MULTICUT GAME

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1. Preliminaries

The *multicut problem* is a combinatorial optimization problem. The objective is to cut a given graph into multiple components such that the sum of the cost of the edges that are cut is minimal.

More formally, let $G = (V, E)$ be an undirected graph. A set of edges $M \subseteq E$ is called a *multicut* of G if and only if for every cycle $C = (V_C, E_V)$ in G it holds that $|M \cap E_C| \neq 1$, i.e. there is no cycle that contains exactly one edge from M. For an example, see Figure 1. The set of all multicuts of G is denoted by $\mathcal{M}(G)$. The multicut problem with respect to a graph $G = (V, E)$ and edge costs $c_e \in \mathbb{R}$ for $e \in E$ is the following optimization problem:

$$\min_{M \in \mathcal{M}(G)} \sum_{e \in M} c_e \quad .$$

The multicut problem was first studied in the late 20th century. First, [6] studied this problem for complete graphs and subsequently [3] generalized it to general graphs. The multicut problem is also known as *correlation clustering* [1] and *coalition structure generation* [13].

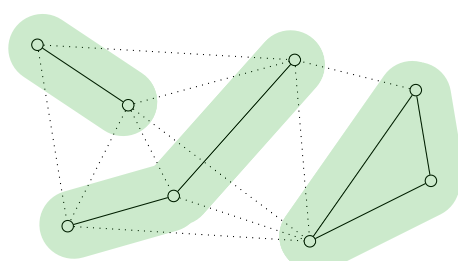

Figure 1. Depicted above is a multicut of a graph that cuts that graph into three induced components. The dotted edges are the edges that are cut. The shaded areas are the induced components.

The multicut problem is NP-hard, meaning that it is unlikely that an efficient algorithm that solves this problem exactly will ever be found. However, there are many interesting applications where the multicut problem produces state of the art results [2, 12]. Because of this, a lot of effort has gone towards understanding the

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multicut problem from a theoretical [7, 5, 3, 11] and algorithmic [6, 1, 4, 9, 10] perspective.

The Multicut Game

The goal of this project is to implement a game where the player interactively tries to solve a multicut problem. The game should be an intuitive and entertaining puzzle game. For an example, see Figure 2.

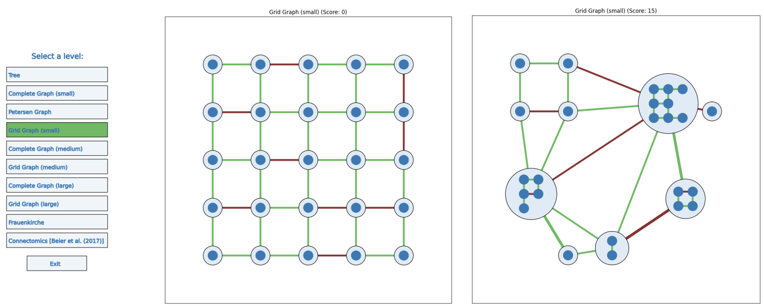

FIGURE 2. Depicted above is a prototype of the game. On the left the player can choose between various levels. Depicted in the middle is a graph for which the player should find the optimal multicut. Depicted on the right is an intermediate clustering that the player has created.

Requirements.

(1) The game should run in a browser and/or on a mobile device. A suitable software framework can be chosen freely (react, react-native, etc.).
(2) The game should be as intuitive as possible. This includes the visual appearance of the game as well as the operations that are available to the player to manipulate the graph.
(3) The game should contain multiple levels of varying difficulty. The levels can be hand designed or automatically generated.
(4) The optimal solution for each level should be computed by an algorithm such that the player can be notified if an optimal solution is found.

Optional features. Besides the main requirements described above the game can be further refined into two directions. Which direction is suitable may depend on the type of the project.

Playful and entertaining.

(5) Lag-free, smooth and graceful animations.
(6) Colorful and eye catching design.
(7) Server side high score lists.
(8) Different game modes (For example: solve a level as fast as possible, solve as many levels as possible in a fixed amount of time, player vs. player, daily challenges).

Educational.

(9) Illustrate how an algorithm solves the problem by animating the operations that the algorithm performs.
(10) Illustrate how the multicut problem is used to solve real world tasks, e.g., image segmentation.
(11) Incorporate different problem variations, e.g., correlation clustering, coalition structure generation, lifted multicut [8], etc.

You are encouraged to get creative and come up with more features to improve the game!

References

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