Fintech HW3

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Part 1: an OLS regression

Note that doing a regression where you lag all the predictors by 1 row is the same as a regression where you 'lead' the response variable by 1. We then regress excess return on the market vs the previous day's factors, and excluding date from the regression model.

Then, we can also calculate a few summary statistics, including the R-Squared value and p-value:

```
lm_fit <- dd_1970 %>%
    mutate(mkt_excess = lead(mkt_excess, 1)) %>%
    select(-Date) %>%
    lm(mkt_excess ~ ., data = .)

lm_fit %>%
    summary %>%
    glance %>%
    knitr::kable(digits = 4)
```

r.squared	adj.r.squared	sigma	statistic	p.value	df
0.3564	0.2011	0.0089	2.2945	0	50

The R-squared value is about 0.36, and the p-value is rounded to 0. This is pretty good!

I would want to know more than just the R-Squared and the p-value before jumping into a trading strategy on this model, for example, checking the regression assumptions, and reducing the number of factors we use in order to avoid overfitting the data.

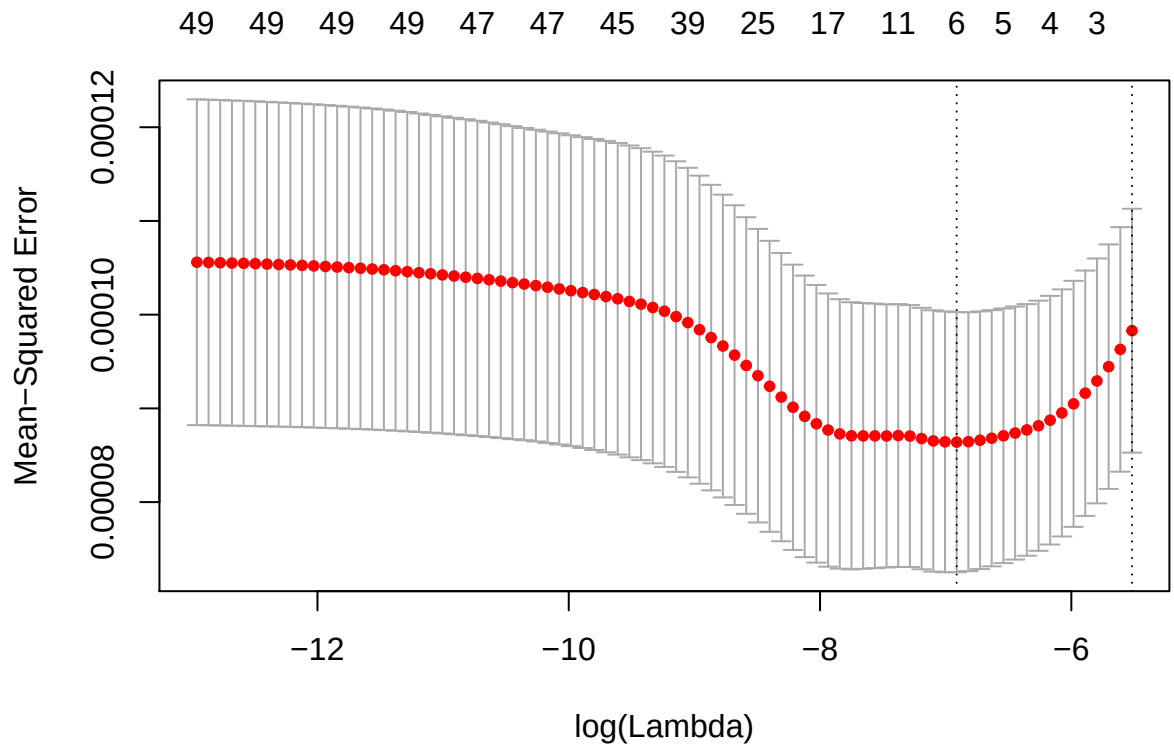
Part 2: A Lasso regression

Lasso is a regularization scheme, using the L_1 norm to 'choose' useful predictors for regression. We perform cross-validation to determine the optimal Lasso λ penalization coefficient. This next graph shows the mean cross-validation error as a function of the λ regularization penalty.

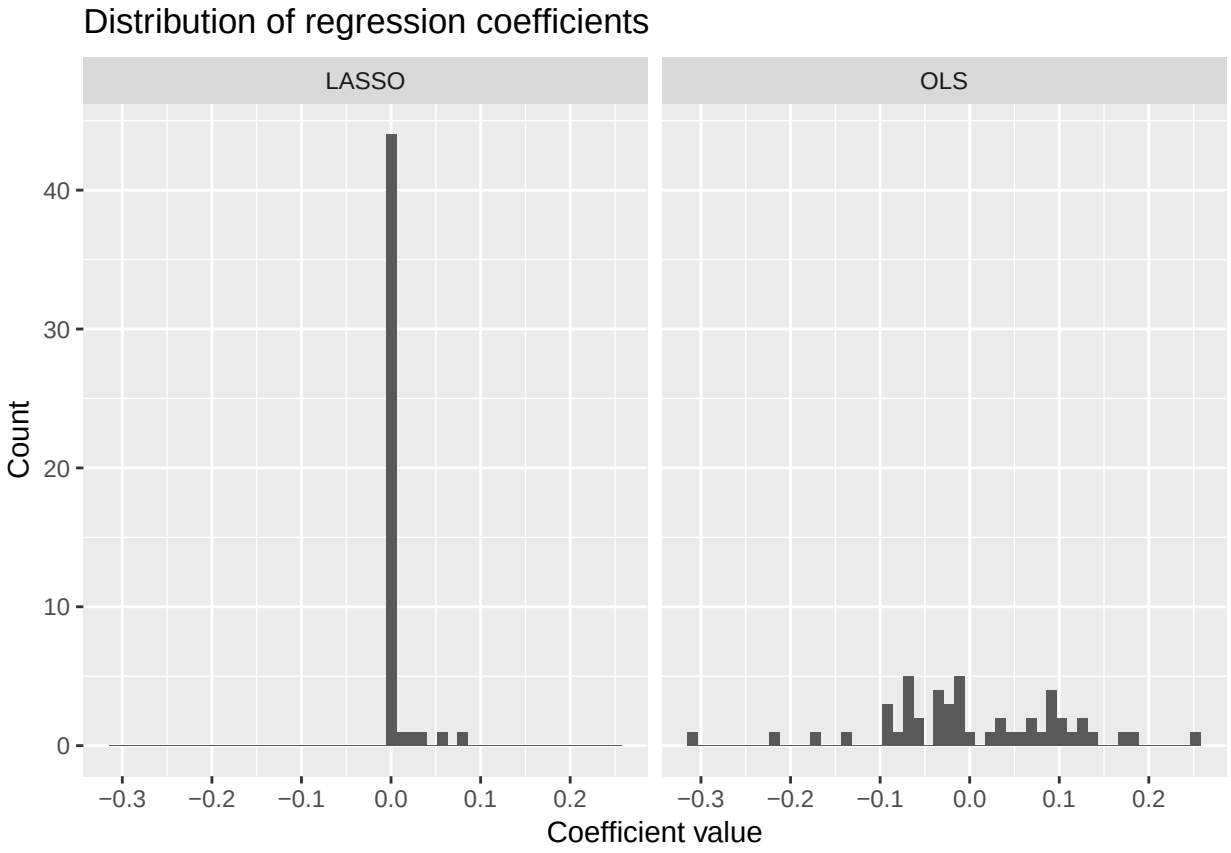
```
lag_mat <- dd_1970 %>%
    mutate(mkt_excess = lead(mkt_excess, 1)) %>%
    select(-Date) %>%
    model.matrix(mkt_excess ~ ., data = .) %>%
    extract(,-1)

lead_y <- dd_1970 %>%
    mutate(mkt_excess = lead(mkt_excess, 1)) %>%
    select(mkt_excess) %>%
    unlist %>%
    extract(-254)

glm_fit <- cv.glmnet(x = lag_mat, y = lead_y, alpha = 1)
```



The optimal Lasso λ is at 0. Using this lambda, we can get the values of the coefficients, and then plot the distribution of the coefficients under OLS and under L1-regularization.



We notice that most of the coefficients under LASSO are centered at 0. This is because the L1-regularization draws coefficients to 0, and forces some coefficients to be 0 to create a more parsimonious model.

Part 3: Making trades, making moves

Q1: OLS

```
dd_1971 <- read.csv("Data_Daily_1971.csv")
lag_1971 <- dd_1971 %>% mutate(mkt_excess = lead(mkt_excess, 1))
```

We can determine if a day will be positive or not via the `predict` function. Then we will decide to go long or short depending on if that prediction is positive or negative.

We do a bit of clever (or hacky) math here. If we think a day is positive, then we go long, and our return is the same as the market's return. If we think a day is negative, then we go short, and our return is $-1 \times R$. Then, we can represent our returns as $sign(\hat{R}_i) \times R_i$, where $sign$ is the sign of our prediction for some day i, and R_i is the actual return.

```
pred_lm <- predict(lm_fit, lag_1971)
long_lm <- (pred_lm > 0) * 2 - 1  # hacky, I know
returns_lm <- lag_1971$mkt_excess * long_lm
returns_lm <- returns_lm[complete.cases(returns_lm)]
daily_lm <- cumprod(1+returns_lm)
```

We guess the correct direction on 61.11% of the days. Absent transaction costs, this gives us a total gross return of 0.36. The Sharpe ratio is 0.19.

Q2: Lasso

```
coef_glm2 <- as.matrix(coef(glm_fit, s="lambda.min")) %>% as.vector
mat_glm <- as.matrix(select(lag_1971, -Date, -mkt_excess))
pred_glm <- predict(glm_fit, newx = data.matrix(mat_glm), s = "lambda.min", type = "response")

long_glm <- (pred_glm > 0) * 2 -1
returns_glm <- lag_1971$mkt_excess * long_glm
returns_glm <- returns_glm[-which(is.na(returns_glm))]
daily_glm <- cumprod(1 + returns_glm)
```

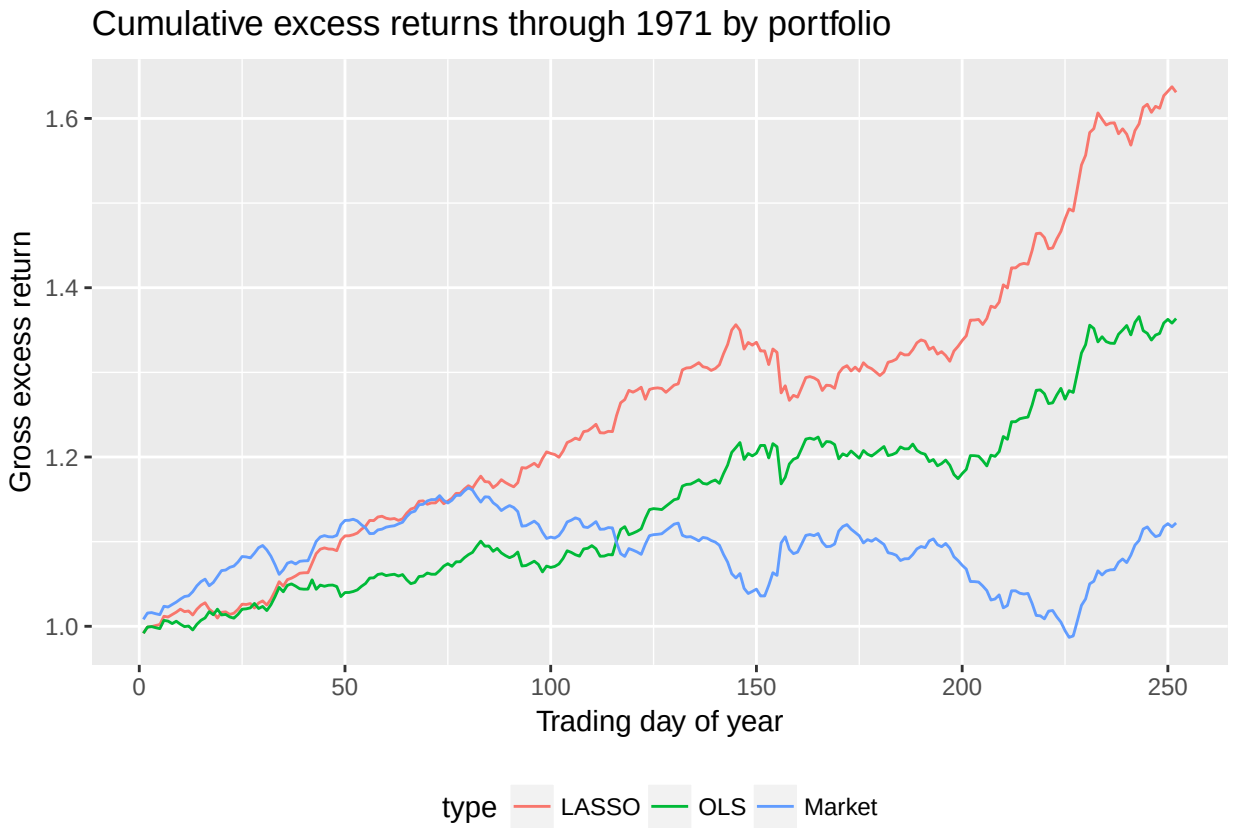
We guess correct direction on 64.68% of the days. Absent transaction costs, this gives us a total gross return of 0.63. The Sharpe ratio is 0.31.

This is significantly better than the original OLS formulation. The LASSO regularization makes the resulting model more robust to overfitting by 'drawing down' the more extreme coefficients and making the model more parsimonious by also forcing some values to 0.

We can plot these returns together:

```
df_glm_gross <- data.frame(type = "LASSO", value = daily_glm, day = 1:252)
df_lm_gross <- data.frame(type = "OLS", value = daily_lm, day = 1:252)
df_mkt_gross <- data.frame(type = "Market",
                           value = cumprod(lag_1971$mkt_excess[1:252] + 1),
                           day = 1:252)
df_gross <- rbind(df_glm_gross, df_lm_gross, df_mkt_gross)
```

```
df_gross %>%
    ggplot(aes(x = day, y= value, color = type)) +
    geom_line() + theme(legend.position = "bottom") +
    ggtitle("Cumulative excess returns through 1971 by portfolio") +
    xlab("Trading day of year") + ylab("Gross excess return")
```



The LASSO-regularized model performs the best through the year, the OLS model performs worse, and the equity portfolio performs the worst. Alpha, baby!