ETG Turbulence Isotropization

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Outline

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Drift Wave Instabilities

- Drift waves are characterized as density, temperature or pressure fluctuations in plasmas. Mostly simply they are involved electrostatically in low-β plasmas.
- Modes relevant to tokamak physics include ion-temperature-gradient modes (ITG), electron-temperature-gradient modes (ETG), and trapped electron modes (TEM).
- Low-frequency drift wave turbulence is largely responsible for the anomalous transport of plasma particles across magnetic field lines.

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Ion-Temperature-Gradient Mode Growth

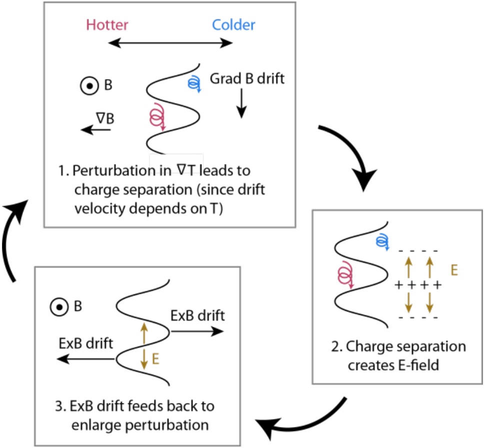

Figure: Simple picture of ITG instability.

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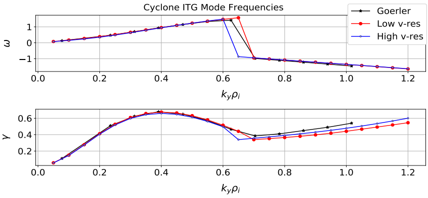
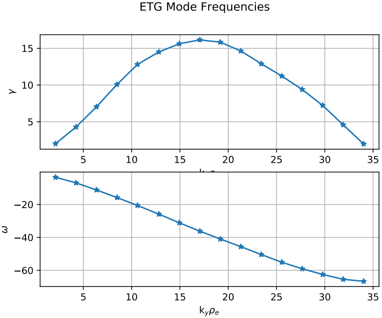
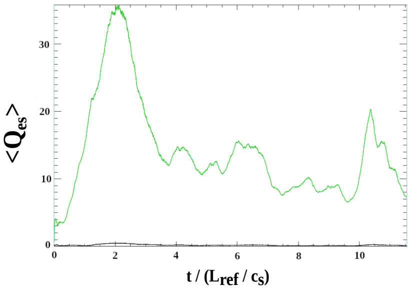
ETG Simulation in GENE

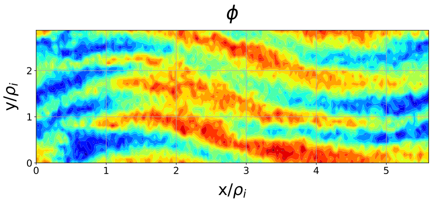




- Successful reproduction of Goerler benchmark with kinetic ions and electrons, showing ITG mode and trapped electron mode growth.
- Conversion of ITG to ETG flux-tube case prior to running non-linear simulations.

ETG "Streamers" in GENE





- ETG turbulence in toroidal gyrokinetic simulations is associated with elongated "streamers".
- Multiscale turbulence simulations have shown that streamers dominate electron heat flux and lead best reproduced experimental heat fluxes within experimental uncertainties.

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Hasegawa-Mima Fluid ETG Model

- Partial differential equation derived from fluid continuity and momentum equations.
- Approximations made that are useful to describing turbulence in tokamak plasmas.
  - Cyclotron motion periods much smaller than time scales that quantities of interest change on (B, Φ, n).
  - Long length scales along $\hat{b}$-direction - $k_{||}/k_{\perp} \equiv \epsilon \ll 1$.
  - Quasi-neutrality of particle densities is enforced.
  - Isothermal equation of state, with adiabatic ions that have negligible temperatures.
- Shown to cause isotropic behavior for long wavelength modes as well as an inverse energy-cascade.

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We start with the fluid continuity and momentum equations and $\tau = T_e/T_i$, where we have already taken the ion approximations discussed on the previous slide:

$$\frac{\partial n_e}{\partial t} + \nabla \cdot (n_e \vec{v}_e) = 0, \tag{1}$$

$$m_e \frac{d\vec{v}_e}{dt} = (1+\tau)\, e\nabla\delta\Phi - \frac{e}{c}\vec{v}_e \times \vec{B} - \frac{\nabla P_e}{n_e}\ . \tag{2}$$

We break equation (2) up into parallel and perpendicular components by taking a dot product with $\hat{b}$ to find,

$$\frac{d\vec{v}_{e,\parallel}}{dt} = (1+\tau)\frac{e}{m_e}\partial_t^{-1}\nabla_\parallel \delta\Phi \Rightarrow v_\parallel \simeq (1+\tau)\frac{k_\parallel e\delta\Phi}{m_e\omega}, \tag{3}$$

Hasegawa-Mima Equations

$$\frac{d\vec{v}_{e,\perp}}{dt} = (1+\tau)\frac{e}{m_e}\nabla_\perp\delta\Phi - \omega_{c,e}\vec{v}_{e,\perp} - \frac{\hat{b}\times\nabla P_e}{m_e n_e}\ . \tag{4}$$

Then we can split up $\vec{v}_e$ by ordering

$$\begin{aligned}
\vec{v}_{e,0} &= \vec{v}_{\|} + \vec{v}_{\perp,0} = \vec{v}_{\|} + (1+\tau)\,\vec{v}_E + \vec{v}_D, \\
\vec{v}_{e,1} &= \vec{v}_{\perp,1} = -\frac{1}{\omega_{c,e}}(\partial_t + \vec{v}_{e,0}\cdot\nabla)(\hat{b}\times\vec{v}_{e,0}) \\
&\simeq \frac{e(1+\tau)}{m_e\omega_{c,e}^2}\partial_t\nabla_\perp\delta\Phi - [\frac{\vec{b}\times\nabla P_e}{n_e}\cdot\nabla_\perp]\frac{e(1+\tau)}{m_e^2\omega_{c,e}^3}\nabla_\perp\delta\Phi \\
&+ \frac{e^2(1+\tau)^2}{m_e^2\omega_{c,e}^3}[\hat{b}\times\nabla_\perp\Phi\cdot\nabla_\perp]\nabla_\perp\delta\Phi\ .
\end{aligned} \tag{5}$$

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Now, with incompressibility, $\nabla \cdot \vec{v}_{e,0,\perp} = 0$, equation (1) becomes,

$$\partial_t \delta n_e + n_e \nabla \cdot (v_{\|} + \vec{v}_{e,1}) + \nabla \delta n_e \cdot \vec{v}_D + (1+\tau)\delta n_e \cdot \vec{v}_E = 0, \tag{6}$$

and plugging in $\delta n_e = \delta n_i$, we find that to order ϵ,

$$\begin{aligned} &- n_e \partial_t \frac{e\delta\Phi}{T_i} + n_e(1+\tau)\frac{e}{m_e}\partial_t^{-1}\nabla_{\|}^2 \delta\Phi + \frac{en_e(1+\tau)}{m_e\omega_{c,e}^2}\partial_t \nabla_\perp^2 \delta\Phi \\ &- \frac{e(1+\tau)}{m_e^2\omega_{c,e}^3}[\hat{b} \times \nabla P_e \cdot \nabla_\perp]\nabla_\perp^2 \delta\Phi + \frac{e^2 n_e(1+\tau)^2}{m_e^2\omega_{c,e}^3}[\hat{b} \times \nabla_\perp \delta\Phi \cdot \nabla_\perp]\nabla_\perp^2 \delta\Phi \\ &+ \nabla_\perp \frac{e\delta\Phi}{T_i} \cdot \frac{\hat{b} \times \nabla P_e}{m_e\omega_{c,e}} + (1+\tau)\frac{e}{m_e\omega_{c,e}}\nabla n_e \cdot \hat{b} \times \nabla\Phi = 0\,. \end{aligned} \tag{7}$$

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Taking the standard electron dyanamic normalization,

$$\Phi = \frac{e\delta\Phi}{T_i}, \; -\frac{1}{r_n} = \frac{\partial_x n_e}{n_e}, \; -\frac{1}{r_t} = \frac{\partial_x T_e}{T_e}, \; \eta_e = \frac{r_n}{r_t}, \\ \rho_e = \sqrt{\frac{\tau m_e}{m_i}}\rho_i, \; \vec{x} = \frac{\vec{x}}{\rho_e}, \; t = \frac{\rho_e}{r_n}\omega_{ce} t \tag{8}$$

and plugging into equation (7) gives the form of the H-M ETG model,

$$-(1 - \frac{1+\tau}{2\tau}\nabla_\perp^2)\partial_t\Phi + \frac{1+\tau}{2\tau}\frac{r_n^2}{\rho_e^2}\partial_t^{-1}\nabla_\parallel^2\Phi + \frac{(1+\tau)(1+\eta_e)}{4\tau}\partial_y\nabla_\perp^2\Phi \\ + \frac{1+\eta_e}{2\tau}\partial_y\Phi + \frac{(1+\tau)^2}{\tau^2}\frac{r_n}{4\rho_e}(\hat{b}\times\nabla_\perp\Phi\cdot\nabla_\perp)\nabla_\perp^2\Phi = 0\,. \tag{9}$$

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Hasegawa-Mima Equations

Finally we drop the parallel gradient term since $k_\parallel^2/k_\perp^2 \sim \epsilon^2$, and simplify the bracketed expression for a 2-D slab geometry to find the final form of our model,

$$\begin{aligned}\partial_t[\Phi - \frac{1+\tau}{2\tau}\zeta] = \frac{(1+\tau)(1+\eta_e)}{4\tau}\zeta_y + \frac{1+\eta_e}{2\tau}\Phi_y \\ + \frac{(1+\tau)^2}{\tau}\frac{r_n}{4\rho_e}[\Phi_x\zeta_y - \zeta_x\Phi_y],\end{aligned} \tag{10}$$

where $\zeta = \nabla^2\Phi$ and x, y subscripts denote partial derivatives. The nonlinear term, due to the polarization drift, is responsible for isotropizing the turbulence.

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Pseudo-Spectral Solver

- Equation (6) is solved numerically using the pseudo-spectral method.
  - Fourier transform the equation and get $\zeta = (k_x^2 + k_y^2)\Phi$.
  - Inverse Fourier transform $\zeta_{x,y}$ and $\Phi_{x,y}$ back into real space.
  - Calculate the non-linear products between ζ and Φ in real space and then Fourier transform the products so they can be added to the other terms in Fourier space.
  - Time advance Φ discretely.

- Time advancement is done using the 4^{th} order Runge-Kutta method.

- The pseudo-spectral method requires aliasing of modes to avoid over-emphasizing the contribution of lower modes due to non-linear terms - add new slide.

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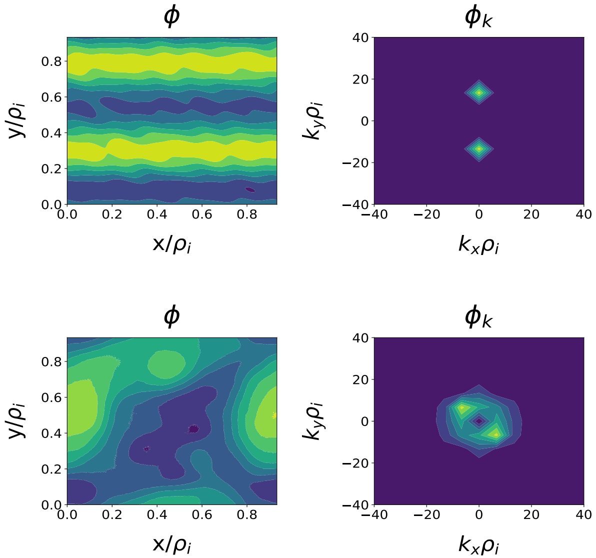
ETG H-M Results



- Results of H-M code. Initial condition (above) isotropizes at later time (below).
- Add parameters?

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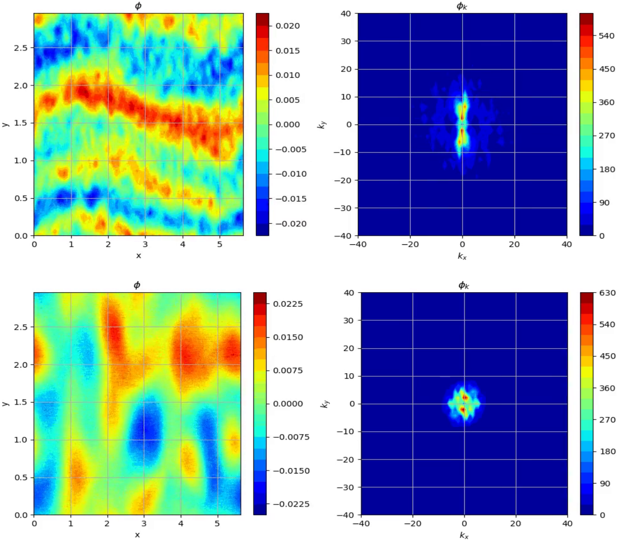
GENE ETG Streamer Test



- Test of GENE saturated ETG results into H-M code. Initial conditions (above) isotropize at later times (below).
- Inverse energy cascade observed.

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Zonal Flow Excitation

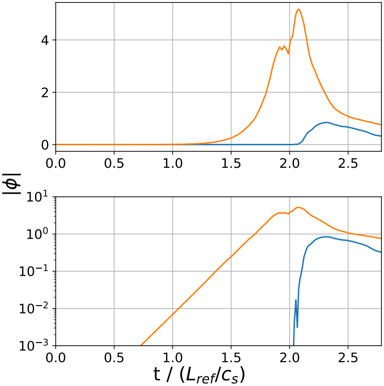

Figure: A non-linear GENE sim initialized with two modes - a default ZF mode ($k_y = 0$) and the orange ETG ($k_y \rho_i = 6.36$) mode. This excites the blue ZF mode ($k_y = 0, k_x \rho_i = 4.955$).

- Zonal flow can be spontaneously excited by intermediate-scale ETG turbulence in tokamak geometries - $k_\perp \rho_e \ll 1 \ll k_\perp \rho_i$.
- Plan to carry out large-scale ETG simulations with GENE to compare to theory of zonal flow excitation by ETG modes.

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References

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