

Università degli Studi di Trieste

Dipartimento di Scienze Economiche, Aziendali, Matematiche e Statistiche Corso di Laurea Magistrale in Scienze Statistiche e Attuariali

Tesi di Laurea

Application of GLM Advancements to Non Life Insurance Pricing

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The data scientist is a person who is better at statistics than any software engineer and better at software engineering than any statistician.

Josh Wills

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Abstract

This is my abstract ...

Table of Contents

| Lı | st of I | rigures | | XI |
|----|---------|---------------|--|------|
| Li | st of T | Fables | | xiii |
| In | trodu | ction | | 1 |
| | Thes | sis aim . | | . 1 |
| | Actu | ary and | datascientist figure | . 1 |
| | Thes | sis struc | ture | . 1 |
| 1 | Non | -Life In | nsurance Pricing | 3 |
| | 1.1 | What a | a Non-Life Insurance is | . 3 |
| | 1.2 | Non-L | ife insurance pricing | . 5 |
| | | 1.2.1 | Compound distribution hypotheses | . 5 |
| | | 1.2.2 | Distribution of the total cost of claims | . 7 |
| | | 1.2.3 | Risk premium and Technical Price | . 8 |
| | 1.3 | Model | ling and Personalization | . 8 |
| | | 1.3.1 | Pricing variables | . 9 |
| | | 1.3.2 | Pricing variables encoding | 10 |
| | | 1.3.3 | Pricing Rule and Modeling | 11 |
| | | 1.3.4 | Response variables and distribution | . 12 |
| | | 1.3.5 | Model fitting and data available | . 12 |
| | | 1.3.6 | Data available | . 12 |
| | 1.4 | Non-L | ife Insurance in Italy | . 13 |
| | 1.5 | The ac | ctuary role | . 13 |
| 2 | Stat | istical n | nodels for Non Life Insurance Pricing | 15 |
| | 2.1 | Statist | ical Models | . 15 |
| | | 2.1.1 | GLM | 16 |
| | | 2.1.2 | Elastic Net | 16 |
| | | 2.1.3 | GAM | . 17 |
| | | 2.1.4 | GBM | . 17 |

TABLE OF CONTENTS

| | 2.2 | Model comparison | 18 |
|----|--------|------------------------|----|
| | 2.3 | The actuary importance | 18 |
| | 2.4 | Implementation | 19 |
| 3 | Prac | ctical application | 21 |
| | 3.1 | Data description | 21 |
| | 3.2 | Model used | 22 |
| | 3.3 | Model assessment | 22 |
| | 3.4 | Results | 23 |
| Bi | bliogi | raphy | 25 |

List of Figures

| 1.1 | Insurance Contract cash flows. | | | | | | | | | | | | | | 5 |
|-----|--------------------------------|--|--|--|--|--|--|--|--|--|--|--|--|--|---|

List of Tables

| 1 1 | Dummer vanishlas an as din a | | | | | | | | | | | | | | | | | | | | | | | 1 | 1 |
|-----|------------------------------|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|------|------|--|--|---|---|---|-----|
| 1.1 | Dummy variables encoding. | • | • | • | • | • | • | • | • | • | • | • | • | • | • | • | • | | | | | • | • | 1 | . 1 |

Introduction

| La mia introduzione |
|----------------------------------|
| Thesis aim |
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| Lorem ipsum |
| Thesis structure |
| Lorem ipsum |
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1

Non-Life Insurance Pricing

In this chapter I am going to provide an overview on how non-life insurance works from an actuarial point of view with a specific focus on the pricing process.

1.1 What a Non-Life Insurance is

The Italian Civil Code provides the following definition of insurance contract:

Definition 1.1 (Insurance Contract, Art. 1882, Italian Civil Code). The insurance is the contract with which an insurer, in exchange of the payment of a certain premium, obliged himself, within the agreed limits:

- 1. to pay an indemnity to the insured equivalent to the damage caused by an accident;
- 2. or to pay an income or a capital if a life-related event occurs.

This definition identifies two parties: the *Insurer* and the *Policyholder*. The policyholder pays to the Insurer a certain *Premium* at the beginning of the insurance coverage and the insurer will pay a benefit if a certain event (*Claim*) occurs. This event could happen zero, one or more than one times, so it is possible to have more than one claim.

Usually, in non-life insurance, the benefit is the payment of a sum. This sum could be predetermined (e.g. in motor theft insurance, where the benefit is usually the value of the insured vehicle) or defined by the entity of the claims (e.g. in motor third party liability insurance, it depends on the damage the policyholder has provided to a third party). Regarding the "agreed limits", another peculiarity of non-life insurances is that the coverage period is defined as a fixed amount of time, usually corresponding to 1 year.

Starting from this legal definition, we can formalize a non-life insurance contract as follows. Let's:

- $[t_1, t_2]$, with $t_1 < t_2$, be the coverage period;
- P > 0 be the premium payed by the policyholder to the insurer;
- $N \in \mathbb{N}$ be the number of claims occurred during the coverage period (*claims count*);
- $\tau_1, \tau_2, \dots, \tau_N$, with $t_1 < \tau_1 < \tau_2 < \dots < \tau_N < t_2$, be the timing of each claim;
- $Z_1, Z_2, \dots, Z_N > 0$ be the amount of each claim (claims severities or claims sizes).

The total cost of claims for the insurance is:

$$S = \begin{cases} 0 & \text{if } N = 0\\ \sum_{i=1}^{N} Z_i & \text{if } N > 0 \end{cases}$$

For semplicity, in the following we are going to just use the notation $S = \sum_{i=1}^{N} Z_i$ with the meaning of 0 if N = 0.

Figure 1.1 shows the cash flows corresponding to the insurance contract. From this representation we can interpret the entering into an insurance contract by the policyholder as a way to exchange the negative cash flows $-Z_1, -Z_2, \ldots, -Z_N$ with one single negative cash flow -P. On the other hand, the insurer undertakes the negative cash flows $-Z_1, -Z_2, \ldots, -Z_N$ in exchange for a positive cash flow +P.

The major difference between these cash flows is that P is a certain amount, while Z_1, Z_2, \ldots, Z_N , at the time t_1 , are uncertain in the amount, in the count (N) and in the timing $(\tau_1, \tau_2, \ldots, \tau_N)$. So, the policyholder, paying a premium P, is giving his risk to the insurer.

This representation points out the inversion of the production cycle typical of the insurance activity. From the insurer point of view, the revenue emerges at the beginning of the economic activity, in t_1 , while the costs will emerge later. In most of other economic activities, the costs emerge before the selling of the product, so the agent can choose the selling price taking into account how much that product costed him. In insurance activity, the insurer, when is selling his product (the insurance coverage) doesn't know the amount of costs he is going to pay for that product. Thus, it is crucial to properly estimate the future costs and determine an adequate premium.

From a statistical point of view, we can translate this uncertainty saying that N and Z_1, Z_2, \ldots, Z_N are random variables. Therefore, we can say that $\{N, Z_1, Z_2, \ldots\}$ is a stochastic process. Usually, in non-file insurance pricing, the variables $\tau_1, \tau_2, \ldots, \tau_N$ are not taken into account because the coverage span is short and from a financial point of view the timing of the claims occurrences is negligible.

CHAPTER 1. NON-LIFE INSURANCE PRICING

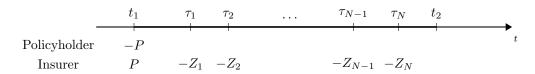


Figure 1.1: Insurance Contract cash flows.

Previously we said that $Z_1, Z_2, ..., Z_N$ are all > 0. This assumption corresponds to the fact that we are excluding the null claims, i.e. the claims that have been opened, but result in no payment due by the insurer. For the values of Z_i with N < i we can use the rule that $N < i \Rightarrow Z_i = 0$, so $Z_{N+1} = 0, Z_{N+2} = 0, ...$ Therefore, we can say that $N < i \Leftrightarrow Z_i = 0$.

1.2 Non-Life insurance pricing

In insurances, the premium that the the insurer offers to the policyholder in exchange for the insurance coverage is not the same for every policyholder. The insurer evaluates the risk related to that policy and determine a "proper" premium taking into account risk related factors and commercial related factor. The process of *pricing* corresponds in defining the set of rules for determining this "proper" premium P_i for a specific policyholder i, given the known information on him. In the next sections I am going to better explain what "proper" means.

1.2.1 Compound distribution hypotheses

The first step for evaluating the stochastic process $\{N, Z_1, Z_2, ...\}$ is to introduce some probabilistic hypotheses. The usual hypotheses assumed are the following:

Definition 1.2 (Compound distribution). Let's assume that:

- 1. for each n > 0, the variables $Z_1|N = n$, $Z_2|N = n$, ..., $Z_n|N = n$ are stochastically independent and identically distributed;
- 2. the probability distribution of $Z_i|N=n$, $i \le n$ does not depend on n.

Under these hypotheses we say that:

$$S = \sum_{i=1}^{N} Z_i$$

has a compound distribution.

The variables $Z_i|N=n$ used in this definition can be interpreted as the *claim severity for the ith claim under the hypothesis that n claims occurred*. The two hypotheses provided in definition 1.2 imply that the distribution of $Z_i|N=n$, i < n does not depend from i nor from n. In the following, we are going to use the notation Z to represent a random variable with the $Z_i|N=n$, i < n distribution and $F_Z(\cdot)$ for its cumulative distribution function (i.e. $F_Z(z) = P(Z \le z)$).

Let's consider the variabile $Z_i|N>i$. We can interpret it as the *claim severity for the* i^{th} *claim under the hypothesis that the* i^{th} *claim occurred*. From the hypotheses provided in definition 1.2 we can obtain that also $Z_i|N>i$ has the same distribution of $Z_i|N=n$, i< n. This can be easily obtained as follows:

$$P(Z_i|N \ge i) = P\left(Z_i \middle| \bigvee_{n=i}^{+\infty} (N=n)\right) = \tag{1.1}$$

$$= \sum_{n=i}^{+\infty} \underbrace{P(Z_i \le z | N=n)}_{=F_Z(z)} P(N=n | N \ge i) =$$
 (1.2)

$$= \sum_{n=i}^{+\infty} F_Z(z) P(N=n|N \ge i) =$$
 (1.3)

$$= F_Z(z) \underbrace{\sum_{n=i}^{+\infty} P(N=n|N \ge i)}_{=1} =$$

$$= F_Z(z)$$
(1.4)

Where:

- the step (1.1) and the step (1.2) are given by the fact that the event {N ≥ i} can be decomposed as {N ≥ i} = {V_{n=i}^{+∞}(N = n)} and that the events {N = n}, n ∈ {i, i+1, i+2,...} are two-by-two disjoint, so they constitute a partition of {N ≥ i}, that allows us to use the disintegrability property of the probability;
- the step (1.3) is due to the fact that the distribution of $Z_i \le z | N = n$ depends neither on i nor on n;
- the equivalence $\sum_{n=i}^{+\infty} P(N=n|N\geq i)=1$ at step (1.4) is due to the fact that the events $\{N=n\}, n\in\{i,i+1,i+2,\ldots\}$ are a partition of $\{N\geq i\}$.

Therefore, Z can be considered as the *claim severity for a claim under the hypothesis* that that claim occurred.

1.2.2 Distribution of the total cost of claims

Under the hypotheses defined in definition 1.2, it is possible to obtain the full distribution of S given the distribution of N and Z. In this chapter we are going to provide only the formula of the expected value E(S), but, with the same approach one can obtain all the moments.

The expected value of the total cost of claims E(S) can be obtained from the expected value of the claims count E(N) and the expected value of the claim severity E(Z) as follows:

$$E(S) = \sum_{n=0}^{+\infty} P(N=n)E(S|N=n) =$$
 (1.5)

$$= \sum_{n=0}^{+\infty} P(N=n)E\left(\sum_{i=1}^{n} Z_{i} \middle| N=n\right) =$$
 (1.6)

$$= \sum_{n=0}^{+\infty} P(N=n) \sum_{i=1}^{n} \underbrace{E(Z_i|N=n)}_{=E(Z)} =$$
 (1.7)

$$= \sum_{n=0}^{+\infty} P(N=n) n E(Z) =$$
 (1.8)

$$= E(Z) \underbrace{\sum_{n=0}^{+\infty} nP(N=n)}_{=E(N)} =$$

$$(1.9)$$

$$= E(N)E(Z) (1.10)$$

Where:

- the step (1.5) is given by the fact that the events $\{N=0\}, \{N=1\}, \{N=2\}, \dots$ constitute a partition of the certain event Ω , that allows us to use the disintegrability property of the expected value;
- the step (1.6) is due to the definition of S;
- the step (1.7) is due to the linearity of the expected value;
- the steps (1.8) and (1.9) are due to the fact that, as assumed by the compound distribution hypotheses, $E(Z_i|N=n)$ does not depends on i and n;
- the step (1.10) is due to the definition of the expected value $E(N) = \sum_{n=0}^{+\infty} nP(N=n)$.

This result tells us that, under the hypotheses of the compound distribution, it is possible to easily obtain E(S) from E(N) and E(Z). That means that we can model separately E(N) and E(Z) and, from them, obtain E(S). That result is particularly useful in personalization (paragraph 1.3), because, for each individual i, given the information we have on him $x_i = (x_{i1}, x_{i2}, \dots, x_{ip})$,

we can estimate his expected claim size $E(N_i)$ and his expected claim severity $E(Z_i)$ and obtain his expected total cost of claims as $E(S_i) = E(N_i)E(Z_i)$.

1.2.3 Risk premium and Technical Price

The expected cost of claims E(S) is important because it gives us a first interpretation of what "proper" premium means.

Definition 1.3 (Risk Premium). Said S the total cost of claims of a policyholder, his *Risk Premium* is given by:

$$P^{(risk)} = E(S)$$

The *Risk Premium* is the premium that on average covers the total cost of claims. As mentioned above, as the coverage spans are usually short, we are not taking into account the timing of the claims and we don't discount the fact that the claims occur later than the premium payment.

It is clear that this premium, that only cover the cost of claims, is not "proper" in the practice.

First of all, the insurer has to cover also the expenses related to the policy (commission on sales and expenses related to the claim settlement) and the general expenses of the company. Adding the expenses, we obtain what is called *Technical Price*.

Definition 1.4 (Technical Price). Said S the total cost of claims of a policyholder and E the expenses related to his policy, his *Technical Price* is given by:

$$P^{(tech)} = E(S) + E = P^{(risk)} + E$$

Secondly, even if the policyholder would pay a premium that on average covers claims and expenses, it would not make sense for the insurer undertaking that risk with nothing in return. So, to the Technical Price, a loading must be added. This loading can be justified as a loading for the risk or as a profit loading.

The result of the Technical Price with these loadings can be further modified based on business logic, as I am going to discuss later.

1.3 Modeling and Personalization

In this section We are going to better explain how pricing based on policyholder information works.

1.3.1 Pricing variables

Usually for every policyholder we have a certain amount of information on him that is considered relevant for his risk evaluation. This information must be reliable and observable at the moment of the underwriting of the policy.

In motor insurances, this information could be:

- Information on the insured vehicle: make, model, engine power, vehicle mass, age of the vehicle;
- General information of the policyholder: age, sex, address (region, city, postcode), ownership of a private box where he parks the car;
- Insurance specific information of the policyholder: number of claims caused in the previous years, how long he has been covered, bonus-malus class;
- Policy options: amount of the maximum coverage, presence and amount of a deductible, presence of other insurance guarantees, how many drivers will drive the vehicle;
- Customer information on the policyholder: how many years he has been a customer of the insurer, how many other policies he owns.
- Telematics data: how many kilometers per year the policyholder travelled in the previous years, how many sharp accelerations and decelerations per kilometer the policyholder performed in the previous years.

These pieces of information are usually called *pricing variables*.

We must observe that some of these variables are available for every potential customer (such as his age and address), while others are only available for policyholder that are already customers (such as telematics data that is available only if the policyholder agreed on installing on their car the device that collects this data).

Moreover, even considering the variables that are available for every customer, it is important to be aware on how reliable they are. Some of them comes from official documents (as customer age and address or bonus-malus class), but others could be declared by the customer and his statements are not easily verifiable by the insurer (as the ownership of a private box or how many drivers will drive the vehicle).

This topic of variables reliability fits in the wider framework of fraud detection. Usually insurance companies put a lot of effort in preventing frauds. This is done with active actions, such as documents checks and inspections, and with predictive fraud detections models. The two most common categories of frauds are underwriting frauds (such as false declaration on insurance related data) and settlement frauds (such as faking an accident). The customer information on the policyholder are usually important to predict both underwriting fraud and settlement

fraud. Usually customers that have a longer relationship with the company and own many policies are less likely to commit frauds.

Regarding the topic of variables reliability, the Italian Insurance Associations (ANIA) in the last years made some big steps forward by collecting in its databases a lot of information about policyholders and vehicles and making it available to insurance companies. For example, by logging in these databases it is possible, at the moment of the quote request, to retrieve useful insurance specific information such as the number of claims caused by the customer in the previous years or how long he has been covered and useful information on his vehicle as when it has been registered or how many changes of ownership did it experienced.

One of the roles of the actuary is to understand how reliable the information on the policyholder is and to decide how to use that information.

1.3.2 Pricing variables encoding

Formally the pricing variables can be encoded as a vector of real numbers. $x_i = (x_{i1}, x_{i2}, \dots, x_{ip}) \in \mathcal{X} \subseteq \mathbb{R}^p$. In the modeling framework they can be also called explanatory variables, covariates, predictors or features.

The pricing variables can be of two types:

- 1. *Quantitative variables*: variables, like policyholder age or vehicle mass, that can be easily represented as a number;
- 2. *Qualitative variables*: variables, like policyholder sex or vehicle make, that represent a category and are usually represented with strings.

The quantitative variables, with eventual transformations, are already suitable to be used.

To facilitate the use of the qualitative variables, they are usually encoded as sets of binary variables.

If a variable x has only 2 possible modalities, it can be easily encoded in a binary variable z that assigns 0 to one modality and 1 to the other. For example, if $x = \sec x$, it can be encoded this way:

$$z = \begin{cases} 1 & \text{if sex } = \text{`Male'} \\ 0 & \text{if sex } = \text{`Female'} \end{cases}$$

In general, if a variable x has K modalities, it can be encoded in K-1 binary variables $z_1, z_2, \ldots, z_{K-1}$. For example, if x = make and it can have 4 possible modalities ('Fiat', 'Alfa-Romeo', 'Lancia', 'Ferrari') it can be encoded this way:

$$z_{1} = \begin{cases} 1 & \text{if make} = \text{'Fiat'} \\ 0 & \text{otherwise} \end{cases}$$

$$z_{2} = \begin{cases} 1 & \text{if make} = \text{'Alfa-Romeo'} \\ 0 & \text{otherwise} \end{cases}$$

$$z_{3} = \begin{cases} 1 & \text{if make} = \text{'Lancia'} \\ 0 & \text{otherwise} \end{cases}$$

The variables z_1 , z_2 , z_3 are called dummy variables. We can observe that all the information about the make is embedded in just these 3 variables, so a fourth dummy variable that indicate the modality 'Ferrari' is not needed. Indeed:

make = 'Ferrari'
$$\iff z_1 = z_2 = z_3 = 0$$

In table 1.1 the dummy variable encoding is illustrated.

Table 1.1: Dummy variables encoding.

| make | z1 | z2 | z 3 |
|------------|----|----|------------|
| Fiat | 1 | 0 | 0 |
| Alfa-Romeo | 0 | 1 | 0 |
| Lancia | 0 | 0 | 1 |
| Ferrari | 0 | 0 | 0 |

For some models it is suggested to use also the dummy variable that indicates the K^{th} modality. This encoding is called one-hot encoding and it is mainly used in Neural Networks. For the models considered in this paper it is preferred the K-1 dummy variables encoding, so we will always consider it.

In the following, when I use the notation $x_i = (x_{i1}, x_{i2}, ..., x_{ip})$, I'll always consider that the qualitative variables have been already encoded as dummy variables, so $(x_{i1}, x_{i2}, ..., x_{ip}) \in \mathcal{X} \subseteq \mathbb{R}^p$

1.3.3 Pricing Rule and Modeling

The pricing variables are used as input of a Pricing Rule.

Definition 1.5 (Pricing Rule). A *Pricing Rule* is a function $f(\cdot)$ that from an instance of a set of pricing variables $x_i \in \mathcal{X}$ returns a price:

$$f: \ \mathcal{X} \longrightarrow R_+$$
$$x_i \longmapsto P_i$$

The process of pricing consists in defining a Pricing Rule based on observed data from the past and assumptions on the future.

The first step for defining a Pricing Rule is to model the total cost of claims S and obtain a pricing rule for the Risk Premium $P^{(risk)}$.

In general, in this context, modeling a *response variable Y* means finding a function r: $\mathcal{X} \to \mathcal{C}$ that, given a set of explanatory variables $x_i = (x_{i1}, x_{i2}, \dots, x_{ip}) \in \mathcal{X} \subseteq \mathbb{R}^p$, returns the expected value of the response variable E(Y) and eventually other moments of Y or even the full distribution of Y. I used a generic \mathcal{C} as codomain of the function $r(\cdot)$ to not specify whether in the model describe just E(Y) (and so $\mathcal{C} = \mathbb{R}$) or something more.

As we observed in section 1.2.2, under the compound distribution hypotheses, it is not needed to model directly the total cost of claims S, but we can separately model N and Z.

1.3.4 Response variables and distribution

Usually in statistical modeling, the response variables are seen as random variables with a distribution belonging to a certain family.

1.3.5 Model fitting and data available

We will discuss some of the most widespread models for claims count and claims severity in chapter 2.

1.3.6 Data available

1.4 Non-Life Insurance in Italy

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1.5 The actuary role

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CHAPTER 1. NON-LIFE INSURANCE PRICING

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Questa è una citazione (Shea et al., 2014; Lottridge et al., 2012)

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2

Statistical models for Non Life Insurance Pricing

2.1 Statistical Models

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2.1.1 GLM

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2.1.4 **GBM**

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2.2 Model comparison

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2.4 Implementation

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3

Practical application

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3.1 Data description

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3.2 Model used

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3.3 Model assessment

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CHAPTER 3. PRACTICAL APPLICATION

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3.4 Results

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