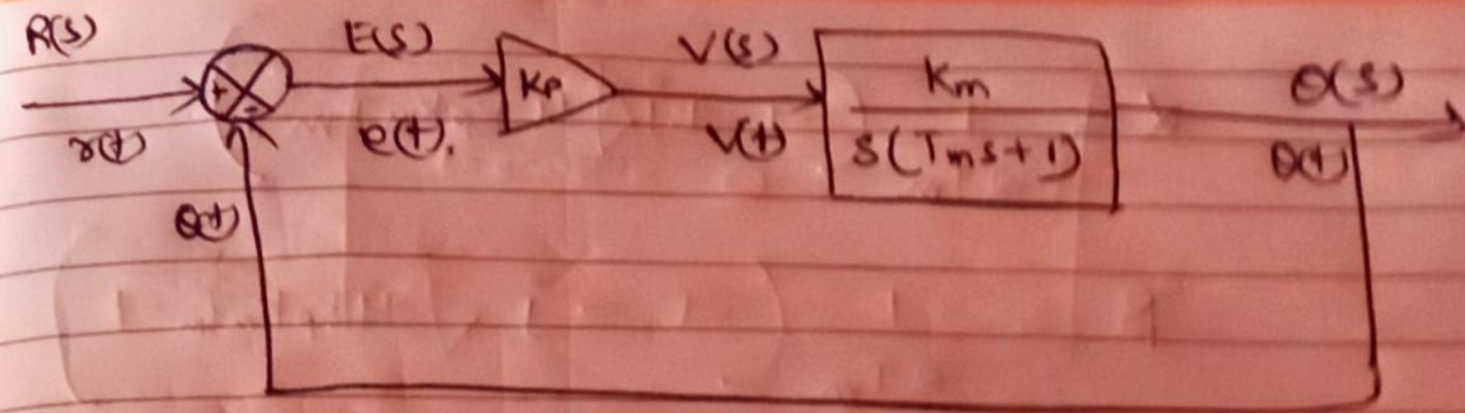
$$e(s) = u(s) \frac{R_1}{(1+R_1C_1s)R_2}$$

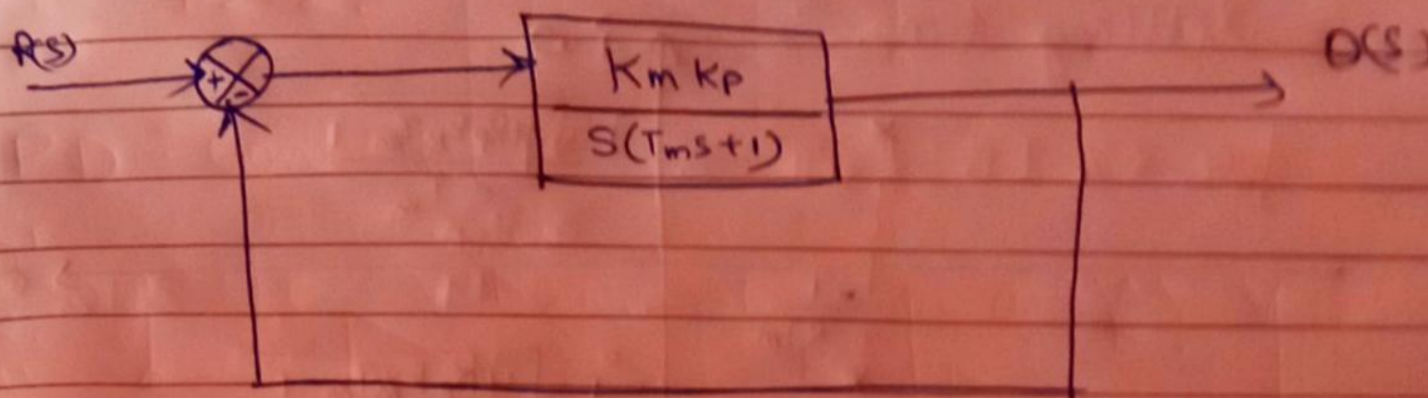
$$T(s) = \frac{u(s)}{e(s)} = \frac{(1+R_1C_1s)R_2}{R_1}$$

$$= \frac{R_2}{R_1} + R_2C_1s$$



9)



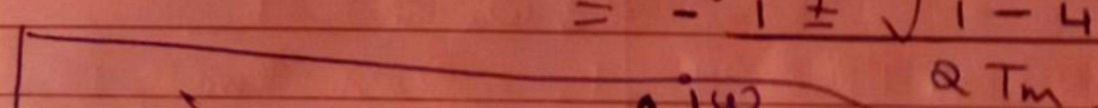


$$T(s) = \frac{\Theta(s)}{R(s)} = \frac{\dfrac{K_m K_p}{s(T_m s+1)}}{1 + \left(1 \times \dfrac{K_m K_p}{s(T_m s+1)}\right)}$$

$$T(s) = \frac{K_m K_p}{s^2 T_m + s + K_m K_p}$$

Root of c/s eqⁿ = Poles of Transfer function of system.

$$= \frac{-1 \pm \sqrt{1 - 4T_m K_m K_p}}{2T_m} = P_1, P_2$$



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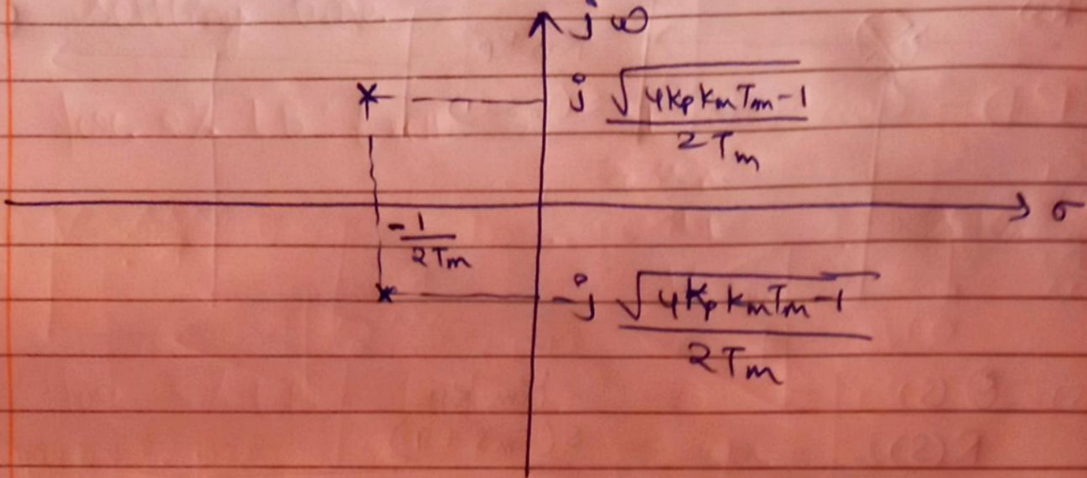
$$\alpha = \frac{1}{2T_m}\left(-1 + j\sqrt{4K_pK_mT_m - 1}\right)$$

$$\beta = \frac{1}{2T_m}\left(-1 - j\sqrt{4K_pK_mT_m - 1}\right)$$



b) Time domain specifications :-

Standard form : $s^2 + 2\xi\omega_n s + \omega_n^2$

C/S Eqⁿ : $s^2 + \frac{s}{T_m} + \frac{K_pK_m}{T_m}$

natural frequency $\omega_n = \sqrt{\frac{K_pK_m}{T_m}}$

$$2\xi\omega_n = \frac{1}{T_m}$$

$$\xi = \frac{1}{2T_m}\sqrt{\frac{T_m}{K_pK_m}}$$

$$\xi = \frac{1}{2\sqrt{T_mK_pK_m}}$$

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raise time $t_r = \frac{\pi - \theta}{\omega_d}$

$$\omega_d = \omega_n \sqrt{1 - \xi^2}$$

$$= \sqrt{\frac{K_p K_m}{T_m}} \sqrt{1 - \frac{1}{4 T_m K_p K_m}}$$

$$= \frac{\sqrt{4 T_m K_p K_m - 1}}{2 T_m}.$$

$$\theta_{(radian)} = \cancel{\tan^{-1}} \left[(\cos^{-1} \xi) \frac{\pi}{180^\circ} \right] \text{ radian.}$$

$$\boxed{t_r = \frac{2T_m \left[\pi - \frac{\pi}{180} \cos^{-1} \xi \right]}{\sqrt{4 T_m K_p K_m - 1}} \text{ sec.}}$$

peak time $t_p = \frac{n\pi}{\omega_d}$

$n = 1$

$$\boxed{t_p = \frac{2\pi T_m}{\sqrt{4 T_m K_p K_m - 1}}}$$

Peak overshoot $M_o = \cancel{e^{-\pi \cot\theta}}$

$$= e^{-\pi \frac{\xi}{\sqrt{1-\xi^2}}}$$

$$= e^{-\pi \frac{1/2\sqrt{T_m K_p K_m}}{\sqrt{1 - \frac{1}{4 T_m K_p K_m}}}}$$

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$$\boxed{M_0 = e^{-\frac{\pi}{\sqrt{4K_pK_mT_m - 1}}}}$$

Settling time ;

for 2% EB ; $t_{sett} = \dfrac{4}{\xi \omega_n}$

$= \dfrac{4}{1/2T_m} = 8T_m$

For 5% EB, $t_{sett} = \dfrac{3}{\xi \omega_n}$

$= \dfrac{3}{1/2T_m} = 6T_m$

Delay time,

$$T_d = \frac{1 + 0.7\xi}{\omega_n}$$

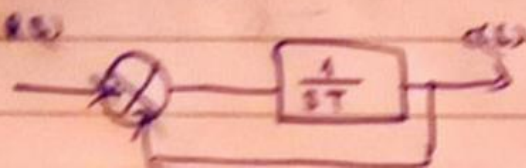
$$= \frac{1 + 0.7\left(\frac{1}{2\sqrt{K_p K_m T_m}}\right)}{\sqrt{\frac{K_m K_p}{T_m}}}$$

$$= \sqrt{\frac{T_m}{K_m K_p}} + \frac{0.35}{K_m K_p}$$

$$= \frac{0.35 + \sqrt{K_m K_p T_m}}{K_m K_p}$$

2) Write down basic differences b/w 1st order syst. & 2nd order system.

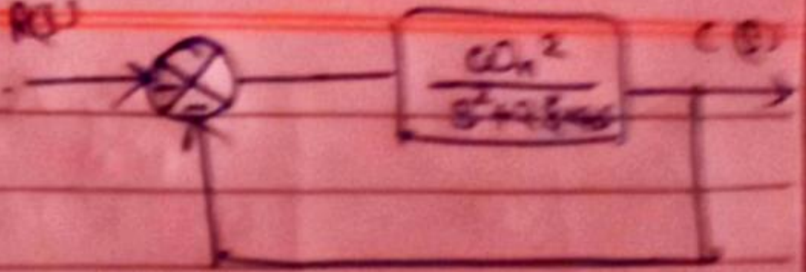
1st order system	2nd order system
1. Systems whose i/p & o/p relationship is a first order differential eqn.	1. Systems whose i/p & o/p relationship is a second order differential eqn.
2. Systems containing only single energy storage element.	2. System containing 2 independent energy storage elements

Ex:- 

$$\frac{C(s)}{R(s)} = K \frac{1}{s\tau + 1}.$$

here K is DC gain,
τ is time constant

Ex:



$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$