Nonlinear interaction of toroidicity-induced drift modes

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Drift modes in toroidal geometry are destabilized by trapped electron inverse dissipation and evolve to a nonlinearly saturated state. Using renormalized one-point turbulence theory for the nonlinear gyrokinetic equation in the ballooning representation, it is shown that ion Compton scattering is an effective saturation mechanism. Ion Compton scattering transfers wave energy from short to long perpendicular wavelength, where it is absorbed by ion resonance with extended, linearly stable, long-wavelength modes. The fluctuation spectrum and fluctuation levels are calculated using the condition of nonlinear saturation. Transport coefficients and energy confinement time scalings are determined for several regimes. Specifically, the predicted confinement time density scaling for an Ohmically heated discharge increases from $n^{3/8}$ in the collisionless regime to $n^{9/8}$ in the dissipative trapped electron regime.

I. INTRODUCTION

Drift waves have frequently been associated with the observed low-frequency density fluctuations and energy confinement degradation in tokamaks. Previous investigation[1–7] has focused on the development of theoretical models for the nonlinear evolution and saturation of drift mode instabilities and on the comparison of the theoretical predictions with experimental observations. In all cases, a shearless or sheared slab model of a tokamak has been used in the theory.

In toroidal geometry, there exist two branches of the basic electron drift mode.[8] The slab-like branch is the toroidal analogue of the familiar Pearlstein–Berk mode structure in the sheared slab model, and is characterized by a rapid eigenfunction variation along the magnetic field line. The slab-like branch is damped by energy convection toward the ion Landau resonance point. The toroidicity-induced branch, which results from the inclusion of ion magnetic drifts, has a bound wave function which is slowly varying over the scale of the connection length. The toroidicity-induced modes do not experience shear damping. Hence, this branch is destabilized by any inverse electron dissipation.[9] For this reason, the toroidicity-induced branch appears to be more relevant to considerations of drift wave stability and transport.

Our purpose in this paper is to elucidate the mechanism of nonlinear evolution and saturation of the toroidicity-induced branch.

Many possible mechanisms for saturation of drift wave instabilities have been advanced.[1,5,10] Here, nonlinear ion–wave interaction is considered. Starting from the nonlinear ion gyrokinetic equation, a nonlinear dielectric operator is constructed by a perturbative expansion to third order in field amplitude. It is important to note that the toroidicity induced mode structure has significant effects on the nonlinear interaction of the modes. Specifically, the nonlinear dielectric operator is nonlocal in the extended poloidal coordinate and the beat fluctuation-ion resonance operator must be renormalized. The latter point follows from the fact that typically $d_k > k_{\|} \omega_{ti}$, where d_k is the ion turbulent decorrelation frequency and ω_{ti} is the ion transit frequency.

Using the saturation condition, a spectral intensity equation is derived. The spectral intensity is determined by the balance of the electron-induced growth with transfer of wave energy to long wavelengths by the ion Compton scattering process. Ultimately, wave energy is deposited in the longest wavelengths, which are not bounded in poloidal angle and hence are shear damped.

The spectral intensity equation is solved to obtain the wavenumber spectrum and the total fluctuation intensity at saturation. Using the saturated state spectral intensity, the electron radial diffusion coefficient is calculated for a number of collisionality regimes.

Using the assumption of balance of electron heat diffusion losses with Ohmic heating, the electron temperature is eliminated. Final forms for the electron energy confinement time density scaling increases from $n^{3/8}$ in the collisionless regime to $n^{9/8}$ in the dissipative trapped electron regime.

The effects of finite β_e corrections and transport due to magnetic braiding are discussed. At low β ($\beta_p < 1$), magnetic braiding does not contribute substantially to electron energy transport, which is dominated by electrostatic convection. However, electron conduction losses may be more significant at higher β ($\beta_p > 1$).

The remainder of this paper is organized as follows. In Sec. II, we describe the physical model to be considered, the geometry, and the set of nonlinear equations used. In Sec. III, relevent aspects of the linear theory are discussed. In Sec. IV, the renormalized equations are derived and the spectral equation at saturation is established. In Sec. V, the various nonlinear transfer processes are discussed. In Sec. VI, the spectral equation is solved in different regimes, and the turbulent intensity is evaluated. In Sec. VII, these results are used to determine electron transport coefficients and the scaling of the associated confinement times. In Sec. VIII, the electron heat conduction due to magnetic fluttering is evaluated and compared with the previous transport coefficients. In Sec. IX, we conclude and discuss the results.

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II. BASIC MODEL

The basic model is the one of a tokamak with circular concentric flux surfaces. The nonlinear equations are conveniently written in toroidal geometry using the high-n ballooning mode formalism. This formalism exploits the separation of the fluctuation scale lengths across field lines and along field lines, which are typically of the order of the ion gyroradius and of the connection length, respectively. The electrostatic potential fluctuations ϕ will therefore be written as a sum of quasimodes, which have the eikonal form $\hat{\phi}_n(\theta)\exp\{-in[\zeta - q(r)\theta + S(r)]\}$, where n is the toroidal mode number, ζ is the toroidal angle, θ is the poloidal angle, $q(r) = rB_\zeta/RB_\theta$ is the safety factor, and $S(r)$ is the eikonal phase which exhibits the fast radial variation of the fluctuations. The definition of the shear parameter $s \equiv d(\ln q)/d(\ln r)$ and of $\theta_\kappa(r) \equiv dS/dq$ are customary and will be used in the following equations. The nonadiabatic part h of the ion distribution function obeys the nonlinear gyrokinetic equation with finite Larmor radius effects[11]:

$$\begin{aligned}\Big(-i\omega + \frac{v_\parallel}{Rq}\frac{\partial}{\partial\theta} + i\omega_{Di}(\mathbf{v},\theta)\Big)&h(v_\parallel,v_\perp,\theta)\\ = -i(\omega-\omega_{*i})&J_0(v_\perp k_\perp/\Omega_i)\\ &\times F_M(v_\parallel,v_\perp)(e/T_i)\phi(\theta)\\ &-\frac{c}{B}\sum_{k'_\theta,m',m'',S,S'}(2\pi m' + \theta'_\kappa - 2\pi m'' - \theta''_\kappa)k'_\theta\, k''_\theta\, s\\ &\times\exp[2i\pi q(m'n' + m''n'') + i(nS - n'S' - n''S'')]\\ &\times[J_0(v_\perp k''_\perp/\Omega_i)\phi''(\theta - 2\pi m'')h'(\theta - 2\pi m')].\end{aligned} \tag{1}$$

Here, $\omega_{*i} \equiv -k_\theta cT_i/eBL_n$ is the ion diamagnetic frequency, $k_\theta \equiv nq/r$ is the poloidal wave vector, m_i, T_i, Ω_i, and e are the ion mass, temperature, gyrofrequency, and charge, L_n is the density scale length, J_0 is the Bessel function of order zero, F_M is the Maxwellian average ion distribution, $k_r(\theta) \equiv k_\theta s(\theta - \theta_\kappa)$ is the local radial wave vector, $k_\perp^2(\theta) \equiv k_\theta^2[1 + s^2(\theta - \theta_\kappa)^2]$ is the square of the local perpendicular wave vector,

$$\begin{aligned}\omega_{Di} \equiv [\omega_{*i}(L_n/R)(v_\parallel^2 &+ v_\perp^2/2)/v_i^2]\\ &\times[\cos\theta + s(\theta - \theta_\kappa)\sin\theta]\end{aligned}$$

is the ion magnetic drift frequency, and $v_i \equiv (T_i/m_i)^{1/2}$ is the ion thermal velocity. The parallel ion velocity is assumed to be a constant during the ion trajectory along the field line, and trapped ions are neglected. The second term on the right-hand side of Eq. (1) is the gyroaveraged nonlinear $E\times B$ drift $\langle\mathbf{v}_{E\times B}\rangle\cdot\nabla h$, and has the form of a multiple sum over k'_θ, m' and eikonals S, of the product of quasimodes. In that term, the primes refer to fluctuations of wave vectors $\mathbf{k}'$ and the double primes to fluctuations of wave vectors $\mathbf{k}''$, and the sum is limited to terms such that locally $k_\theta = k'_\theta + k''_\theta$ and $k_r(\theta) = k'_r(\theta) + k''_r(\theta)$. The electron response will be chosen linear, of the form

$$n_{e_k} = [1 - i\delta(k,\omega)]e\phi_k/T_e. \tag{2}$$

The arbitrariness in δ facilitates the examination of a variety of electron destabilization mechanisms. For the dissipative trapped electron regime, $\omega < \nu_{\rm eff} < \omega_{be}$, which corresponds to the case of principal interest in this paper, the nonadiabatic electron response is modeled by

$$\delta(k,\omega)\phi_k \equiv [(\omega_{*e} - \omega)/\nu_{\rm eff}](2\epsilon)^{1/2}\overline{\phi}_k. \tag{3}$$

The field $\overline{\phi}_k$ is the trapped particle bounce averaged potential, ω_{be} is the electron bounce frequency, ω_{*e} is the electron diamagnetic frequency, $\nu_{\rm eff} = \nu_{ei}/\epsilon$ is the effective collision frequency for trapped electrons, $\epsilon = r/R$ is the inverse aspect ratio, and $(2\epsilon)^{1/2}$ is the trapped electron population fraction. Note that in that collisionality regime, the scattering of waves by trapped electrons is negligible, and the linear electron response is well justified.

The nonlinear system of equations is closed using the quasineutrality equation:

$$\frac{e}{T_e}(1 - i\delta)\phi = -\frac{e}{T_i}\phi + \int d\mathbf{v}\, J_0(v_\perp k_\perp/\Omega_i)\, h(v_\perp,v_\parallel,\theta). \tag{4}$$

III. LINEAR ANALYSIS

The ion equation Eq. (1) is linearized and the linear propagator is furthermore expanded in the small parameter $\omega_{ti}/(\omega - \omega_{Di})$, where $\omega_{ti} \equiv v_i/Rq$ is the ion transit frequency. The ion density fluctuation is thus obtained[12]

$$\begin{aligned}n_i = -\frac{e}{T_i}\phi(\theta) &+ \frac{e}{T_i}(\omega - \omega_{*i})\int d\mathbf{v}\, F_M\, J_0(v_\perp k_\perp/\Omega_i)\\ &\times\Big\{\frac{J_0(v_\perp k_\perp/\Omega_i)}{\omega - \omega_{Di}}\phi - \frac{(v_\parallel/Rq)^2}{\omega - \omega_{Di}}\\ &\times\frac{\partial}{\partial\theta}\Big[\frac{1}{\omega - \omega_{Di}}\frac{\partial}{\partial\theta}\Big(\frac{J_0(v_\perp k_\perp/\Omega_i)}{\omega - \omega_{Di}}\phi\Big)\Big]\Big\},\end{aligned}$$

which is further approximated by

$$\begin{aligned}n_i = -\frac{e}{T_i}\phi(\theta) &+ \frac{\omega - \omega_{*i}}{\omega - \overline{\omega}_{Di}}\frac{e}{T_i}\Gamma_0(\rho_i^2 k_\perp^2)\\ &\times\Big(\phi - \frac{\omega_{ti}^2}{(\omega - \overline{\omega}_{Di})^2}\frac{\partial^2}{\partial\theta^2}\phi\Big),\end{aligned} \tag{5}$$

where $\overline{\omega}_{Di}(\theta) = 2\epsilon\omega_{*i}[\cos\theta + s(\theta - \theta_\kappa)\sin\theta]$ is the ion magnetic drift frequency for a thermal particle, $\Gamma_0(x) \equiv \exp(-x)I_0(x)$, $I_0(x)$ is the modified Bessel function of order zero, $\rho_i = v_i/\Omega_i$ is the ion Larmor radius. Using the ion response equation (5) together with quasineutrality and the electron response equation (4) one obtains the eigenmode equation for ϕ and ω

$$\frac{d^2}{d\theta^2}\phi + Q(\theta)\phi = 0, \tag{6}$$

where

$$\begin{aligned}Q(\theta) \equiv \frac{T_i}{T_e}&\Big(\frac{\omega - \overline{\omega}_{Di}(\theta)}{\omega_{ti}}\Big)^2\\ &\times\Big\{1 - \Big(1 + \frac{T_i}{T_e}\Big)\frac{\omega_{*e} - \overline{\omega}_{De}(\theta)}{\omega - \omega_{*i}}\\ &+ \frac{\omega - \overline{\omega}_{Di}(\theta)}{\omega - \omega_{*i}}\Big[\frac{1 - \Gamma_0}{\Gamma_0}\Big(1 + \frac{T_e}{T_i} - i\frac{\delta(k,\omega)}{\Gamma_0}\Big)\Big]\Big\}.\end{aligned} \tag{7}$$

Here the symbol Γ_0 stands for $\Gamma_0(\rho_i^2 k_\perp^2)$, and $\omega_{*e} = -\omega_{*i} T_e/T_i$, $\overline{\omega}_{De} = -\overline{\omega}_{Di} T_e/T_i$ are the electron diamagnetic and magnetic drift frequencies.

The nonlinear analysis requires a complete understanding of the linear spectrum which is composed of a great variety of modes. For each k_θ, i.e., for each toroidal mode number, there exists a slab-like branch mode and a toroidicity induced branch mode with various θ_κ, a parameter related to the global radial structure. The modes are further distinguished by their eigenmode number, e.g., their number of nodes between turning points in θ. In order to simplify the analysis, we retain, for each k_θ, only the most unstable—or least stable—modes. Here, these correspond to the toroidicity induced modes, centered at the outside of the torus (i.e., $\theta_\kappa = 0$), with the lowest eigenmode number. This choice is motivated by the fact that at saturation these modes have the largest fluctuation level and dominate the spectrum. This is because:

(a) the slab-like modes are not significantly excited by the trapped electrons, because their fast variation along θ is averaged over a bounce orbit, and because of their large rate of shear damping;

(b) the toroidicity-induced modes localized inside of the torus (i.e., $\theta_\kappa = \pi$) are only feebly excited by the trapped electrons, localized on the outside of the torus;

(c) the toroidicity-induced modes with $\theta_\kappa \neq 0 \pmod{\pi}$, which correspond to the components of higher-order modes for the global envelope in r, are also more stable;

(d) the toroidicity-induced modes with $\theta_\kappa = 0$, but with one or more nodes along the field line have an oscillatory structure along the bounce orbit of the electron, hence are not significantly excited.

In summary, the dominant quasimodes of the system resemble the one shown in Fig. 1, and are extended along the field line but bounded within $\pm\pi$. One dispersion relation $\omega(k_\theta)$ for these modes is shown in Fig. 2.

For small k_θ (e.g., $k_\theta \rho_s \lesssim 0.1$), the modes are shear damped and hence are stable. They provide a sink at long wavelength, where energy is ultimately transferred to the ions.

IV. RENORMALIZED TURBULENCE THEORY

Here, the direct interaction response functions h_k and ϕ_k describe the relaxation of perturbations with toroidal mode number n. Their equations are obtained conveniently by calculating the phase coherent part of the nonlinear terms. Accordingly, they are solutions of the renormalized gyrokinetic equation

$$\left(-i\omega + i\omega_{Di} + \frac{v_\parallel}{Rq}\frac{\partial}{\partial\theta} + d_k\right)h_k = [-i(\omega-\omega_{*i}) + b_k](e/T_i)J_0(v_\perp k_\perp/\Omega_i)F_M\,\phi_k \quad (8)$$

solved consistently with the quasineutrality equation

$$\int d\mathbf{v}\, J_0(v_\perp k_\perp/\Omega_i)\, h_k = \frac{e}{T_i}\phi_k + \frac{e}{T_e}(1-i\delta)\phi_k. \quad (9)$$

The nonlinear operators d_k and b_k in Eq. (8) follow from the expression

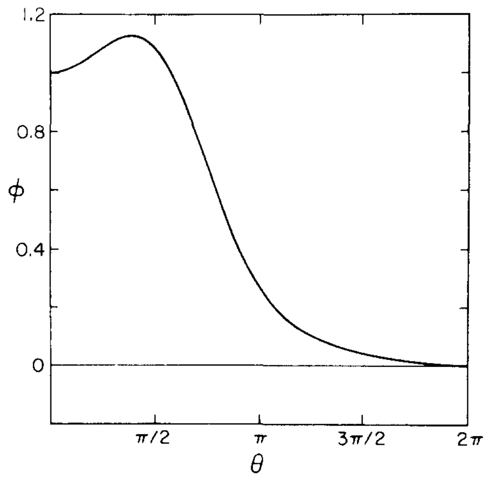

FIG. 1. Structure of the electrostatic potential component ϕ of the quasimode, along the field line. Here $T_i/T_e = 0.5$, $s = 0.8$, $q = 2.0$, $\epsilon = 0.1$, $k_\theta \rho_s = 0.2$, $\omega_{*e}/\omega_s = 4.0$, and $\omega/\omega_s = 2.4$.

$$-d_k h_k(\theta) + b_k(e/T_i)F_M J_0(v_\perp k_\perp/\Omega_i)\phi_k(\theta) \equiv -\frac{c}{B}\sum_{k'_\theta, m'}(2\pi m' + \theta'_k - 2\pi m'' - \theta''_k) \times k'_\theta k''_\theta\, s \exp(i\xi)\big[J_0(v_\perp k''_\perp/\Omega_i)\phi^{(2)}_{k''}(\theta - 2\pi m'') \times h_{k'}(\theta - 2\pi m') - J_0(v_\perp k'_\perp/\Omega_i)\phi_{k'}(\theta-2\pi m') \times h^{(2)}_{k''}(\theta - 2\pi m'')\big]_{\text{phase coherent}}, \quad (10)$$

where the phase ξ is defined by

$$\xi = nS - n'S' - n''S'' - 2\pi q(m'n' + m''n''). \quad (11)$$

Here, the driven fluctuations $h^{(2)}$ and $\phi^{(2)}$ are solutions of the system

$$\left(-i\omega'' + i\omega''_{Di}(\theta - 2\pi m'') + \frac{v_\parallel}{Rq}\frac{\partial}{\partial\theta} + d_{k''}\right) \times h^{(2)}_{k''}(\theta - 2\pi m'') = [-i(\omega'' - \omega''_{*i}) + b_{k''}](e/T_i)F_M J_0(v_\perp k''_\perp/\Omega_i)\phi^{(2)}_{k''} - (c/B)2\pi m' k_\theta k'_\theta\, s \exp(-i\xi)$$

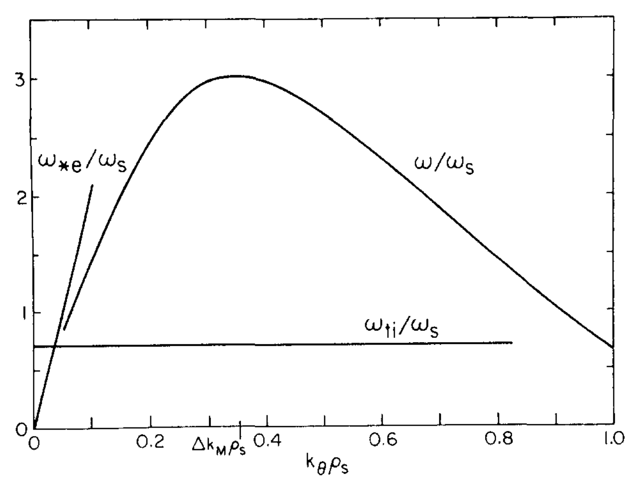

FIG. 2. Dispersion relation for the toroidicity-induced drift modes, for $T_i/T_e = 0.5$, $s = 0.8$, $q = 2.0$, $\epsilon = 0.1$.

$$\times [J_0(v_\perp k'_\perp/\Omega_i)\phi_{-k'}(\theta-2\pi m')h_k(\theta) - J_0(v_\perp k_\perp/\Omega_i)\phi_k(\theta)h_{-k'}(\theta-2\pi m')], \tag{12}$$

and

$$\int d\mathbf{v}\, J_0(v_\perp k''_\perp/\Omega_i)\, h^{(2)}_{k''} = \frac{e}{T_e}(1-i\delta'')\phi^{(2)}_{k''} + \frac{e}{T_i}\phi^{(2)}_{k''}. \tag{13}$$

In Eq. (12), $\theta_k=\theta'_k=0$, ξ is defined by Eq. (11), and only the terms that give a phase coherent contribution in Eq. (10) have been retained. The driven potential $\phi^{(2)}_{k''}$ in the equation describes the shielding of the driven ion fluctuations by the plasma. The induced scattering effect associated with $\phi^{(2)}_{k''}$ will reduce the direct ion Compton effect associated with $f^{(2)}_{k''}$. Indeed, for the case of adiabatic electrons, and to lowest order in ion Larmor radius, these two effects cancel, yielding no net contribution. However, this exact balance is broken when finite ion Larmor radius and nonadiabatic electron response effects are included. Note that the eigenmode extent in θ guarantees that $k_\perp\rho_s\sim 1$ even for $k_\theta\rho_s<1$.

In the following discussion, we will focus our attention on Compton scattering, due to the resonant interaction of the ions with the beat fluctuations $f^{(2)}_{k''}$ and $\phi^{(2)}_{k''}$ of two modes, and the associated shielding by induced potential fluctuations.

The renormalized ion propagator is formally the solution $G(\theta,\theta')$ of the equation

$$\left(-i(\omega-\omega_{Di}) + \frac{v_\parallel}{Rq}\frac{\partial}{\partial\theta} + d(\theta)\right)G(\theta,\theta') = \delta(\theta-\theta') \tag{14}$$

that vanishes for $|\theta-\theta'|\to\infty$. Note that the ions which participate in a modal fluctuation are mostly nonresonant since $|\omega-\omega_{Di}|\gg\omega_{ti}$. The nonresonant propagator is nearly local in θ and is given to lowest order by

$$G(\theta,\theta') = \{1/(-i)[\omega-\omega_{Di}(\theta)]\}\delta(\theta-\theta'), \tag{15a}$$

where $\delta(x)$ is the Dirac function. For particles which resonate with beat fluctuations, the expression of the propagator is considerably more complicated and is nonlocal in θ. Nevertheless, it remains true that when the operator $G(\theta,\theta')$ acts on a smooth function of θ' and v, it is approximately equivalent to a local expression

$$G(\theta,\theta') = \{1/[-i(\omega-\omega_{Di})+\Delta\omega]\}\delta(\theta-\theta'), \tag{15b}$$

where $\Delta\omega$ includes the various particle-fluctuation decorrelation frequencies. Particle decorrelation is due to ion parallel motion ($\Delta\omega\sim\omega_{ti}$), ion magnetic drift ($\Delta\omega\sim\omega_{Di}$), and nonlinear decorrelation ($\Delta\omega\sim d$). The nonlinear decorrelation occurs because of stochastic $E\times B$ motion of the ions in the turbulent electric field. The rate of decorrelation for the k_θ fluctuation is approximately

$$d_k = \left(\frac{c}{B}\right)^2 \sum_{k'_\theta,m'} (2\pi m')^2 k_\theta^2\, k'^2_\theta s^2 [J_0(v_\perp k'_\perp/\Omega_i)]^2 \times\{1/[-i(\omega''-\omega''_{Di})+d_{k''}]\}\langle\phi\phi\rangle_{k'_\theta}(\theta-2\pi m'). \tag{16}$$

Since the ions do not resonate with the primary fluctuations ϕ_k and $\phi_{k'}$, the driving term of Eq. (12) [i.e., the second term of the right-hand side of Eq. (12)] is

$$N'' \equiv (c/B)\,2\pi m' k_\theta k'_\theta s \times\exp(-i\xi)\,J_0(v_\perp k_\perp/\Omega_i)\,J_0(v_\perp k'_\perp/\Omega_i) \times(e/T_i)\,F_M\,\phi_k(\theta)\phi_{-k'}(\theta-2\pi m') \times\left(\frac{\omega'_{Di}-\omega'_{*i}}{\omega'-\omega'_{Di}} - \frac{\omega_{Di}-\omega_{*i}}{\omega-\omega_{Di}}\right). \tag{17}$$

Hence,

$$h^{(2)}_{k''} = \frac{\omega''-\omega''_{*i}}{\omega''-\omega''_{Di}+i\Delta\omega''}\frac{e}{T_i}F_M J_0(v_\perp k''_\perp/\Omega_i)\,\phi^{(2)}_{k''} + \frac{N''}{-i(\omega''-\omega''_{Di})+\Delta\omega''}. \tag{18}$$

It follows from quasineutrality [Eq. (13)] that

$$\frac{e}{T_i}\phi^{(2)}_{k''} = \frac{1}{\alpha''}\int dv\, J_0(v_\perp k''_\perp/\Omega_i)\frac{N''}{-i(\omega''-\omega''_{Di})+\Delta\omega''}, \tag{19}$$

where

$$\alpha'' \equiv 1 + (T_i/T_e)(1-i\delta'') - \Gamma_0(\rho_i^2 k''^2_\perp) - \Gamma_0(\rho_i^2 k''^2_\perp)\frac{\overline{\omega}''_{Di}-\omega''_{*i}}{\omega''-\overline{\omega}''_{Di}+i\Delta\omega''} \tag{20}$$

can be inverted provided that the three-wave decay-type relations are not satisfied. This condition requires finite frequency dispersion.

The expression (20) for α'' is equivalent to a local approximation for the dielectric operator, which is justified by the small ion transit frequency. As a check, one can show that the dispersion relation is well approximated by the local equation $Q_\omega(\theta)=0$ evaluated at $\theta\simeq\pi/2$. However, the derivative terms of Eq. (6) are essential to determine the structure of the mode. Note that the full kinetic treatment of α'' is not needed, because $d_k>\omega_{ti}$ renders ion particle streaming effects irrelevant.

The nonlinearly modified eigenmode equation for the potential is

$$\text{(a)}\quad \frac{\partial^2\phi_k}{\partial\theta^2} + [Q(\theta)+\Delta Q(\theta)]\phi_k = 0, \tag{21}$$

where the nonlinear term is given by

$$\text{(b)}\quad \frac{e}{T_i}\Delta Q\,\phi_k = -i\frac{(\omega-\overline{\omega}_{Di})^2}{\omega_{ti}^2}\frac{1}{\Gamma_0}\frac{1}{\omega-\omega_{*i}} \times\int d\mathbf{v}\{-d_k J_0(v_\perp k_\perp/\Omega_i)\,h_k + b_k[J_0(v_\perp k_\perp/\Omega_i)]^2(e/T_i)F_M\phi_k\}_{\text{phase coherent}}. \tag{22}$$

The last factor in Eq. (22) is evaluated using the expressions (10), (18), and (19). Hence,

$$\frac{e}{T_i}\Delta Q\ \phi_k(\theta) = -i\frac{(\omega-\overline{\omega}_{Di})^2}{\omega_{ti}^2}\frac{1}{\Gamma_0}\frac{1}{\omega-\omega_{*i}}\frac{e}{T_i}\phi_k(\theta)\left(\frac{c}{B}\right)^2\sum_{m',k'_\theta}(2\pi m' k_\theta k'_\theta s)^2$$
$$\times\langle|\phi_{k'}(\theta-2\pi m')|^2\rangle\left(\frac{\omega_{*i}-\overline{\omega}_{Di}}{\omega-\overline{\omega}_{Di}}-\frac{\omega'_{*i}-\omega'_{Di}}{\omega'-\overline{\omega}'_{Di}}\right)\frac{1}{-i(\omega''-\overline{\omega}''_{Di})+\Delta\omega''}$$
$$\times\left[\langle J_0^2 J_0'^2\rangle_v-\frac{1}{\Gamma_0}(\langle J_0 J'_0 J''_0\rangle_v)^2+\frac{(\langle J_0 J'_0 J''_0\rangle_v)^2}{\alpha''}\right.$$
$$\left.\times\left(\frac{\overline{\omega}'_{Di}-\omega'_{*i}}{\omega'-\overline{\omega}'_{Di}}-\frac{1+T_i/T_e-\Gamma''_0-i\delta''T_i/T_e}{\Gamma''_0}\right)\right], \tag{23}$$

where $\langle\cdots\rangle_v$ denotes velocity average over the equilibrium ion distribution function.

It is important to note that because of finite temperature ratio T_i/T_e and nonadiabatic electron response, the induced scattering effect [associated with the second term in the large brackets of Eq. (23)] reduces, but does not cancel the Compton scattering term [associated with the first term in the large brackets of Eq. (23)].

The width $\Delta\omega''$ of the propagator in Eq. (23) includes linear and nonlinear broadenings. This width determines the rate of transfer of energy from mode to mode in the long-wavelength part of the spectrum, and hence ultimately the saturation amplitude. In these transfer processes, two modes can exchange energy, provided that the ions can resonante with the beat fluctuation, and absorb momentum. This is possible whenever the beat frequency $\omega''=\omega-\omega'$ is of the order of the broadening $\Delta\omega''$. The dispersion curve (Fig. 2) shows that two types of interaction, the distant and close,[6] can exist when the interacting modes have disparate or similar wavenumbers k_θ, respectively.

We will successively examine the two kinds of interaction, noting that the sign of Im ΔQ is such that they both transfer energy to longer wavelengths. An important characteristic of the nonlocal theory appears in the quasimode structure which is localized within a connection length. Consequently the nonlinear potential $\Delta Q(\theta)$ is localized away from $\theta=0$ because the $m'=0$ term of Eq. (23) vanishes. The $m'=\pm 1$ contributions are most significant. The interaction of the quasimodes k_θ and k'_θ will then be strongest at $\theta=\pm\pi$ where their overlap is maximal (Fig. 3). Note that only a fraction of the spectrum extent will determine the transfer of energy.

V. NONLINEAR TRANSFER PROCESSES

In this section, the various types of nonlinear transfer processes are discussed. The distant interaction is due to the beating of a high-k_θ and a low-k'_θ fluctuation of similar frequencies ($\omega_{*e}\gg\omega\sim\omega'\sim\omega'_{*e}$). The beat fluctuation has a large perpendicular wavenumber

$$k''_\perp=[(k_\theta-k'_\theta)^2+(k_\theta s\theta-k'_\theta s\theta-2\pi s k'_\theta)^2]^{1/2}.$$

As a consequence, for even moderate long-wavelength fluctuation levels, the distant interaction is strong and causes a nonlinear scattering rate which exceeds the growth due to trapped electron excitation.

Here the interaction time of the particle with the beat wave is limited by the nonlinear decorrelation d'', where $d''\geqslant|\omega-\omega'|$ so that strong turbulence estimates apply. Hence the decorrelation rate is the eddy turnover rate given by

$$\Delta\omega_k\sim d_k\sim[4\pi^2 s^2 k_\theta^2\langle v_E^2(\theta)\rangle]^{1/2},$$

where $\langle v_E^2\rangle$ is the $E\times B$ velocity autocorrelation

$$\langle v_E^2(\theta)\rangle\equiv\frac{c^2}{B^2}\int dk_\theta\, k_\theta^2\langle|\phi|^2(\theta)\rangle_{k_\theta}.$$

It follows that the nonlinear potential is

$$\mathrm{Im}\,\Delta Q\sim\frac{\omega^2}{\omega_s^2}\frac{1}{\omega}\frac{\omega_{*e}}{\omega}\frac{(2\pi k_\theta s)^2\langle v^2\rangle}{d}\sim\frac{\omega^2}{\omega_s^2}\frac{\omega_{*e}}{\omega}\frac{d}{\omega}.$$

By contrast, the electron excitation corresponds to a linear potential which, for large k_θ, is approximately

$$\mathrm{Im}\,Q\sim(\omega^2/\omega_s^2)\delta.$$

Note that typically the spectrum average of (Im ΔQ − ImQ) is positive for $\delta\leqslant 1$. Hence, the nonlinear transfer rate due to distant interaction significantly exceeds the electron excitation rate. As a consequence, these high-k_θ modes are only feebly excited and do not significantly contribute to the fluctuation spectrum. Thus, the spectrum is populated only for $k_\theta\lesssim\Delta k_N$, which corresponds to $k_\perp\rho_s\sim k_\theta s\pi\rho_s\lesssim 1$.

The close interaction occurs between modes in the long-wavelength region of the spectrum. Energy cascades to lower k_θ, where it is absorbed by shear damping of low-k_θ modes. The saturation criterion, according to which steady-state turbulence is determined by requiring test mode marginal stability, is imposed. Hence, a normal mode spectrum

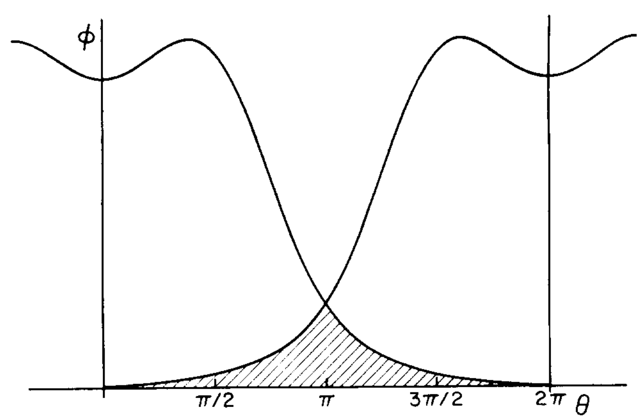

FIG. 3. Overlap of two quasimodes $\phi_{k_\theta}(\theta)$ and $\phi_{k'_\theta}(\theta-2\pi)$, for $k_\theta\simeq k'_\theta$.

is obtained, where for all k_θ the eigenmodes have a nonlinear frequency ω with $\mathrm{Im}\,\omega = 0$.

In order to find estimates and deduce scaling laws, we balance the nonlinear decay with the linear growth, thus obtaining the equation

$$\left\langle \delta \frac{1}{\Gamma_0} \frac{T_i}{T_e} \frac{\omega - \omega_{Di}}{\omega - \omega_{*i}} \frac{(\omega - \omega_{Di})^2}{\omega_{ti}^2} \right\rangle$$
$$= \left\langle \frac{(\omega - \omega_{Di})^2}{\omega_{ti}^2} \frac{1}{\omega - \omega_{*i}} \frac{1}{\Gamma_0} \right.$$
$$\times \sum_{k'_\theta, m} (2\pi m k_\theta s)^2 \left(\frac{\omega'_{*i} - \omega'_{Di}}{\omega' - \omega'_{Di}} - \frac{\omega_{*i} - \omega_{Di}}{\omega - \omega_{Di}} \right) \rho$$
$$\left. \times \frac{\langle |v_{k'_\theta}(\theta - 2\pi m)|^2 \rangle}{-i(\omega'' - \omega''_{Di}) + \Delta\omega''} \right\rangle, \tag{24}$$

where the outer angular brackets refer to an average over the mode structure. Here the numerical factor ρ (of the order of 0.1) [which appears in the last factor of Eq. (23)] takes into account the reduction of Compton scattering by induced scattering. For $k'_\theta - k_\theta$ small, we expand the terms $(\omega'_{*i} - \omega'_{Di})/(\omega' - \omega'_{Di})$ and $\langle |v_{k'_\theta}(\theta - 2\pi m)|^2 \rangle$ of Eq. (24) around k_θ, and find the balance

$$\frac{\omega^2}{\omega_s^2} \delta \simeq \frac{\omega^2}{\omega_s^2} \frac{1}{\omega} \int \frac{dk'_\theta}{2\pi} \sum_m (2\pi m k_\theta s)^2$$
$$\times \Delta k_\theta^2 \rho \frac{d}{dk_\theta} \left(\frac{\omega_{*e} - \omega_{De}}{\omega - \omega_{Di}} \right)$$
$$\times \frac{1}{-i(\omega'' - \omega''_{Di}) + \Delta\omega''} \frac{d}{dk_\theta} \langle |v_{k'_\theta}(\pi)|^2 \rangle,$$

that one can also write

$$\frac{\omega^2}{\omega_s^2} \delta \simeq \frac{\omega^2}{\omega_s^2} \frac{1}{\omega} (2\pi)^2 k_\theta^2 s^2 \frac{1}{2\pi} \frac{(\Delta k_\theta)^3}{\Delta\omega}$$
$$\times \rho \frac{d}{dk_\theta} \left(\frac{\omega_{*e} - \omega_{De}}{\omega - \omega_{Di}} \right) \frac{d}{dk_\theta} \langle |v_{k_\theta}(\pi)|^2 \rangle. \tag{25}$$

Thus, a first-order differential equation for the spectral intensity is obtained. In Eq. (25) it is necessary to specify the quantities $\Delta\omega$ and Δk_θ, which are functions of k_θ, and typical of the particle decorrelation frequency and of the range of mode interaction. The quantities are defined by

$$\Delta\omega = \max\{\omega_{ti}, d_{k_\theta}\}, \tag{26a}$$

and by

$$\Delta k_\theta \sim \min\{\Delta\omega / v_*, \Delta k_{\mathrm{M}}\}, \tag{26b}$$

where $\Delta k_{\mathrm{M}} \sim \rho_s^{-1}/\pi s$ is the width of the spectrum.

VI. SOLUTION OF THE SPECTRUM INTENSITY EQUATION

The spectrum intensity equation (25) is now solved to obtain the spectrum function and intensity. Here, $\langle |v_{k_\theta}|^2 \rangle = c^2 k_\theta^2 \langle |\phi_{k_\theta}|^2 \rangle / B^2$ is the correlation function of the radial $E \times B$ drift.

The equation for the spectrum is thus given by

$$\frac{d}{dk_\theta} \langle |v_{k_\theta}|^2 \rangle \simeq 2\pi \frac{\Delta\omega}{(\Delta k_\theta)^3} \frac{\delta}{\rho} \frac{\omega}{(2\pi)^2 k_\theta^2 s^2}$$
$$\times \left[\frac{d}{dk_\theta} \left(\frac{\omega_{*e} - \omega_{De}}{\omega - \omega_{Di}} \right) \right]^{-1}, \tag{27}$$

for $0 < k_\theta < \Delta k_{\mathrm{M}}$.

The initial condition is provided by $\langle |v_{k_\theta}^2| \rangle = 0$ at $k_\theta = \Delta k_{\mathrm{M}}$, the point where the frequency assumes its largest value (Fig. 2). Several approximations facilitate further the solution of Eq. (27). First,

$$\frac{d}{dk_\theta} \left(\frac{\omega_{*e}}{\omega} \right) \sim \frac{1}{2} \frac{k_\perp^2 \rho_s^2}{k_\theta} \sim \frac{1}{2} \frac{k_\theta}{\Delta k_{\mathrm{M}}^2}$$

gives an estimate of the eigenfrequency dispersion. Second,

$$\Delta k_{\mathrm{M}} \sim \rho_s^{-1}/\pi s$$

gives an estimate for the width of the excited spectrum in wavenumber space. Third,

$$\langle v_{k_\theta}^2(\pi) \rangle / \langle v_{k_\theta}^2(0) \rangle \sim 0.2$$

reflects the fact that the interaction is localized in the region $\theta \simeq \pi$, and that the mode is ballooning. Fourth, the case of collisional trapped particle destabilization [Eq. (3)] is considered and

$$\frac{(\omega_* - \omega)\omega}{k_\theta^2 v_*^2} \sim k_\perp^2 \rho_s^2 \sim \frac{k_\theta^2}{\Delta k_{\mathrm{M}}^2}.$$

Then, the spectral equation becomes [$\rho \simeq 0.1$, $|v_k(\pi)|^2 / |v_k(0)|^2 \simeq 0.2$]:

$$\frac{1}{2\pi} \frac{d}{dk_\theta} \langle |v_{k_\theta}|^2 \rangle \simeq -\frac{1}{2} \frac{k_\theta}{\Delta k_\theta^3} \frac{\Delta\omega}{\nu_{\mathrm{eff}}} \frac{(2\epsilon)^{1/2}}{s^2}. \tag{28}$$

Using Eq. (28), the total turbulent intensity is now obtained. It is measured by

$$\langle v_E^2 \rangle = \int \frac{dk_\theta}{2\pi} \langle |v_{k_\theta}|^2 \rangle$$
$$= \int_0^{\Delta k_{\mathrm{M}}} \frac{dk_\theta}{2\pi} (\Delta k_{\mathrm{M}} - k_\theta) \frac{d}{dk_\theta} \langle |v_{k_\theta}|^2 \rangle. \tag{29}$$

The expressions above Eq. (28) and Eq. (29) are combined with the expressions for the propagator broadening $\Delta\omega$, and the interaction range Δk_θ given by Eqs. (26a) and (26b). The nonlinear decorrelation frequency d''_k has the form

$$d''_k \sim (1/2\pi)(2\pi)^2 k_\theta^2 s^2 (\Delta k_\theta / \Delta\omega) \langle v_{k_\theta}^2(\pi) \rangle. \tag{30}$$

Note that here, use has been made of the equality $k''_r = k''_\theta s\theta + 2\pi s k'_\theta$, so that for $k''_\theta = k_\theta - k'_\theta \ll k_\theta$, one has $|\mathbf{k}''_\perp| \simeq |\mathbf{k}_\perp|$.

Three regimes can be distinguished. There are the weak ($d \lesssim \omega_{ti}$), intermediate ($\omega_{ti} \lesssim d \lesssim v_* \Delta k_{\mathrm{M}}$), and strong turbulence ($v_* \Delta k_{\mathrm{M}} \lesssim d$) regimes.

In the weak turbulent regime, $\Delta\omega = \omega_{ti} > d_k$ and $\Delta k_\theta = \omega_{ti}/v_* < \Delta k_{\mathrm{M}}$. The integration of Eq. (27) yields

$$\frac{\langle v_E^2 \rangle}{v_*^2} \simeq \frac{1}{6} v_*^3 \frac{\Delta k_{\mathrm{M}}^3}{\omega_{ti}^2} \frac{1}{\nu_{\mathrm{eff}}} \frac{(2\epsilon)^{1/2}}{s^2}$$
$$\simeq \frac{1}{6\pi^3} \frac{(2\epsilon)^{1/2}}{s^5} \frac{v_* \rho_s^{-1}}{\nu_{\mathrm{eff}}} \frac{q^2}{\epsilon^2} \frac{T_e}{T_i},$$

and the consistency requirement $d < \omega_{ti}$ demands that

$$\frac{(2\epsilon)^{1/2}}{s^4}\frac{v_* \rho_s^{-1}}{\nu_{\rm eff}} < 75\left(\frac{\epsilon}{q}\right)^3\left(\frac{T_i}{T_e}\right)^{3/2}.$$

The high collisionality that is required by this condition is, for usual tokamak parameters, incompatible with the inequality $\omega_{be} > \nu_{\rm eff}$. As a consequence, the weak turbulence regime is irrelevant.

The intermediate collisionality regime is characterized by $\Delta\omega \simeq d_k > \omega_{ti}$, and $\Delta k_\theta \simeq d_k/v_* < \Delta k_{\rm M}$. For simplicity, we will assume that these inequalities apply over the whole range of the spectrum. The ion turbulent decorrelation frequency equation (30) is then

$$d_k'' \sim (1/2\pi)k_\theta^2(2\pi)^2 s^2(0.2)(1/v_*)\langle|v_{k_\theta}|^2(0)\rangle, \tag{31}$$

and the spectral intensity equation (28) can be written as

$$\frac{d}{dk_\theta}\left(\frac{\langle|v_{k_\theta}|^2\rangle}{v_*^2}\right)^3 \simeq -\frac{3}{2}\frac{(2\epsilon)^{1/2}}{s^2}\frac{1}{\nu_{\rm eff}}\frac{v_*}{(2\pi)^3 k_\theta^3 s^4(0.2)^2}.$$

Hence the spectral intensity is given by

$$\frac{\langle v_{k_\theta}^2(0)\rangle}{v_*^2}\simeq\frac{1}{\Delta k_{\rm M}}\left(\frac{\Delta k_{\rm M}^2}{k_\theta^2}-1\right)^{1/3} \times\left(\frac{75}{(2\pi)^3}\frac{(2\epsilon)^{1/2}}{s^6}\frac{v_*\Delta k_{\rm M}}{\nu_{\rm eff}}\right)^{1/3}. \tag{32}$$

As $k_\theta \to 0$, the spectral intensity is divergent but integrable. This wavenumber spectrum is sketched in Fig. 4. From Eq. (32), the spectrum-integrated intensity is

$$\frac{\langle v_E^2\rangle}{v_*^2}\simeq 0.2\frac{(2\epsilon)^{1/6}}{s^{7/3}}\left(\frac{v_*\rho_s^{-1}}{\nu_{\rm eff}}\right)^{1/3}. \tag{33}$$

The saturation level has thus a weak density dependence, and scales like $n^{-1/3}$. This regime gives fluctuation level predictions in general accord with typical experiments (of the order of 0.2). The domain of applicability of Eq. (33) is limited by the condition $d < v_* \Delta k_{\rm M}$ for all k_θ. From (31) and (32), d is maximized for $k_\theta/\Delta k_{\rm M} = (2/3)^{1/2}$ at which point

$$d/v_* \Delta k_{\rm M} = 0.15 s^2\pi^2(\langle v_E^2\rangle/v_*^2).$$

This ratio is typically less than one for the collisionality regime that we consider. It is important to note that the condition of moderate turbulence level eliminates the ω_{ti} time

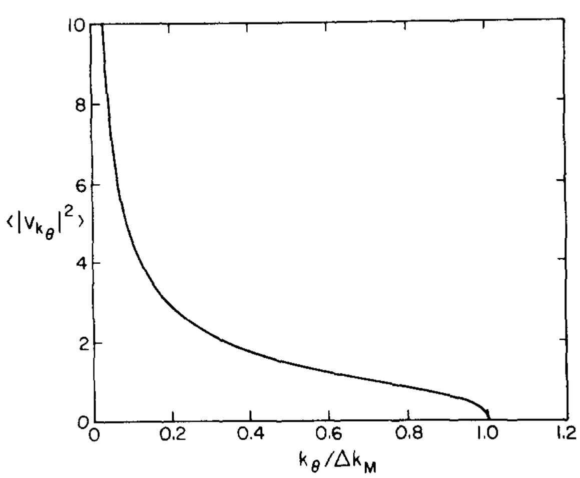

FIG. 4. Saturated spectral intensity, in arbitrary units, versus $k_\theta/\Delta k_{\rm M}$.

scale from the problem and results in a nonlinear interaction proportional to $\langle\phi^2\rangle^3$. Hence, balancing nonlinear interaction with linear destabilization yields $e\phi/T_e \sim (\rho_s/L_n)(\delta)^{1/6}$, where δ is the linear destabilization rate. This is significant, because the theory indicates a *very* weak dependence of fluctuation level on linear destabilization. This result is consistent with the virtually universal experimental observation that $e\phi/T_e \sim \rho_s/L_n$.

The last regime is the strong turbulent regime, characterized by very large nonlinear decorrelations. Here, $\Delta\omega \sim d_k > v_* \Delta k_{\rm M} > \omega_{ti}$ and $\Delta k_\theta \sim \Delta k_{\rm M}$. In the limit of large broadening, the expression of the decorrelation Eq. (30) and the differential spectral equation (28) are no longer valid. Reconsidering the integral equation (24) in the strong turbulence limit, one finds that the spectrum is peaked at $\bar{k}_\theta \ll \Delta k_{\rm M}$, with intensity given by

$$\frac{\langle v_E^2\rangle}{v_*^2}\simeq\frac{(2\epsilon)^{1/2}}{s^2}\frac{\Delta\omega}{\nu_{\rm eff}}. \tag{34}$$

Here,

$$\Delta\omega \simeq \rho_s^{-1}\langle v_E^2\rangle^{1/2}(\bar{k}_\theta/\Delta k_{\rm M}),$$

from which one gets

$$\frac{\langle v_E^2\rangle}{v_*^2}\simeq 1.5\frac{2\epsilon}{s^4}\left(\frac{v_*\rho_s^{-1}}{\nu_{\rm eff}}\right)^2\left(\frac{\bar{k}_\theta}{\Delta k_{\rm M}}\right)^2. \tag{35}$$

We will not pursue the study of this regime because of the large excitation necessary to reach it. Specifically, access to this regime requires $s^2\pi^2\langle v_E^2\rangle/v_*^2 > 1$, which corresponds to $e\tilde{\phi}/T_e > \rho_s/L_n$. In that case, nonlinear mechanisms which have not been included in this analysis should be considered.

In conclusion, for the collisionality that we consider, the intermediate turbulence regime applies and the saturation level is given by Eq. (33).

VII. CONFINEMENT TIMES AND SCALING LAWS

The results of the intermediate turbulent regime are used to determine the test particle diffusion coefficients of passing electrons and trapped electrons. These diffusion coefficients are identified with electron heat transport coefficients. For purpose of comparison, the scaling of the associated energy confinement times is also given.

The untrapped electrons have a test particle diffusion coefficient roughly equal to

$$D_u \simeq \langle v_E^2\rangle/\omega_{te},$$

since they interact with a quasimode during a transit time ω_{te}^{-1}, and since their typical radial step is of the order of v_E/ω_{te}, the radial $E\times B$ displacement. Using the saturation level equation (33), one finds that the corresponding confinement time $\tau_u \equiv a^2/D_u$ scales as

$$\tau_u \propto \epsilon^{1/2}q^{-1}T^{-13/6}s^{7/3}B^2n^{1/3}a^{10/3}, \tag{36}$$

where a is the minor radius and n is the density. Here, $L_n \propto a$, $v_s \propto T^{1/2}$, $\nu_{\rm eff} \propto nT^{-3/2}\epsilon^{-1}$, and $v_* \propto TB^{-1}a^{-1}$.

The trapped particles experience a much larger diffusion, since they remain within the region occupied by a quasimode until they are collisionally detrapped ($\nu_{\rm eff} > \omega$). Hence, the radial step size for trapped particles is $v_E/\nu_{\rm eff}$. Accordingly, the transport coefficient is

$$D_t \simeq (2\epsilon)^{1/2}(\langle v_E^2\rangle/\nu_{\rm eff}),$$

where the first factor takes into account the trapped particle population. Since the ratio of the coefficients $D_t/D_u \simeq \omega_{be}/\nu_{\rm eff} > 1$, the radial $E\times B$ transport due to trapped particles is dominant. As before, the confinement time $\tau_t \equiv a^2/D_t$ scales as

$$\tau_t \propto \epsilon^{-2} n^{4/3} T^{-25/6} B^2 a^{13/3} s^{7/3}. \tag{37}$$

For Ohmically heated discharges, the temperature can be eliminated by using the global power balance: $nT_e/\tau_E = \eta j_\parallel^2$, where η is the plasma resistivity and $j_\parallel$ is the Ohmic current density. This implies

$$\tau_E \propto n q^2 \epsilon^{-2} B^{-2} a^2 T^{5/2}. \tag{38}$$

Therefore, for Ohmically heated plasmas, the scaling laws for τ_u and τ_t become:

$$\tau_u \propto n^{9/14} q^{11/28} a^{19/7} s^{5/4} \epsilon^{-37/56} B^{1/7}, \tag{39}$$

and

$$\tau_t \propto n^{9/8} q^{5/4} a^{23/8} \epsilon^{-2} s^{7/8} B^{-1/2}. \tag{40}$$

For comparison, the scaling for the collisionless trapped particle regime ($\nu_{\rm eff} < \omega_{De}$) is now calculated. Now, the relevant decorrelation frequency is an average value of the electron magnetic drift frequency $\overline{\omega}_{De}$, which scales as $\overline{\omega}_{De} \propto \epsilon T^{1/2}/sa$, when $k_\perp \rho_i \sim 1$. In this highly excited regime, ion Compton scattering[4] as well as mode coupling[13–15] will limit the fluctuations at the level given by the usual mixing length argument

$$\pi^2 s^2 \langle v_E^2\rangle \sim v_*^2. \tag{41}$$

Thus one finds the trapped electron transport coefficient in the collisionless regime

$$D_t \sim (2\epsilon)^{1/2}(\langle v_E^2\rangle/\overline{\omega}_D) \sim (2\epsilon)^{1/2}(v_*^2/\pi^2 s^2 \overline{\omega}_D).$$

Defining the associated confinement time as before, and eliminating the temperature by use of the Ohmic heating relation (38), one finds the scaling

$$\tau \propto n^{3/8} q^{3/4} a^{21/8} s^{-5/8} \epsilon^{-7/16} B^{1/2}. \tag{42}$$

VIII. ELECTRON HEAT CONDUCTION DUE TO MAGNETIC FLUTTERING AT LOW β

Even though at low β (i.e., $\beta_p \lesssim 1$, where β_p is the poloidal β) the modes are predominantly electrostatic, the fluctuating currents induce a radial magnetic field which can be sufficient to cause a cross field transport of heat due to electron conduction.[16] Here we examine the magnetic structure of the toroidicity induced modes, and estimate the magnetic diffusion coefficient and the related electron heat transport.

For a low-β plasma, the perturbed magnetic field may be written as $\widetilde{\mathbf{B}} = \nabla A \times \mathbf{n}_0$, where A is the parallel component of the perturbed potential vector, and $\mathbf{n}_0$ is the unit vector directed along the equilibrium magnetic field. In ballooning representation, the perturbed parallel current, due to the electron motion, is related to the potential vector by the Ampere's law

$$k_\perp^2 A = (4\pi/c) j_\parallel.$$

Linearly, the electron nonadiabatic distribution function $h_e(v_\parallel)$ obeys the magnetic drift-kinetic equation

$$\left(-i(\omega-\omega_{De}) + \frac{v_\parallel}{Rq}\frac{\partial}{\partial\theta}\right)h_e = -i\frac{e}{T_e}(\omega_{*e}-\omega)F_{Me}\left(\phi - \frac{v_\parallel}{c}A\right).$$

The first two velocity moments of this equation, using the isothermal closure for the electron pressure and neglecting electron inertia, yield

$$-i(\omega-\overline{\omega}_D)\overline{n}_e + \frac{1}{Rq}\frac{\partial}{\partial\theta}u_e = -i\frac{e}{T_e}(\omega_{*e}-\omega)\phi,$$

and

$$\frac{v_e^2}{Rq}\frac{\partial}{\partial\theta}\overline{n}_e = i\frac{e}{T_e}(\omega_* - \omega)\frac{v_e^2}{c}A,$$

where

$$\overline{n}_e \equiv \int dv_\parallel\, h_e,$$

and

$$u_e \equiv \int h_e\, v_\parallel\, dv_\parallel = -\frac{j_\parallel}{n_0 e}.$$

Hence, an approximate expression for the parallel Ohm's law is obtained:

$$-\frac{c^2}{R^2q^2}\frac{1}{k_{De}^2}\frac{1}{c}\frac{d^2}{d\theta^2}(k_\perp^2 A) = i(\omega_{*e}-\omega)\left(-\frac{1}{Rq}\frac{\partial}{\partial\theta}\phi + i\frac{\omega-\overline{\omega}_{De}}{c}A\right), \tag{43}$$

where $\overline{\omega}_{De}$ is an average electron magnetic drift frequency, and $k_{De}^2 \equiv 4\pi n_0 e^2/T_e$ is the square of the electron Debye wavenumber.

The Ohm's law expresses here the balance of the electron pressure gradient [left-hand side of Eq. (43)], with the force due to the parallel electric field, and determines the magnetic perturbation A driven by the electrostatic potential ϕ. For an electrostatic toroidicity induced mode, the potential ϕ has the structure shown in Fig. 1, is even in θ, and bounded within $\pm\pi$. The vector potential A is an odd function in θ, and is more extended than ϕ. It has slow, algebraic decay for large θ, and Fig. 5 shows that the typical width W_A of the magnetic perturbation, which is necessary for the computation of the magnetic diffusion coefficient, is larger than the width of ϕ:

$$W_A > \pi.$$

Since trapped electrons do not participate in electron heat transport by conduction, we will examine exclusively the behavior of the passing electrons.

The electron heat transport coefficient depends very strongly on the correlation length L_k of the electrons with the magnetic perturbation. If the correlation length is larger than the extent of the magnetic fluctuation ($L_k > RqW_A/\pi$), then there is weak radial transport. This is indeed a consequence of the odd parity of $A(\theta)$. If on the other hand, the electrons decorrelate rapidly, their radial step $\Delta x \simeq L_k k_\theta A/B$, will be limited by the short decorrelation time, resulting in weak diffusion. The radial transport is therefore maximal when the decorrelation length L_k is of the

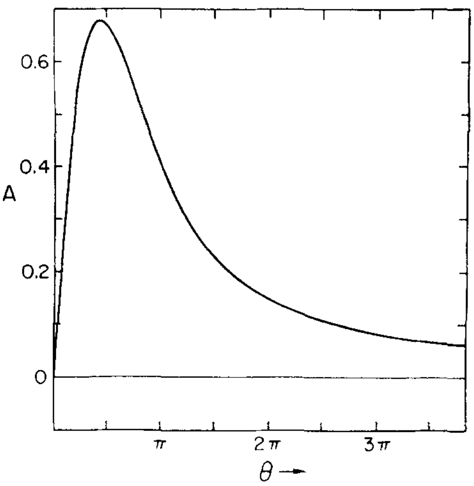

FIG. 5. Structure of the magnetic potential component A of the quasimode, along the field line. The parameters have the same values as for Fig. 1, and $\beta_p = 0.5$.

order of the width of the A fluctuation: $L_k \sim RqW_A/\pi$. Under these conditions the radial magnetic diffusion coefficient is given by

$$D_M \sim (1/L_k^{-1})(k_\theta^2 A^2/B^2),$$

where

$$L_k^{-1} \sim D_M k_\perp^2 \gtrsim (1/Rq)\pi/W_A, \tag{44}$$

and where a spectral average is implied when appropriate.

The Ohm's law relates the magnetic perturbation to the electrostatic perturbation according to

$$\frac{k_\theta}{B} A \simeq \beta \frac{q}{\epsilon} \frac{k_\theta^2}{k_\perp^2} \frac{e}{T_e} \phi,$$

i.e.,

$$\frac{k_\theta}{B} A \simeq \frac{\epsilon}{q} \beta_p \frac{k_\theta^2}{k_\perp^2} \frac{1}{k_\theta L_n} \frac{v_E}{v_*}. \tag{45}$$

It is found that the electron conduction transport coefficient, given by $\chi_e = v_e D_M$ in the small collisionality regime,[16] is

$$\chi_e \simeq (\omega_{te}/k_\theta^2)\beta_p(\langle v_E^2\rangle/v_*^2)^{1/2}. \tag{46}$$

This result requires that Eq. (44) be satisfied. This implies

$$\beta_p \frac{k_\perp^2}{k_\theta^2} \left(\frac{\langle v_E^2\rangle}{v_*^2}\right)^{1/2} \gtrsim \frac{\pi}{W_A}.$$

This inequality holds for large β_p and large levels of fluctuations.

In such cases, the ratio of the conduction χ_e [Eq. (46)] to the electrostatic transport D_u is

$$\frac{\chi_e}{D_u} \simeq \left(\frac{\omega_{te}}{k_\theta v_*}\right)^2 \beta_p \left(\frac{\langle \tilde{v}_E^2\rangle}{v_*^2}\right)^{-1/2},$$

which increases with β_p.

This result illustrates the increasing importance of the heat loss due to electron conduction at higher β_p. The estimate of this effect for $\beta_p > 1$ would, however, require an analysis of the electromagnetic drift modes, since magnetic response can then affect the structure of the eigenmode as well as its frequency.

IX. CONCLUSION

In this paper, ion Compton scattering has been considered as a saturation mechanism for toroidicity-induced drift modes excited by trapped electron inverse dissipation. The saturated state wavenumber spectrum has been calculated and used to obtain the energy confinement time scaling for a number of different collisionality regimes. In particular, the predicted confinement time density scaling for an Ohmically heated discharge increases from $n^{3/8}$ in the collisionless regime to $n^{9/8}$ in the dissipative trapped electron regime.

It is instructive to contrast the Compton scattering process in the sheared slab[4] with that in the torus. In the slab, the spectrum of Pearlstein–Berk eigenmodes extends to x_i, the ion Landau resonance point. Thus, $k_\parallel' \Delta X V_{Ti} \sim \omega$, where ΔX is the spectrum width. Hence, beat wave resonance broadening is irrelevant for moderate turbulence levels. Furthermore, since $k_\parallel' \Delta X V_{Ti} \sim \omega$, the rate of momentum exchange with ions due to Compton scattering is of the order of the rate of energy exchange with the beat waves. This indicates that the rates of nonlinear transfer and dissipation are comparable. Thus, significant nonlinear damping occurs due to beat wave resonance.

In the torus, $k_\parallel' \Delta X V_{Ti} \sim \omega_{Ti} \ll \omega$. Hence, beat wave resonance broadening is important for turbulence levels yielding decorrelation rates $d_{\mathbf{k}}$, such that $d_{\mathbf{k}} > \omega_{Ti}$. In most relevant regimes, this inequality is satisfied. Also, the rate of momentum exchange with ions substantially exceeds the rate of energy exchange with the beat waves. Hence, the rate of nonlinear transfer exceeds the rate of nonlinear damping. Thus, transfer to damped modes is the saturation process in toroidal geometry.

Finally, it should be noted that mode coupling,[13–15] electron and ion clump effects,[17] and nonlinear instabilities[18] have been omitted from this analysis. These processes could partially modify the conclusions of this paper.

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