LG态涡旋电子的轫致辐射

为什么要用 Laguerre-Gaussian 态?

- 在单涡旋散射的情况下，对于贝塞尔态，其散射截面对OAM一般不敏感；而 LG 态则相反，因为其平均横动量随 $\sqrt{l}$ 的增大而增大[1]

- 对贝塞尔态的单涡旋轫致辐射，在宏观靶情况下，散射截面跟相位 $il\phi$ 无关，只跟纵向与横向动量之前的张开角 θ 相关[2]

ref [1]: KARLOVETS D V, SERBO V G, 2020. Effects of the transverse coherence length in relativistic collision.
ref [2]:GROSHEV M E, ZAYTSEV V A, YEROKHIN V A, et al., 2020. Bremsstrahlung from twisted electrons in the field of heavy nuclei.

标量粒子的 LG 波函数[3]

$$\phi_l(\boldsymbol{p}) = (4\pi)^{3/4}\sigma_\perp\sqrt{\sigma_z}\sqrt{2\epsilon(\mathbf{p})}\,\frac{(\sigma_\perp p_\perp)^{|l|}}{\sqrt{|l|!}}\exp[-p_\perp^2\sigma_\perp^2/2-(p_z-\langle p_z\rangle)^2\sigma_z^2/2+il\varphi_p]$$

> $$\int\frac{d^3p}{(2\pi)^3 2\epsilon(\mathbf{p})}|\phi_l(\boldsymbol{p})|^2=1$$

广义拉盖尔多项式有:

$$\int_0^\infty \frac{n!}{\Gamma(n+|l|+1)}\,x^{|l|}\,e^{-x}|L_n^{|l|}(x)|^2\,dx=1$$

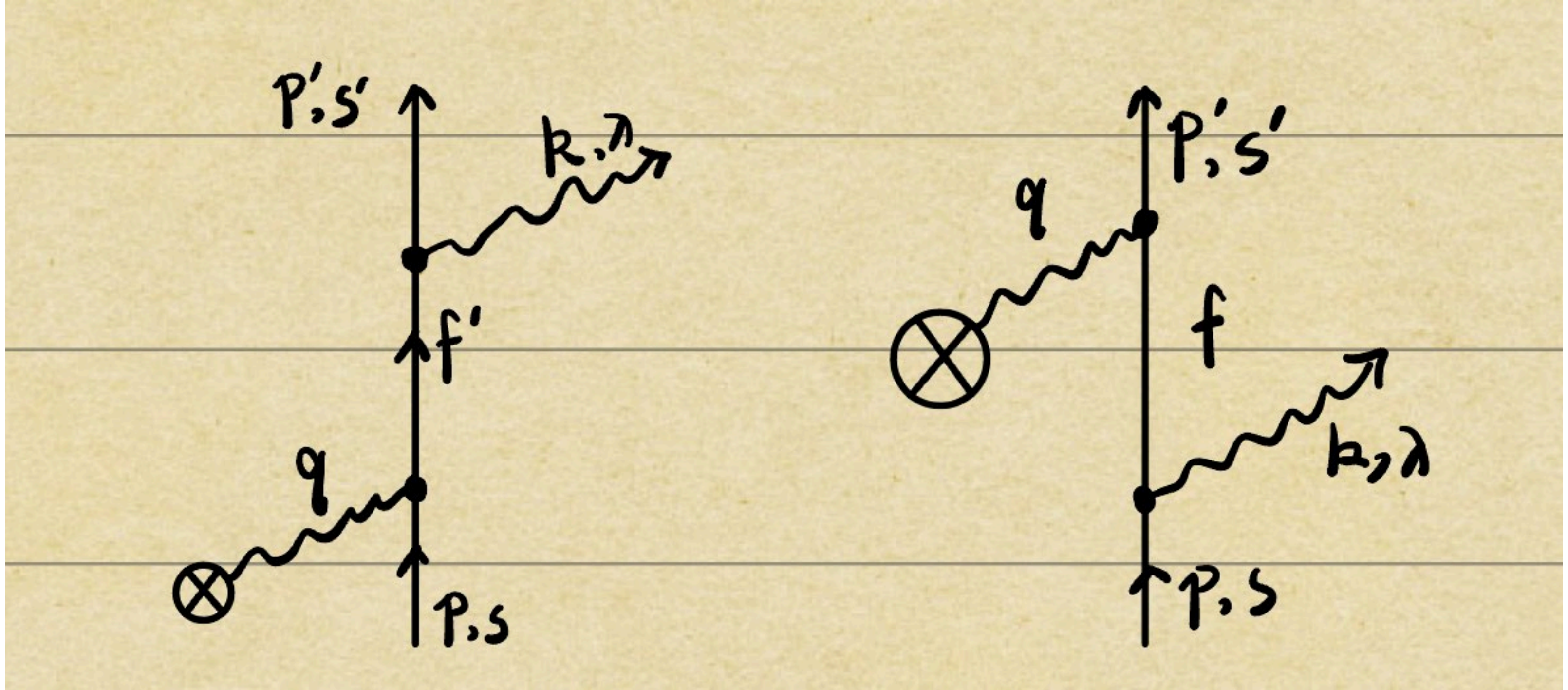
若 $n=0$，则 $\Gamma(|l|+1)=|l|!$ 且 $L_0^{|l|}(x)=1$，再使 $x\to\sigma_\perp^2p_\perp^2$，即可得:

$$\int_0^\infty \frac{1}{|l|!}\,(\sigma_\perp^2p_\perp^2)^{|l|}\,e^{-\sigma_\perp^2p_\perp^2}\,d(\sigma_\perp^2p_\perp^2)=1$$

ref [3]: KARLOVETS D, 2018. Relativistic vortex electrons: Paraxial versus nonparaxial regimes.

非涡旋S矩阵

费曼图



$$\langle f|S|i\rangle = e^2 \int d^4x d^4y\, \bar{\psi}_f(x)[(-i\gamma^\nu A_\nu(y,k))iS_F(x-y)(-i\gamma^0)A_0^{coul}(y)$$
$$+ (-i\gamma^0)A_0^{coul}(x)iS_F(x-y)(-i\gamma^\mu A_\mu(y,k))]\psi_i(y)$$

$$= -Ze^3 \int d^4x d^4y\, \frac{d^3q}{(2\pi)^3}\, \bar{u}(p',s')e^{ip'x}\, [-i\gamma^\nu \epsilon_\nu^* e^{ikx} \int \frac{d^4f'}{(2\pi)^4} \frac{ie^{-if'(x-y)}}{\not{f}' - m_e}(-i\gamma^0)\frac{e^{i\boldsymbol{q}\cdot\boldsymbol{y}}}{\boldsymbol{q}^2}$$
$$- i\gamma^0 \frac{e^{i\boldsymbol{q}\cdot\boldsymbol{x}}}{\boldsymbol{q}^2} \int \frac{d^4f}{(2\pi)^4} \frac{ie^{-if(x-y)}}{\not{f} - m_e}(-i\gamma^\mu \epsilon_\mu^*)e^{iky}]\, u(p,s)e^{-ipy}$$

> 取 $\alpha = \frac{e^2}{4\pi} = \frac{1}{137}$，这里用到了 $A_0^{coul}(x) = -\frac{Ze}{4\pi|\boldsymbol{x}|} = -Ze\int \frac{d^3q}{(2\pi)^3}\, \frac{1}{\boldsymbol{q}^2} e^{i\boldsymbol{q}\cdot\boldsymbol{x}}$

积分后可得 $\langle f|S|i\rangle = 2\pi i\delta(\epsilon' + \omega - \epsilon)M_{pw}$

$$M_{pw} = -Ze^3\, \frac{1}{\boldsymbol{q}^2}\, \bar{u}(p',s')\left[\not{\epsilon}^*(k,\lambda)\frac{\not{p}' + \not{k} + m_e}{(p'+k)^2 - m_e^2}\gamma_0 + \gamma_0\frac{\not{p} - \not{k} + m_e}{(p-k)^2 - m_e^2}\,\not{\epsilon}^*(k,\lambda)\right]u(p,s)$$

- 其中选取 $u(p,s)$ 为:

$$u(p,s)=\begin{pmatrix}\sqrt{\epsilon+m_e}\,\omega^s\\ \sqrt{\epsilon-m_e}\,\boldsymbol{\sigma}\cdot\hat{\boldsymbol{p}}\,\omega^s\end{pmatrix}=\begin{pmatrix}\sqrt{\epsilon+m_e}\,\omega^s\\ \frac{1}{\sqrt{\epsilon-m_e}}\,\boldsymbol{\sigma}\cdot\boldsymbol{p}\,\omega^s\end{pmatrix}$$

这里的 ω^s 简单起见，不选为螺旋度 $\boldsymbol{\sigma}\cdot\hat{\boldsymbol{p}}$ 的本征态，而选为 σ_z 的本证态，即

$$\omega^{s=\frac{1}{2}}=\begin{pmatrix}1\\0\end{pmatrix},\ \omega^{s=-\frac{1}{2}}=\begin{pmatrix}0\\1\end{pmatrix}$$

- $\epsilon^*(k,\lambda)$ 选为:

$$\epsilon^*(k,\lambda)=\frac{1}{\sqrt{2}}\begin{pmatrix}0\\ \lambda\cos\theta_k\cos\varphi_k-i\sin\varphi_k\\ \lambda\cos\theta_k\sin\varphi_k+i\cos\varphi_k\\ \lambda\sin\theta_k\end{pmatrix}$$

单涡旋S矩阵

令初态电子为涡旋态：

$$|i\rangle = \int \frac{d^3p}{(2\pi)^3 2\epsilon}\, \phi_l(\boldsymbol{p})\, e^{-i\boldsymbol{b}\cdot\boldsymbol{p}}\ \ |\boldsymbol{p}\rangle$$

> 这里选取洛伦兹不变的归一化：$\langle \boldsymbol{p}|\boldsymbol{p}'\rangle = 2\epsilon(\boldsymbol{p})\delta^{(3)}(\boldsymbol{p}-\boldsymbol{p}')\delta_{ss'}$

则S矩阵为：

$$\begin{aligned}\langle f|S|i\rangle &= \int \frac{d^3p}{(2\pi)^3 2\epsilon}\, \phi_l(\boldsymbol{p})\, e^{-i\boldsymbol{b}\cdot\boldsymbol{p}}\ \ \langle p'k|S|pq\rangle \\ &= \int \frac{d^3p}{(2\pi)^3 2\epsilon}\, \phi_l(\boldsymbol{p})\, e^{-i\boldsymbol{b}\cdot\boldsymbol{p}}\, 2\pi i\delta(\epsilon'+\omega-\epsilon)\, M_{pw}\end{aligned}$$

则跃迁概率:

> 这里对末态粒子极化求和，而初态电子固定自旋为 $s=\frac{1}{2}$

$$
\begin{aligned}
dP_{fi} &= \frac{d^3p'}{(2\pi)^3 2\epsilon'}\frac{d^3k}{(2\pi)^3 2\omega}\sum_{s'}\sum_{\lambda}|\langle f|S|i\rangle|^2 \\
&= \omega\sqrt{\epsilon'^2-m_e^2}\,\frac{d\epsilon' d\Omega'}{2(2\pi)^3}\frac{d\omega\, d\Omega_k}{2(2\pi)^3}\sum_{s'}\sum_{\lambda}\left|\int\frac{\pi d\Omega}{(2\pi)^3}\sqrt{\epsilon_f^2-m_e^2}\,\phi_l(\epsilon_f,\Omega)\,e^{-i\boldsymbol{b}\cdot\boldsymbol{p}(\epsilon_f,\Omega)}\,M_{pw}\right|^2_{\epsilon_f=\epsilon'+\omega}
\end{aligned}
$$

关于初态电子波函数的选取

由于取不同的 l 值的 ϕ_l 的动量和坐标空间分布并不相同，为了确定结果中的不同是由相位 $il\varphi$ 引起的，我们选取动量与坐标空间没有相位的函数做对比。

首先在动量与坐标空间， ϕ_l 分别为：

$$\phi_l(\boldsymbol{p}) = (4\pi)^{3/4}\sigma_\perp\sqrt{\sigma_z}\sqrt{2\epsilon(\mathbf{p})}\ \frac{(\sigma_\perp p_\perp)^{|l|}}{\sqrt{|l|!}}\ \exp[-p_\perp^2\sigma_\perp^2/2-(p_z-\langle p_z\rangle)^2\sigma_z^2/2+il\varphi_p]$$

$$\phi_l(\boldsymbol{r}) = (\pi)^{-3/4}\ \frac{i^l r^{|l|}}{\sqrt{\sigma_z|l|!}\,\sigma_\perp^{|l|+1}}\ \exp[-r^2/(2\sigma_\perp^2)-z^2/(2\sigma_z^2)+i\langle p_z\rangle z+il\varphi_r]$$

- 所以对应的在动量空间没有相位的波函数为:

$$\phi_p(\boldsymbol{p}) = (4\pi)^{3/4}\sigma_\perp\sqrt{\sigma_z}\sqrt{2\epsilon(\mathbf{p})}\ \frac{(\sigma_\perp p_\perp)^{|l|}}{\sqrt{|l|!}}\ \exp[-p_\perp^2\sigma_\perp^2/2-(p_z-\langle p_z\rangle)^2\sigma_z^2/2]$$

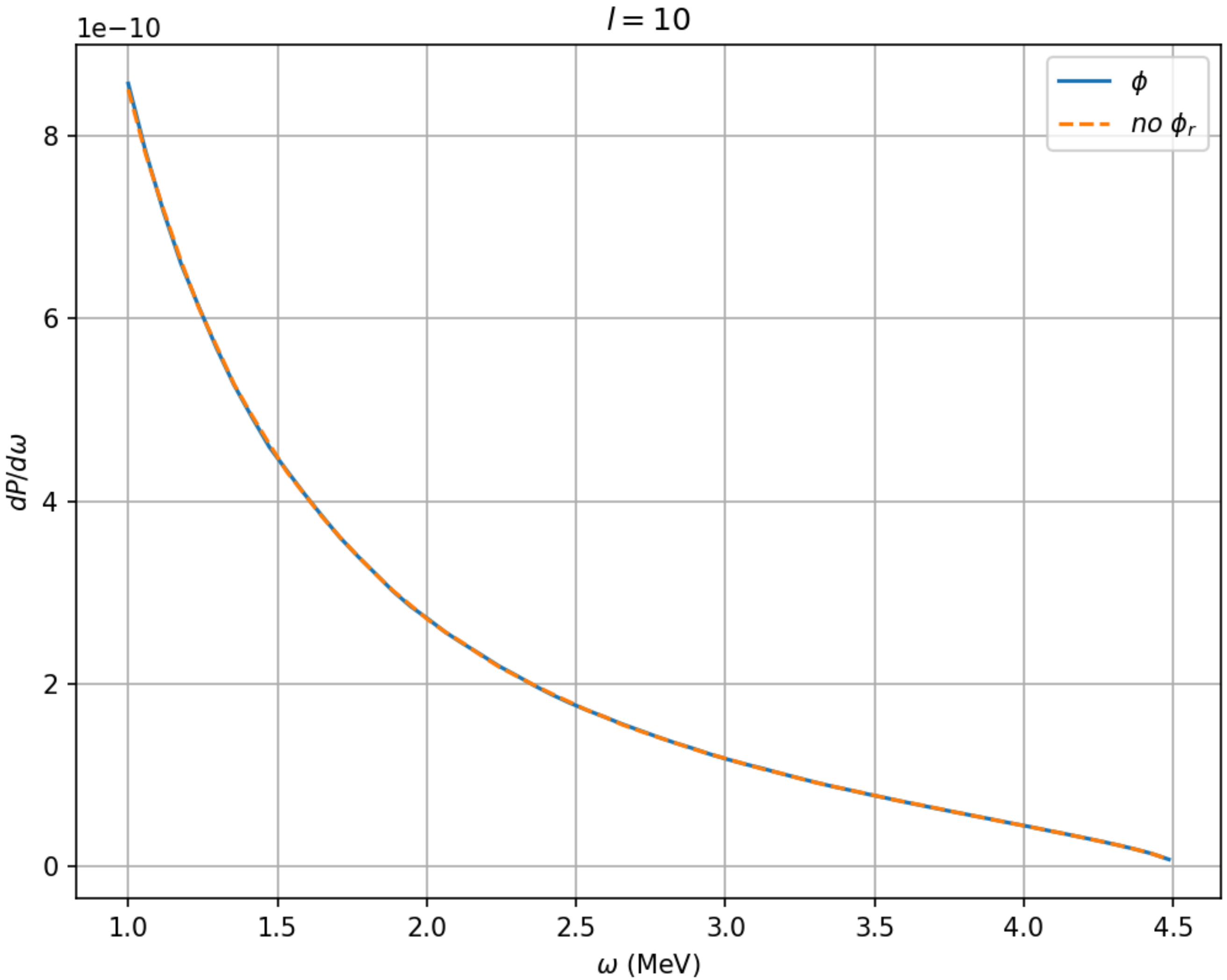
- 对应的坐标空间函数为:

$$\phi_r(\boldsymbol{r}) = (\pi)^{-3/4}\ \frac{i^l \boldsymbol{r}^{|l|}}{\sqrt{\sigma_z|l|!}\ \sigma_\perp^{|l|+1}}\ \exp[-r^2/(2\sigma_\perp^2)-z^2/(2\sigma_z^2)+i\left\langle p_z\right\rangle z]$$

将其傅里叶变换，便能直接应用到之前的公式:

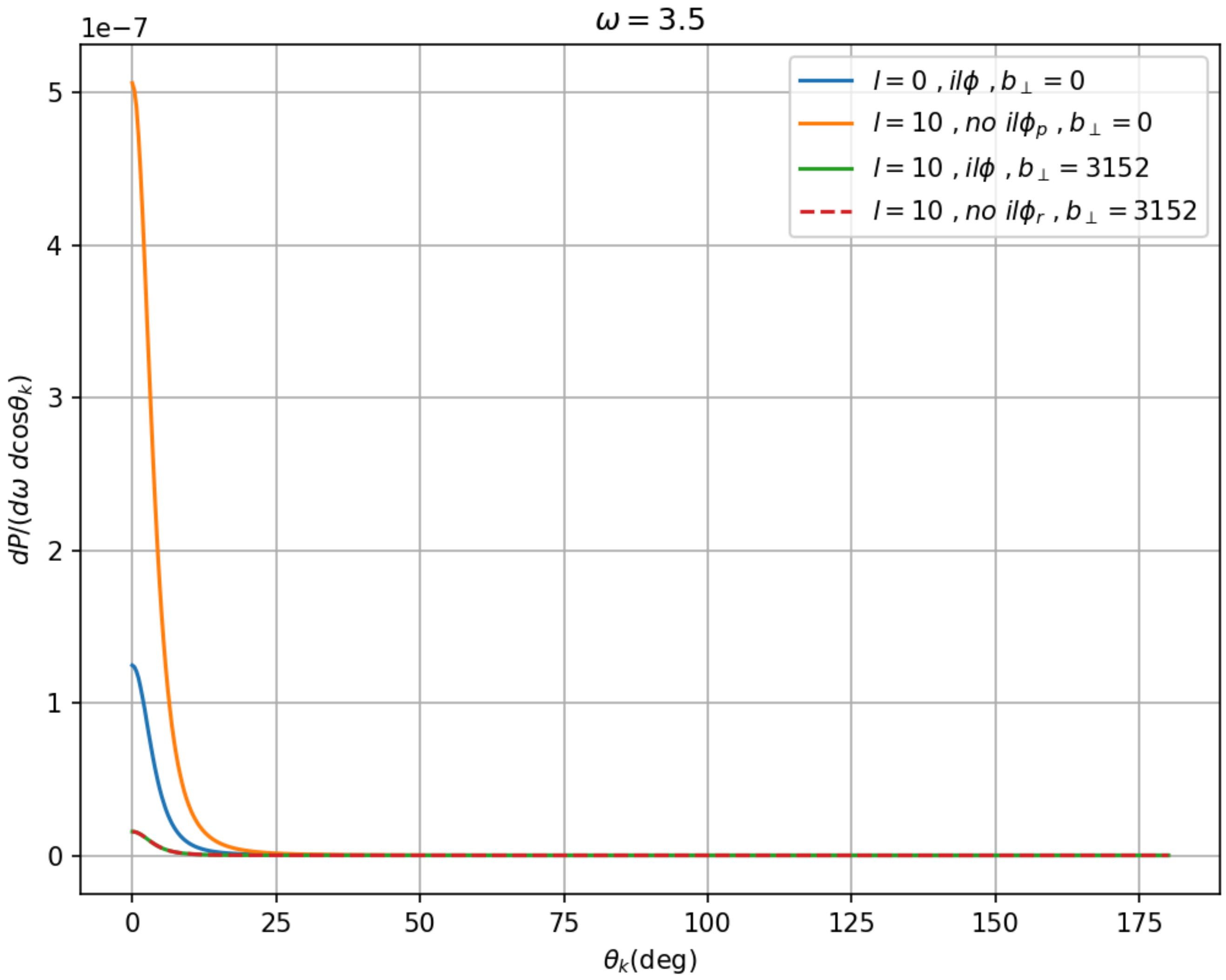
$$\phi_r(\boldsymbol{p}) = (4\pi)^{3/4}\sigma_\perp\sqrt{\sigma_z}\sqrt{2\epsilon(\mathbf{p})}\ \sqrt{\frac{2^l}{l!}}\ \Gamma(\frac{l}{2}+1)\ L_{-1-l/2}(-p_\perp^2\sigma_\perp^2/2)\ \exp[-(p_z-\langle p_z\rangle)^2\sigma_z^2/2]$$

能谱



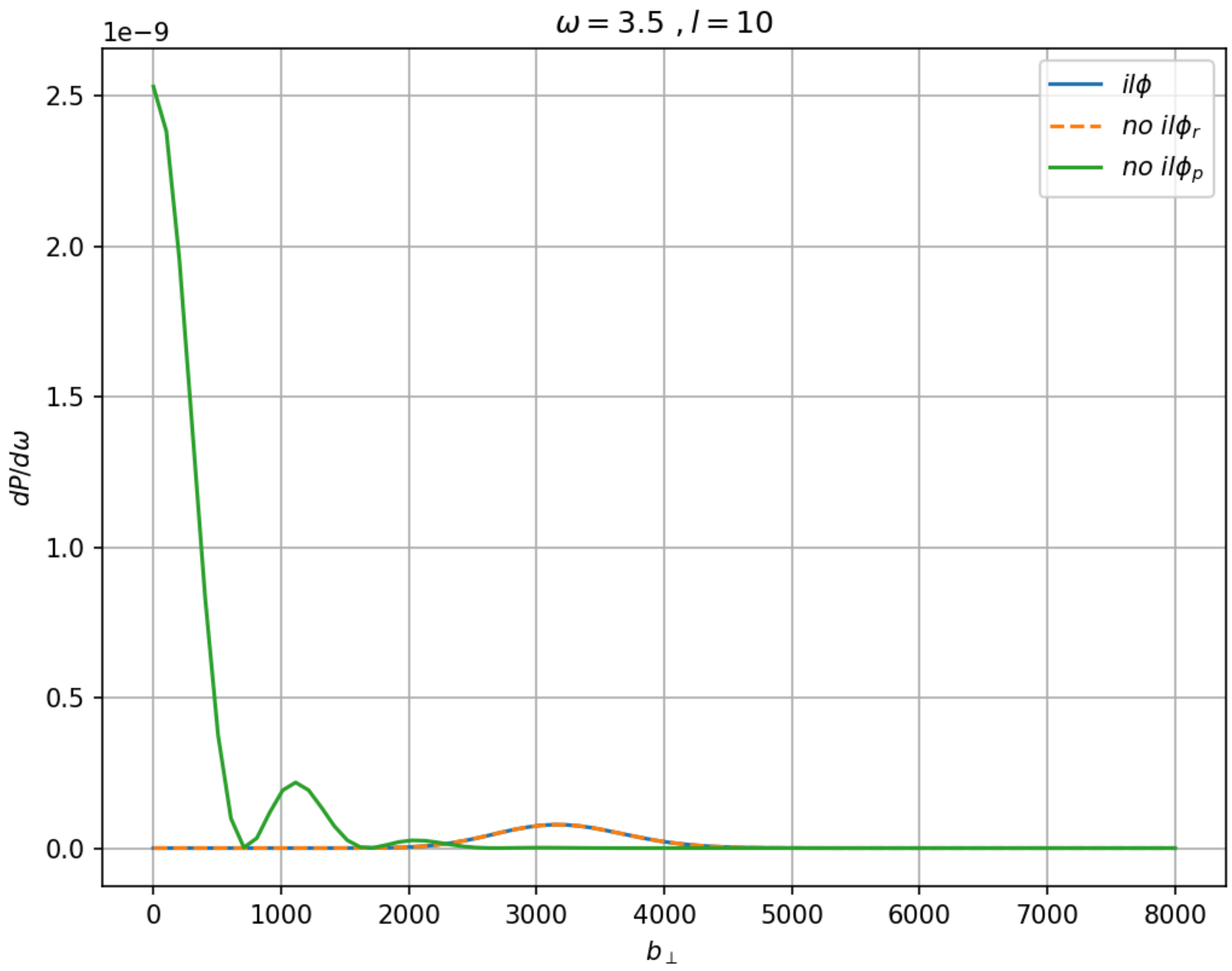
$l = 10$
1e−10
8
6
4
2
0
$dP/d\omega$
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
ω (MeV)
ϕ
$no\ \phi_r$

角度分布



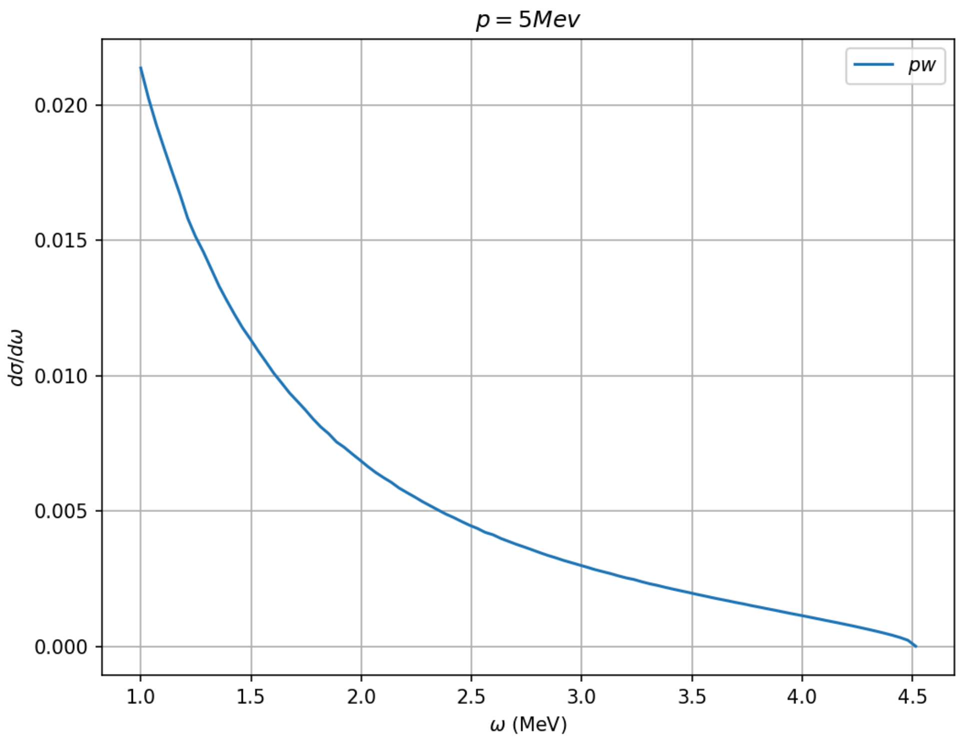
$\omega = 3.5$
1e−7
5
4
3
2
1
0
$dP/(d\omega \; d\cos\theta_k)$
0
25
50
75
100
125
150
175
θ_k(deg)
$l = 0$, $il\phi$, $b_\perp = 0$
$l = 10$, $no \; il\phi_p$, $b_\perp = 0$
$l = 10$, $il\phi$, $b_\perp = 3152$
$l = 10$, $no \; il\phi_r$, $b_\perp = 3152$

关于散射参数的分布

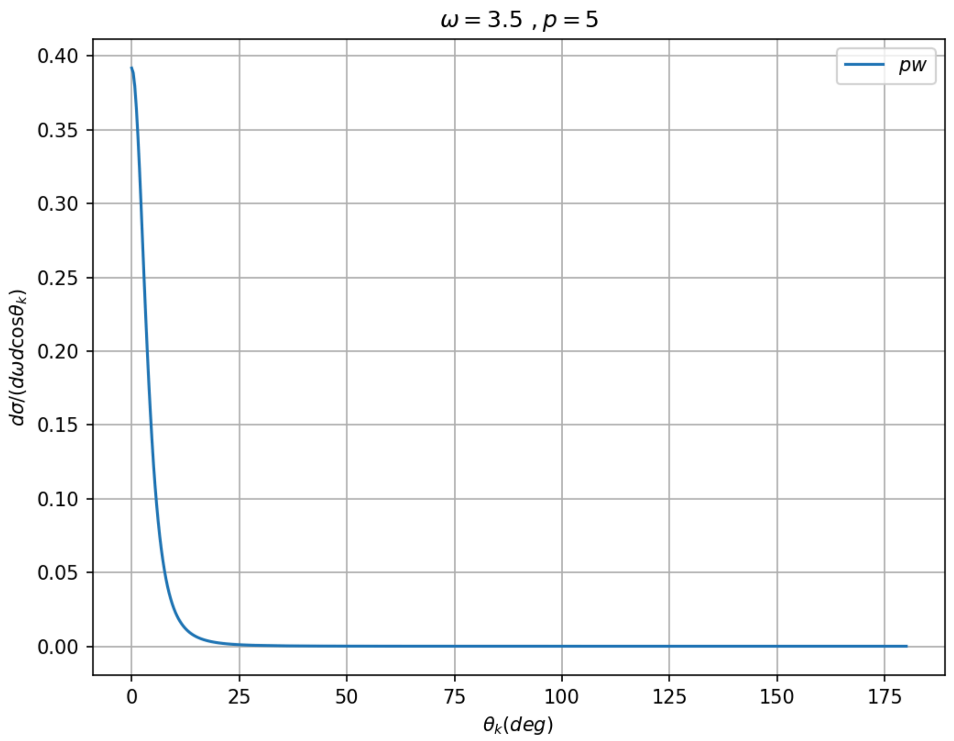


$\omega = 3.5$, $l = 10$
1e−9
2.5
2.0
1.5
1.0
0.5
0.0
$dP/d\omega$
0
1000
2000
3000
4000
5000
6000
7000
8000
$b_\perp$
$il\phi$
no $il\phi_r$
no $il\phi_p$

与平面波结果对比



$p = 5Mev$
pw
0.020
0.015
0.010
0.005
0.000
$d\sigma/d\omega$
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
ω (MeV)



$\omega = 3.5\ , p = 5$
pw
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
$d\sigma/(d\omega d\cos\theta_k)$
0
25
50
75
100
125
150
175
$\theta_k(deg)$