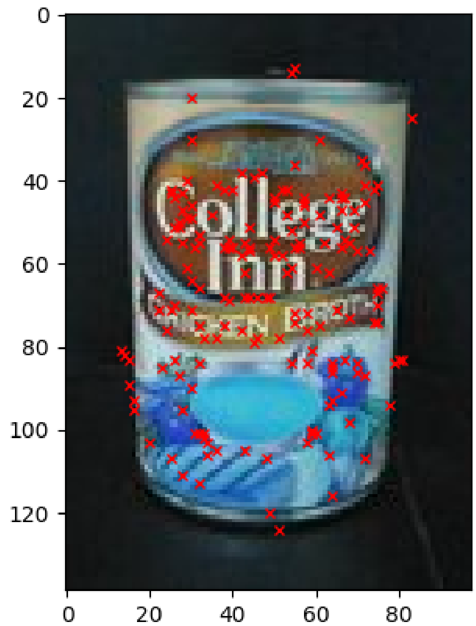
Assignment 2

Problem 1

Q1.1

Q1.2

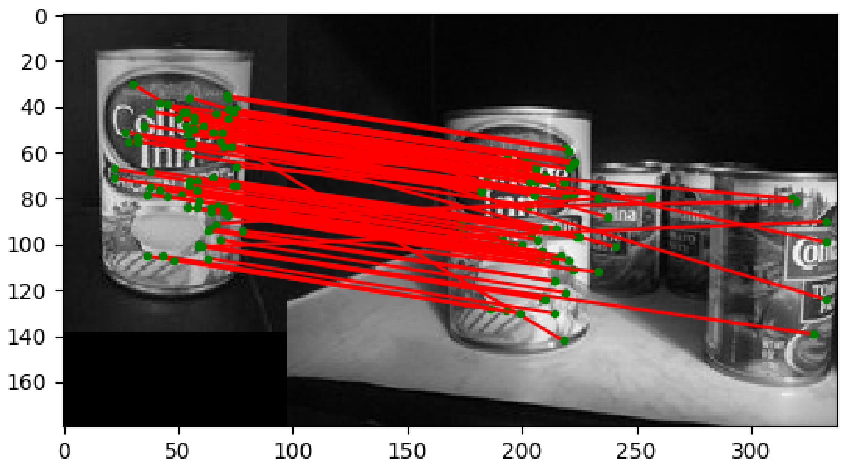
Q1.5

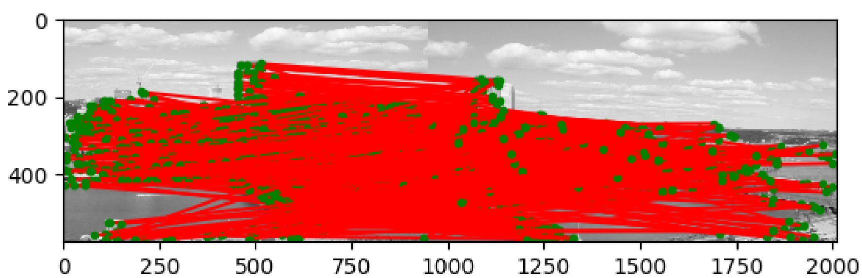


Problem 2

Q2.4

The following images show BRIEF matching on the chicken image, the incline and the pf images.


0
20
40
60
80
100
120
140
160
0
50
100
150
200
250
300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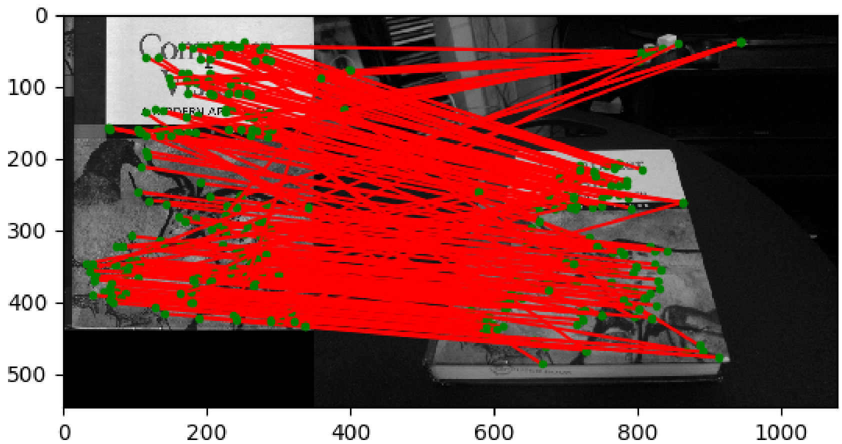
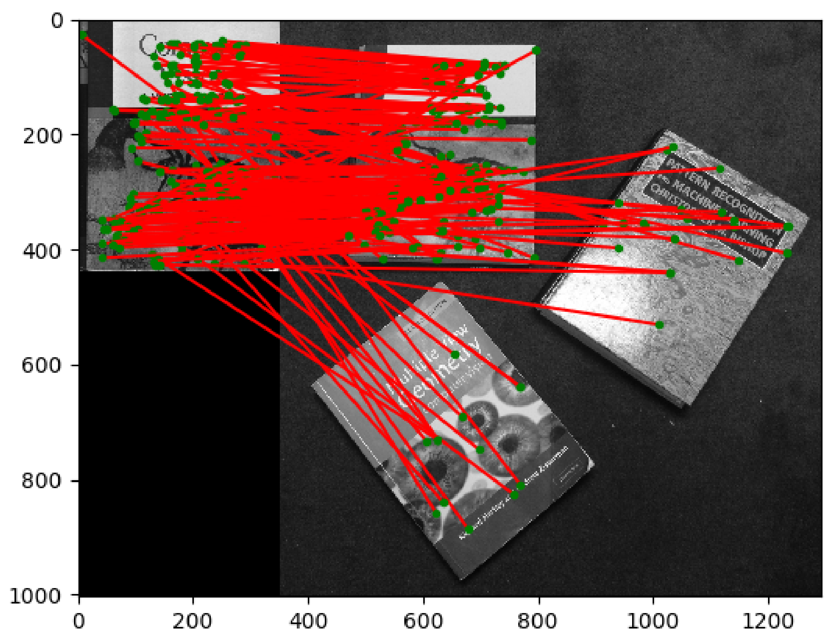
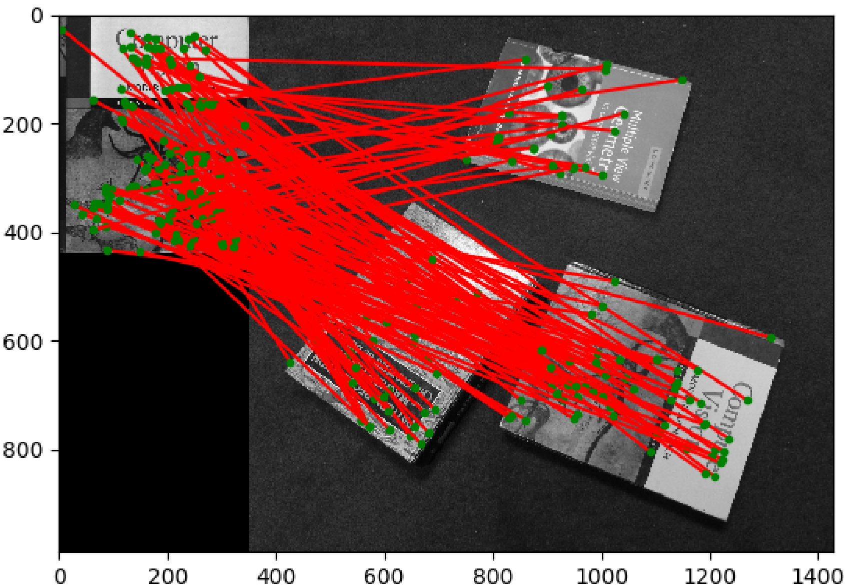
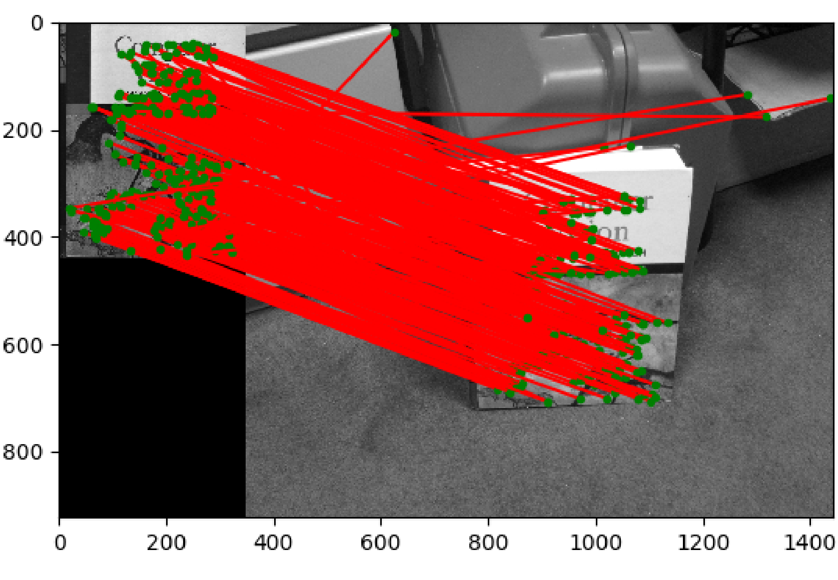
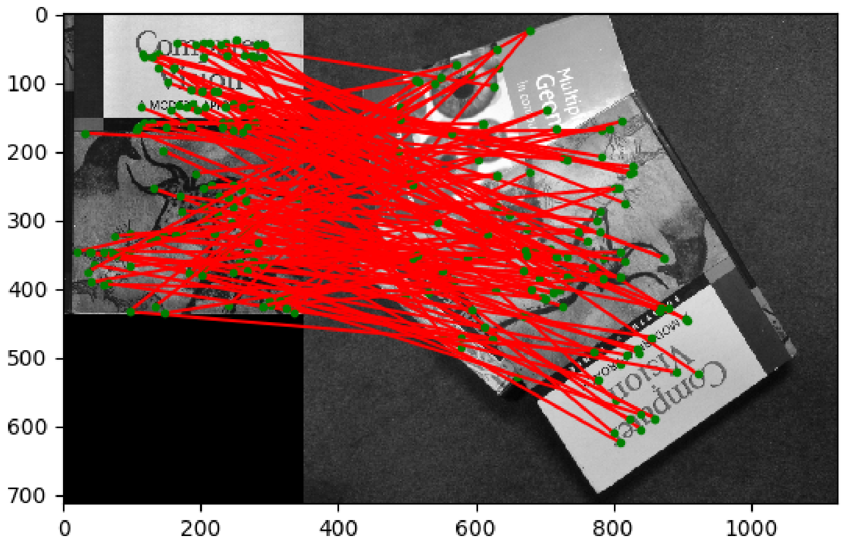
200
400
0
250
500
750
1000
1250
1500
1750
2000


0
100
200
300
400
500
0
200
400
600
800
1000


0
200
400
600
800
1000
0
200
400
600
800
1000
1200


0
200
400
600
800
0
200
400
600
800
1000
1200
1400

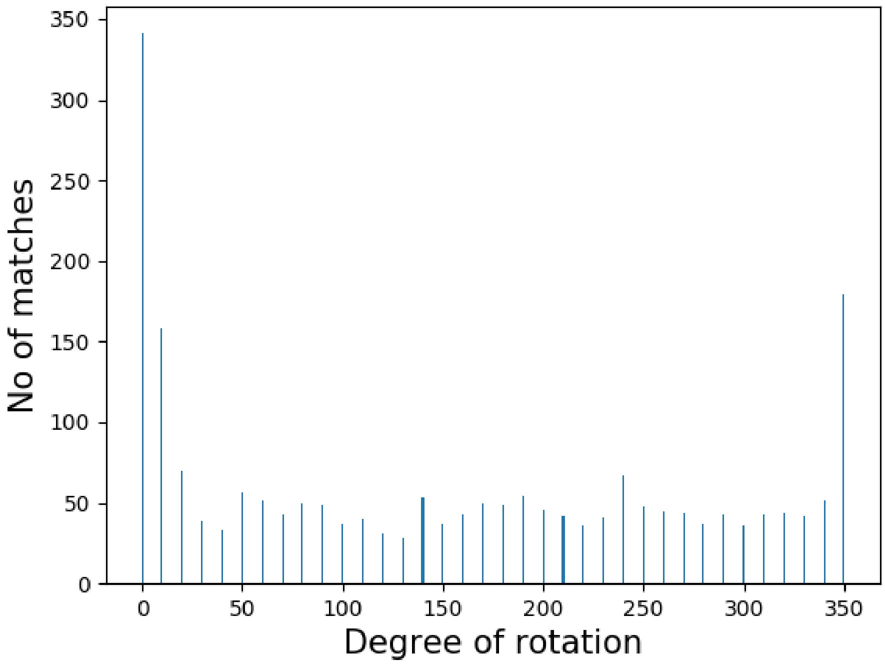

0
200
400
600
800
0
200
400
600
800
1000
1200
1400



As mentioned in the assignment, rotations tend to have bad matches than images without any rotation. Also, images where multiple books exist often have spurious matches although the chicken image where multiple cans exist does not have many false matches. This might have to do with the cans being different from each other in intensity values and alignment whereas some of the books are very similar to the template image.

Q2.5

The following is the plot of the variation of number of matches with the degree of rotation. The number of BRIEF matches significantly drops in the first 10 degree rotation itself and then subsequently drops to a lower value for the next iterations of the degree of rotation. The match count remains similar across the whole spectrum until the rotation has brought back the image to its original position. BRIEF descriptor is not rotation invariant and thus performs poorly. The script briefRotTest.py can be run directly (python briefRotTest.py) to see the same plot.



Problem 3

Q3.1

The homography equation with source point u and destination points x is written as follows:

$$\lambda \mathbf{x} = H\mathbf{u}$$

Writing them in cartesian form with h_{ij} as the values in H matrix:

$$x = \frac{h_{11}u + h_{12}v + h_{13}}{h_{31}u + h_{32}v + h_{33}}$$
$$y = \frac{h_{21}u + h_{22}v + h_{23}}{h_{31}u + h_{32}v + h_{33}}$$

Rearranging them and writing the equation with values of H as the basis, we get the following matrix equations:

$$Ah = \begin{bmatrix} 0 & 0 & 0 & -u & -v & -1 & yu & yv & y \\ u & v & 1 & 0 & 0 & 0 & -xu & -xv & -x \end{bmatrix} \begin{bmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{21} \\ h_{22} \\ h_{23} \\ h_{31} \\ h_{32} \\ h_{33} \end{bmatrix} = 0$$

We can generalize this for N points by adding 2 rows for each point.

$$Ah = \begin{bmatrix} 0 & 0 & 0 & -u_1 & -v_1 & -1 & y_1u_1 & y_1v_1 & y_1 \\ u_1 & v_1 & 1 & 0 & 0 & 0 & -x_1u_1 & -x_1v_1 & -x_1 \\ . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . \\ 0 & 0 & 0 & -u_N & -v_N & -1 & y_Nu_N & y_Nv_N & y_N \\ u_N & v_N & 1 & 0 & 0 & 0 & -x_Nu_N & -x_Nv_N & -x_N \end{bmatrix} \begin{bmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{21} \\ h_{22} \\ h_{23} \\ h_{31} \\ h_{32} \\ h_{33} \end{bmatrix} = 0$$

Q3.2

There are 9 elements in h

Q3.3

The above set of questions require 4 points of corresponding pairs but of course some conditions like non-collinearity need to be satisfied for those points in order to compute the H matrix.

Q3.4

The above system equation for N points can be minimized to solve for h. The problem is formulated as follows:

$$argmin_h ||Ah||_2^2, st||h||_2^2 = 1$$

The above problem can also be re-written as the minimization of the Rayleigh's Quotient

$$argmin_h \frac{h^T A^T Ah}{h^T h}$$

The Raylegh's Quotient attains a minimum algebraic value when the h vector is the eigenvector corresponding to the smallest eigenvalue of $A^T A$. Alternatively, the same solution can also be found by doing the SVD of matrix A and finding the null space of it.

Problem 6

Q6.1

Q6.3

Problem 7

The script augmented_reality.py can be directly run (python augmented_reality.py) to see the plot of the sphere's points on the book. The variable 'center' can be changed to modify the center of the sphere's points.

Q7.2