ETG Turbulence Isotropization

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Outline

1. Drift Wave Instabilities
2. Hasegawa-Mima Fluid Model
3. Zonal Flow Excitation

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Drift Wave Instabilities

- Drift waves are most simply characterized as density, temperature and electrostatic potential fluctuations in low-β plasmas.
- Modes relevant to tokamak physics include ion-temperature-gradient modes (ITG), electron-temperature-gradient modes (ETG), and collisionally-trapped electron modes (CTEM).
- Low-frequency drift wave turbulence is largely responsible for the anomalous transport of plasma particles across magnetic field lines.

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Ion-Temperature-Gradient Mode

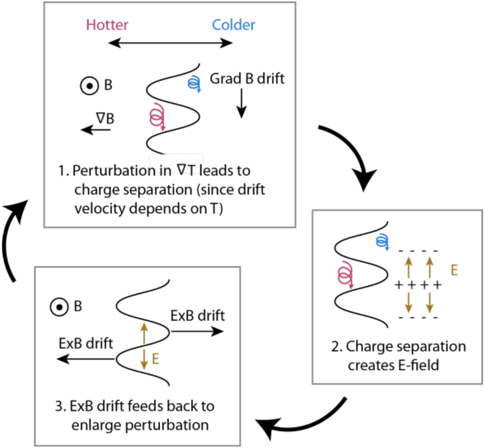
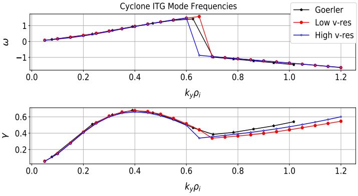
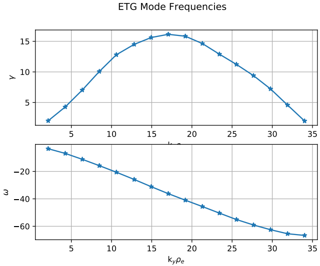
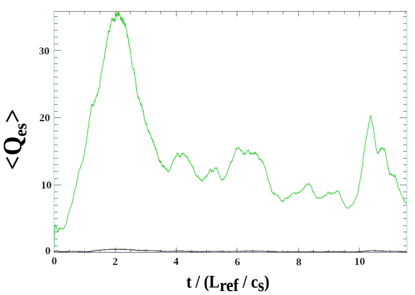


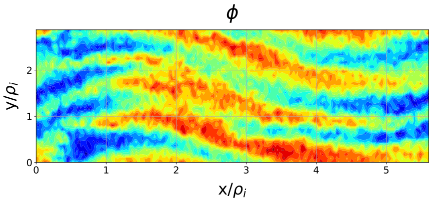
Figure: Simple pciture of ITG instability.

ETG Simulation in GENE









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Hasegawa-Mima Fluid ETG Model

- Partial differential equation derived from fluid continuity and momentum equations.
- Approximations made that are useful to describing turbulence in tokamak plasmas.
  - Cyclotron motion periods much smaller than time scales that quantities of interest change on (B,Φ,n).
  - Long length scales along $\hat{b}$-direction - $k_{||}$ ignorable.
  - Quasi-neutrality of particle densities is enforced.
  - Isothermal equation of state, with adiabatic ions that have negligible temperatures.
- Shown to cause isotropic behavior for long wavelength modes as well as an inverse energy-cascade.

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Hasegawa-Mima Equations

We start with the fluid continuity and momentum equations and $\tau = T_e/T_i$, where we have already taken the ion approximations discussed on the previous slide:

$$\frac{\partial n_e}{\partial t} + \nabla \cdot (n_e \vec{v}_e) = 0, \tag{1}$$

$$m_e \frac{d\vec{v}_e}{dt} = (1+\tau)\, e\nabla\delta\Phi - \frac{e}{c}\vec{v}_e \times \vec{B} - \frac{\nabla P_e}{n_e}\,. \tag{2}$$

We break equation (2) up into parallel and perpendicular components, and break up $\vec{v}_e$ in terms of higher and lower order terms to find,

$$\begin{aligned} \vec{v}_{e,0} &= \vec{v}_{\|} + \vec{v}_{\perp,0} = \vec{v}_{\|} + (1+\tau)\,\vec{v}_E + \vec{v}_D \\ \vec{v}_{e,1} &= \vec{v}_{\perp,1} \end{aligned} \tag{3}$$

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Taking the standard electron dyanamic normalization,

$$\Phi = \frac{e\delta\Phi}{T_i}, \; -\frac{1}{r_n} = \frac{\partial_x n_e}{n_e}, \; -\frac{1}{r_t} = \frac{\partial_x T_e}{T_e}, \; \eta_e = \frac{r_n}{r_t},$$
$$\rho_e = \sqrt{\frac{\tau m_e}{m_i}}\rho_i, \; \vec{x} = \frac{\vec{x}}{\rho_e}, \; t = \frac{\rho_e}{r_n}\omega_{ce} t \tag{4}$$

and plugging into equation () gives the form of the H-M ETG model,

$$-(1 - \frac{1+\tau}{2\tau}\nabla_\perp^2)\partial_t\Phi + \frac{1+\tau}{2\tau}\frac{r_n^2}{\rho_e^2}\partial_t^{-1}\nabla_\parallel^2\Phi + \frac{(1+\tau)(1+\eta_e)}{4\tau}\partial_y\nabla_\perp^2\Phi$$
$$+ \frac{1+\eta_e}{2\tau}\partial_y\Phi + \frac{(1+\tau)^2}{\tau^2}\frac{r_n}{4\rho_e}(\hat{b}\times\nabla_\perp\Phi\cdot\nabla_\perp)\nabla_\perp^2\Phi = 0\,. \tag{5}$$

Finally we drop the parallel gradient term since $k_\parallel^2/k_\perp^2 \sim \epsilon^2$, and simplify the bracketed expression for a 2-D slab geometry to find the final form of our model,

$$\begin{aligned}\partial_t[\Phi - \frac{1+\tau}{2\tau}\zeta] = \frac{(1+\tau)(1+\eta_e)}{4\tau}\zeta_y + \frac{1+\eta_e}{2\tau}\phi_y \\ + \frac{(1+\tau)^2}{\tau}\frac{r_n}{4\rho_e}[\Phi_x\zeta_y - \zeta_x\Phi_y],\end{aligned} \tag{6}$$

where $\zeta = \nabla^2\Phi$.

Pseudo-Spectral Solver

- Equation (6) is solved numerically using the pseudo-spectral method.
  - Fourier transform the equations and get $\zeta = (k_x^2 + k_y^2)\Phi$.
  - Inverse Fourier transform $\zeta_{x,y}$ and $\Phi_{x,y}$ back into real space.
  - Calculate the non-linear products between ζ and Φ in real space and then Fourier transform the products so they can be added to the other Fourier terms.
  - Time advance Φ discretely.
- Time advancement is done using the 4^{th} order Runge-Kutta method.
- The Hasegawa-Mima equations will conserve generalized energy and enstrophy values as a 2-d incompressible fluid conserves the kinetic energy and enstrophy of the fluid.

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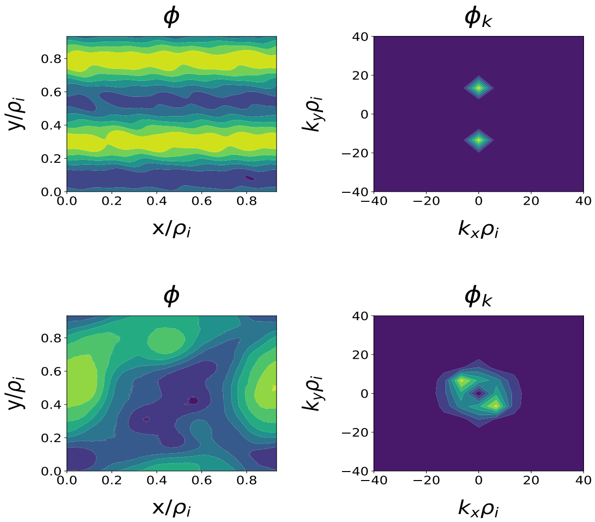
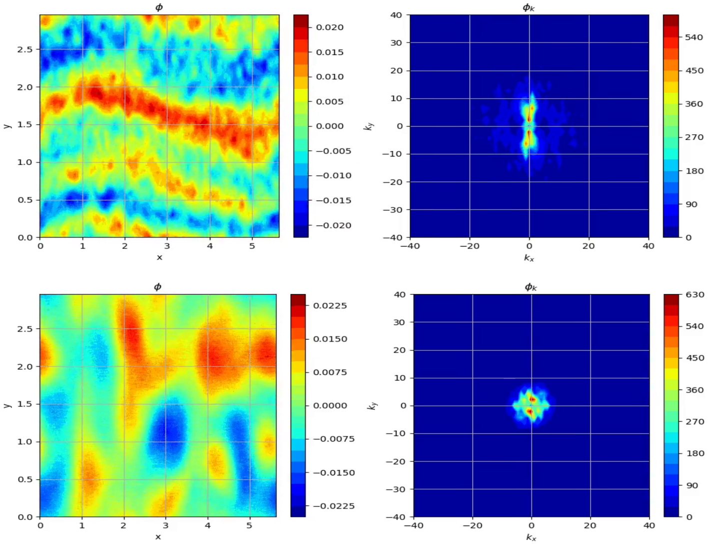
ETG H-M Results



GENE ETG Streamer Test



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Effects of Isotropization

Zonal Flow Excitation

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References