Local Polynomial Regression

Statistical Machine Learning - Individual project

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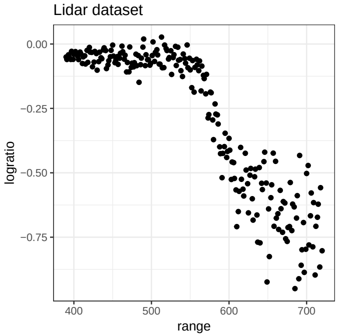
Problem statement: Lidar dataset

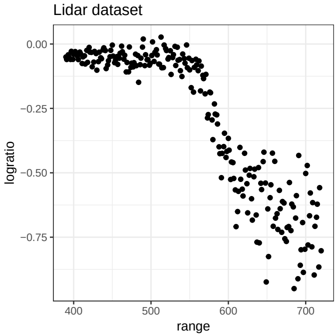


LIDAR = LIght Detection And Ranging

- it is a surveying method that measures distance to a target by illuminating the target with laser light and measuring the reflected light with a sensor
- x: distance travelled before the light is reflected back to its source
- y: logarithm of the ratio of received light from two laser sources

The objective is to estimate:

$$f(x) = E[Y \mid X = x]$$



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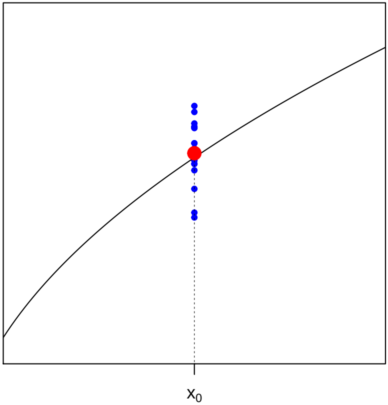

What does local means?

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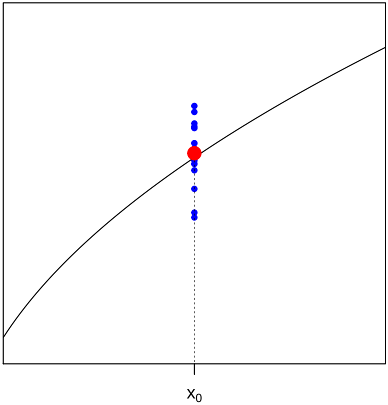
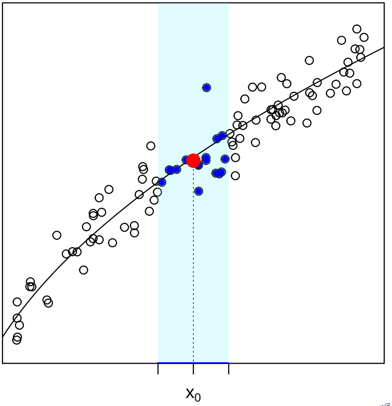
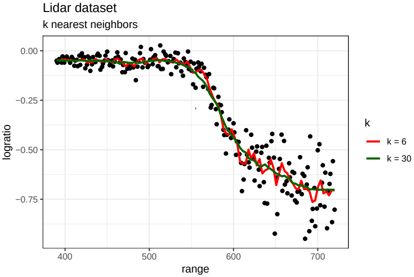


We can consider points "close" to x_0



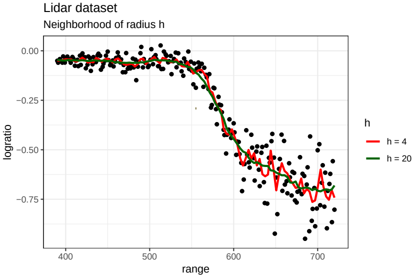
k nearest neighbors

$$\hat{f}(x) = \frac{1}{k}\sum_{i=1}^{n} y_i I_{N_k(x)}(x_i)$$



Neighborhood of radius h

$$\hat{f}(x) = \frac{\sum_{i=1}^{n} y_i I_{[0,h]}(|x - x_i|)}{\sum_{i=1}^{n} I_{[0,h]}(|x - x_i|)}$$



$$\hat{f}(x) = \sum_{i=1}^{n} \ell_i(x) y_i$$

with:

$$\ell_i(x) = \frac{K\left(\frac{x-x_i}{h}\right)}{\sum_{j=1}^{n} K\left(\frac{x-x_j}{h}\right)}$$

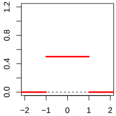
where $K(\cdot)$ is a kernel function that satisfies:

- $K(x) \geq 0$
- $\int K(x)dx = 1$
- $\int xK(x)dx = 0$
- $\int x^2 K(x)dx > 0$

Some proposed kernels

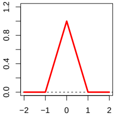
Uniform

$$\frac{1}{2} I_{[-1,1]}(u)$$



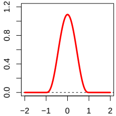
Triangle

$$(1 - |u|) I_{[-1,1]}(u)$$



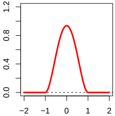
Triweight

$$\frac{35}{32}(1 - u^2)^3 I_{[-1,1]}(u)$$



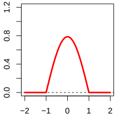
Quartic

$$\frac{15}{16}(1 - u^2)^2 I_{[-1,1]}(u)$$



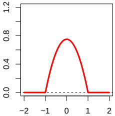
Cosine

$$\frac{\pi}{4} \cos\left(\frac{\pi}{2} u\right) I_{[-1,1]}(u)$$



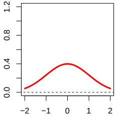
Epanechnikov

$$\frac{3}{4}(1 - u^2) I_{[-1,1]}(u)$$

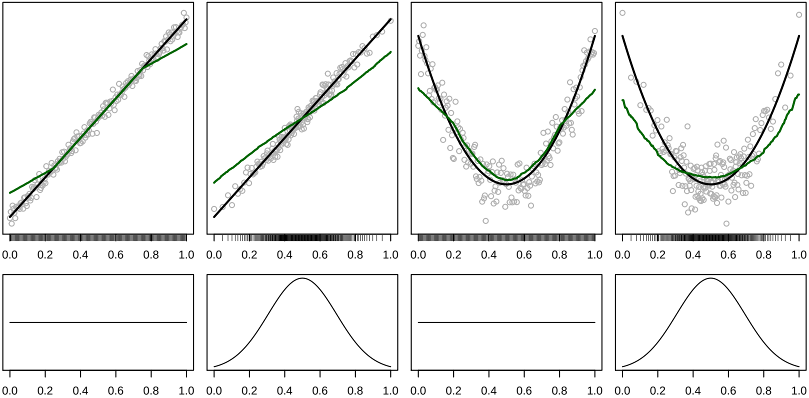


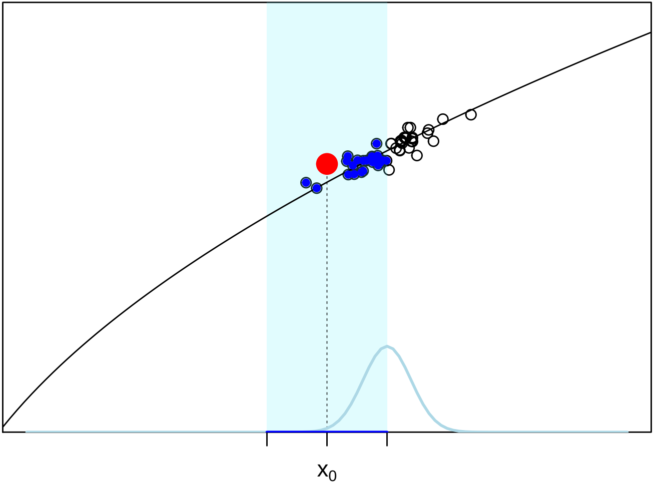
Gaussian

$$\frac{1}{\sqrt{2\pi}} e^{-u^2/2}$$



Design bias, boundary bias and concavity bias




x_0

titolo del frame