Tokamak Turbulence Simulations using GENE

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Research Project

- Run linear and NL ETG with kinetic ions in support of ETG isotropization project (Haotian Chen lead)
- Study impurity pinch in pedestal

What is GENE?

Gyrokinetic Electromagnetic Numerical Experiment

- A gyrokinetic plasma turbulence code that solves nonlinear gyrokinetic equations in a flux-tube or global domain.
- A “continuum” code that solves the gyrokinetic equations using a Eulerian approach (fixed grid) in 5D phase space.
- Efficiently calculates linear mode properties.
- Massively parallelized w/ MPI or OpenMP and ported to a large number of architectures. Written in Fortran2008 w/ IDL diagnostic tool.
- Initial timestep (linear) is determined from eigenvalue analysis. Timestep is adaptive.

Gyrokinetic system of equations I

Gyrokinetic Vlasov equation per species σ with collisions

$$\frac{\partial f_\sigma}{\partial t} + \dot{\mathbf{X}} \cdot \nabla f_\sigma + \dot{v}_{\parallel} \frac{\partial f_\sigma}{\partial v_{\parallel}} + \dot{\mu} \frac{\partial f_\sigma}{\partial \mu} = C(f_\sigma, f_{\sigma'}) \tag{1}$$

gyrocenter position $\mathbf{X}$

$$\dot{\mathbf{X}} = v_{\parallel} \mathbf{b}_0 + \frac{B_0}{B_{0\parallel}^*} \mathbf{v}_{\perp}$$

with the combined drift velocities

$$\mathbf{v}_{\perp} \equiv \frac{c}{B_0^2} \bar{\chi}_1 \times \mathbf{B}_0 + \frac{\mu}{m_\sigma \Omega_\sigma} \mathbf{b}_0 \times \nabla B_0 + \frac{v_{\parallel}^2}{\Omega_\sigma} (\nabla \times \mathbf{b})_{\perp}$$

and the generalized potential
$\bar{\chi} = \bar{\phi}_1 - \frac{v_{\parallel}}{c} \bar{A}_{1\parallel} + \frac{\mu}{q_\sigma} \bar{B}_{1\parallel}$

parallel velocity $v_{\parallel}$

$$\dot{v}_{\parallel} = \frac{\dot{\mathbf{X}}}{m_\sigma v_{\parallel}} \cdot \left(q_\sigma \bar{\mathbf{E}}_1 - \mu \nabla (B_0 + \bar{B}_{1\parallel}) \right)$$

with the electric field
$\mathbf{E}_1 = -\nabla \phi_1 - \frac{\mathbf{b}_0}{c} \frac{\partial}{\partial t} A_{1\parallel}$

magnetic moment μ

$$\dot{\mu} = 0$$

... which is δf-splitted in GENE

Gyrokinetic system of equations II

Gyrokinetic field equations

Poisson equation

$$\nabla_\perp^2 \phi_1 = -4\pi \sum_\sigma q_\sigma n_{1\sigma}$$

Ampère's law

$$\nabla_\perp^2 A_{1\|} = -\frac{4\pi}{c} \sum_\sigma j_{1\|\sigma}$$

$$B_{1\|} = -4\pi \sum_\sigma \frac{p_{1\perp,\sigma}}{B_0}$$

... all in all a nonlinear, 5D partial integro-differential system of equations

Relevant moments (in local approx./Fourier space)

$$n_{1\sigma,\mathbf{k}} = \frac{2\pi B_0}{m_\sigma} \int \mathrm{d}v_\| \mathrm{d}\mu \left[J_0 h_{1\sigma,\mathbf{k}} - q_\sigma \phi_{1,\mathbf{k}} \frac{F_{0\sigma}}{T_{0\sigma}} \right]$$

$$j_{1\|\sigma,\mathbf{k}} = q_\sigma \frac{2\pi B_0}{m_\sigma} \int \mathrm{d}v_\| \mathrm{d}\mu \; v_\| \left[J_0 h_{1\sigma,\mathbf{k}} - q_\sigma \phi_{1,\mathbf{k}} \frac{F_{0\sigma}}{T_{0\sigma}} \right]$$

$$p_{1\perp\sigma,\mathbf{k}} \equiv \frac{2\pi B_0}{m_\sigma} \int \mathrm{d}v_\| \mathrm{d}\mu \; \mu B_0 I_1 h_{1\sigma,\mathbf{k}}$$

with the nonadiabatic part of f_1

$$h_{1\sigma} \equiv f_{1\sigma} + \left[q_\sigma J_0 \phi_1 + \mu I_1 B_{1\|} \right] \frac{F_{0\sigma}}{T_{0\sigma}}$$

and the Bessel functions

$$J_0 = J_0(k_\perp \rho)$$

$$I_1 = I_1(k_\perp \rho) = 2 J_1(k_\perp \rho)/(k_\perp \rho)$$

Algorithm Overview

- Each particle species is described by a time-dependent distribution function in a five-dimensional phase space: $\mathrm{f}(\mathbf{R},\mathrm{v}_{||},\mu,t)$
- Eulerian method is applied by employing a fixed grid in 5-D phase space. The *z* and $v_{||}$ operators are discretized using a 4th order Arakawa scheme, while the *x* and *y* terms are treated using a pseudo-spectral approach – linear terms are evaluated in k-space and non-linear terms in real space.
- The velocity space integrations are done via Gauss and trapezoidal rules in μ and $v_{||}$ space, respectively.
- The time step is evaluated via a 4th order Runge-Kutta scheme.
- In addition, one has to advance purely spatial, scalar quantities characterizing the electromagnetic fields by solving modified versions of Maxwell's equations. These quantities are:

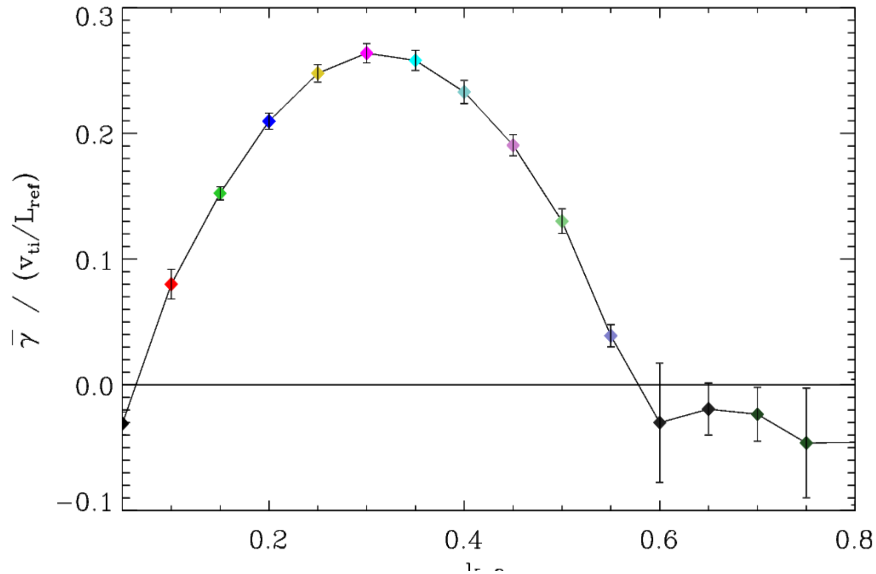
$$\phi(\mathbf{x},t) \qquad \mathrm{A}_{||}(\mathbf{x},t) \qquad \mathrm{B}_{||}(\mathbf{x},t)$$

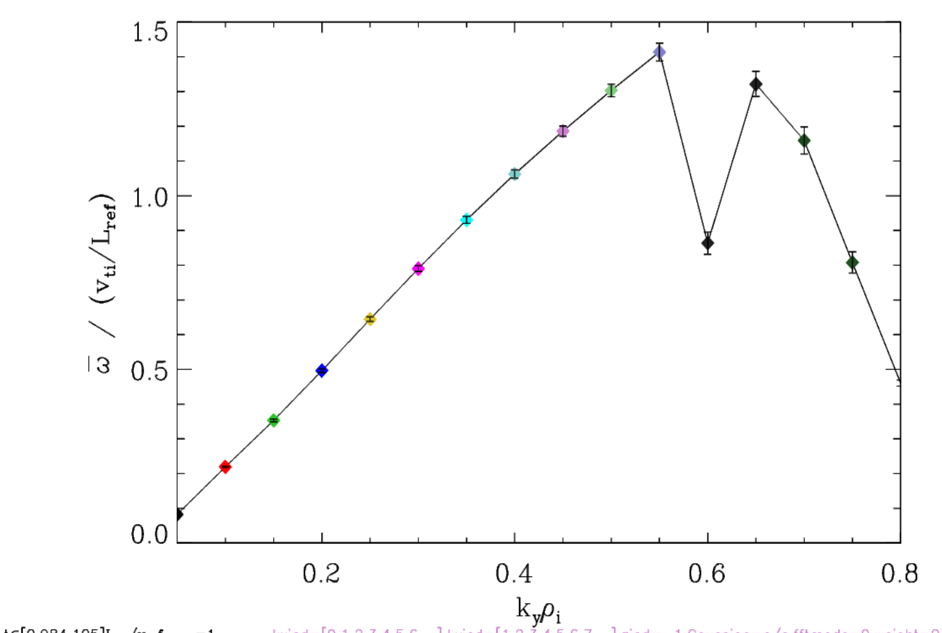
Extras

- Magnetic shaping: EFIT, Miller, Circular, etc.
- Fairly rigorous collision operator
- Linear k_y scans can be done on a workstation overnight.
- IDL diagnostics
- Straight-forward scripts for parameter scans.

Linear Runs

Cyclone Base Case





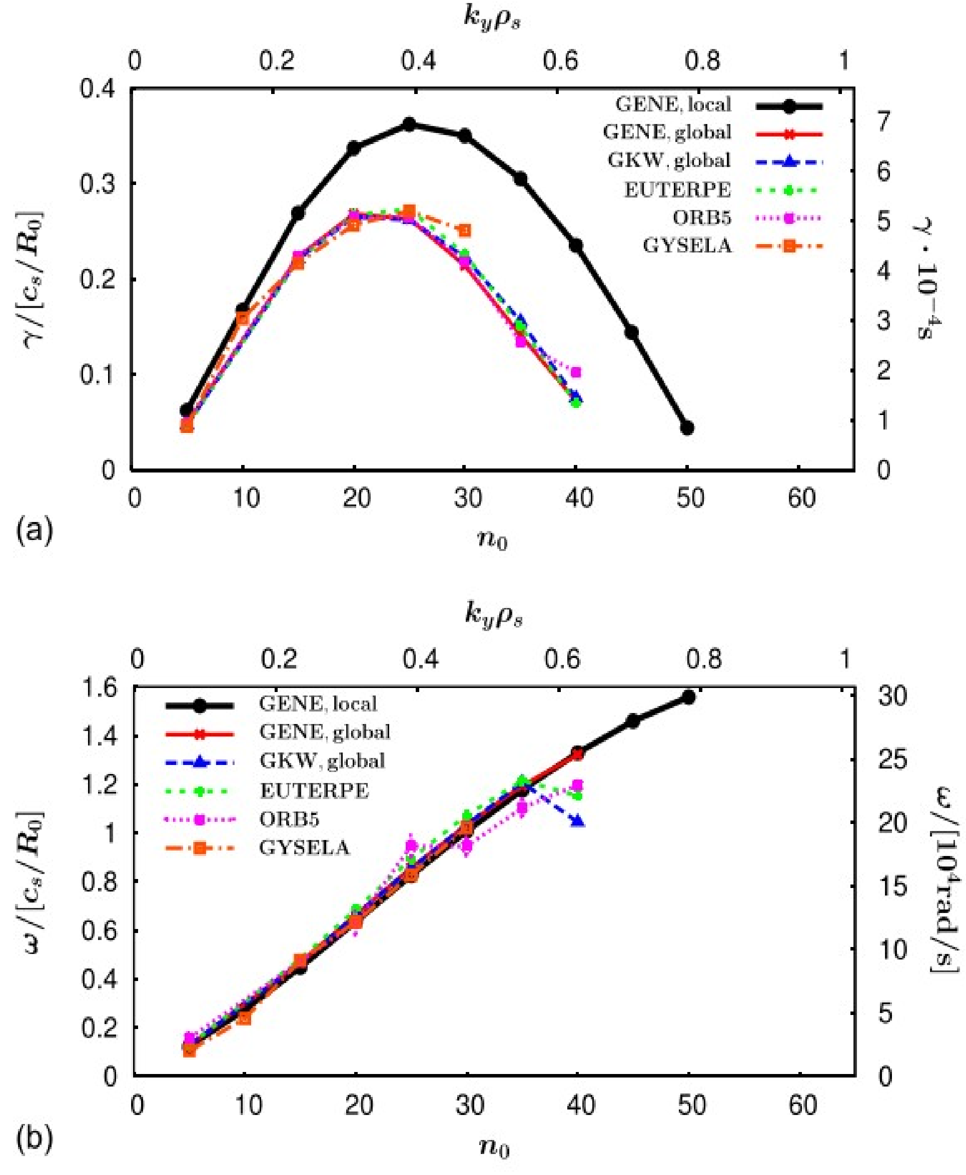


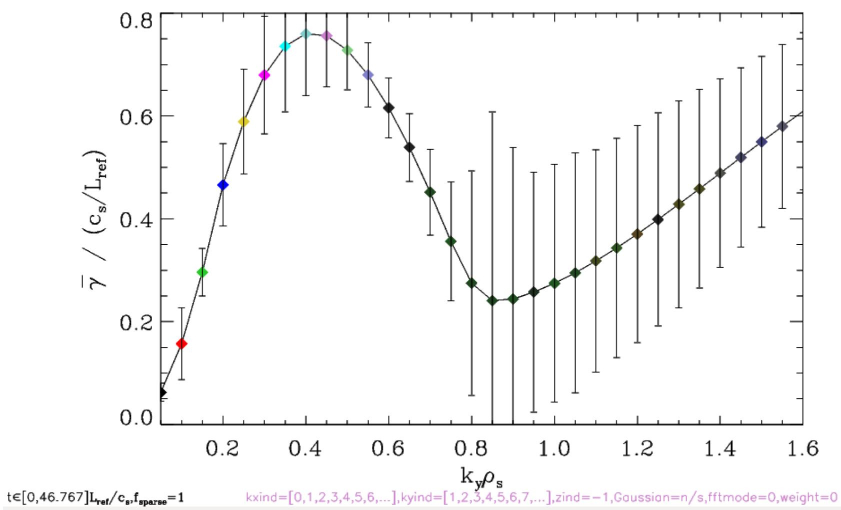
FIG. 3. Linear (a) growth rate and (b) real frequency spectra for the one-species, adiabatic electrons case from EUTERPE, GENE, GKW, GYSELA, and ORB5. The local (flux tube) are plotted for reference, as well.

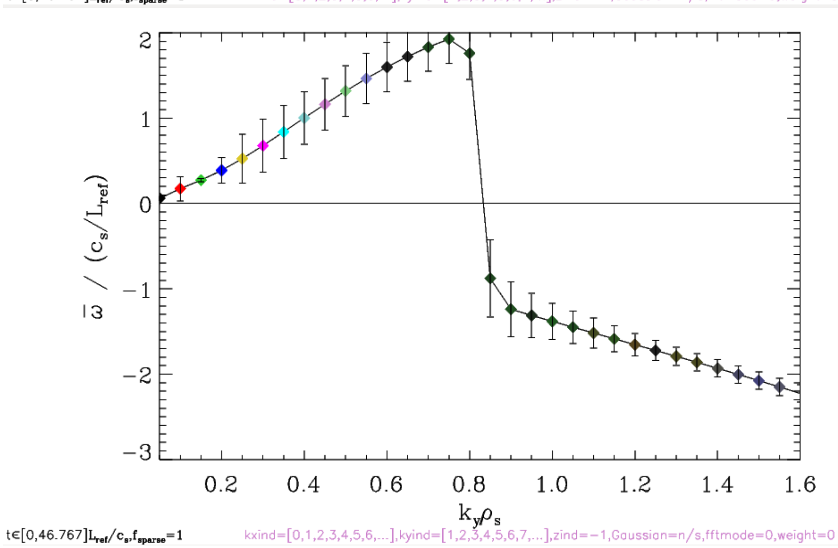
ITG Case

w/ kinetic electrons

$$q(r) = 2.52(r/a)^2 - 0.16(r/a) + .086, \text{ s} = \text{r} * (\text{q'}/\text{q})$$

$$r = a/2, \text{ s} = 0.837$$





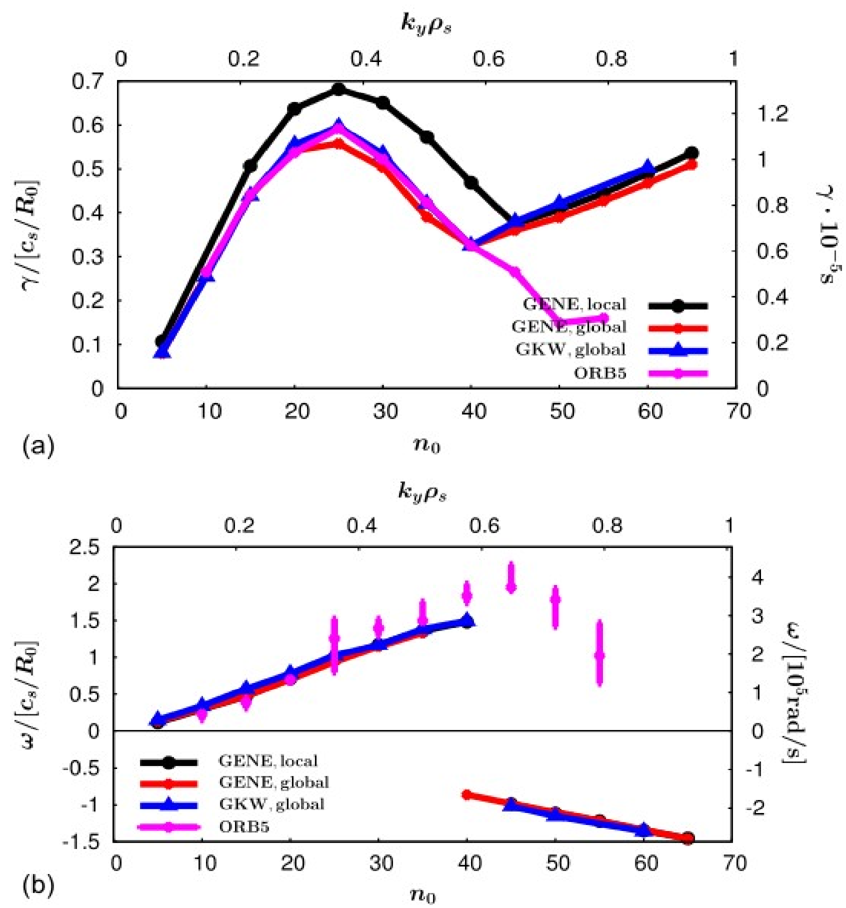


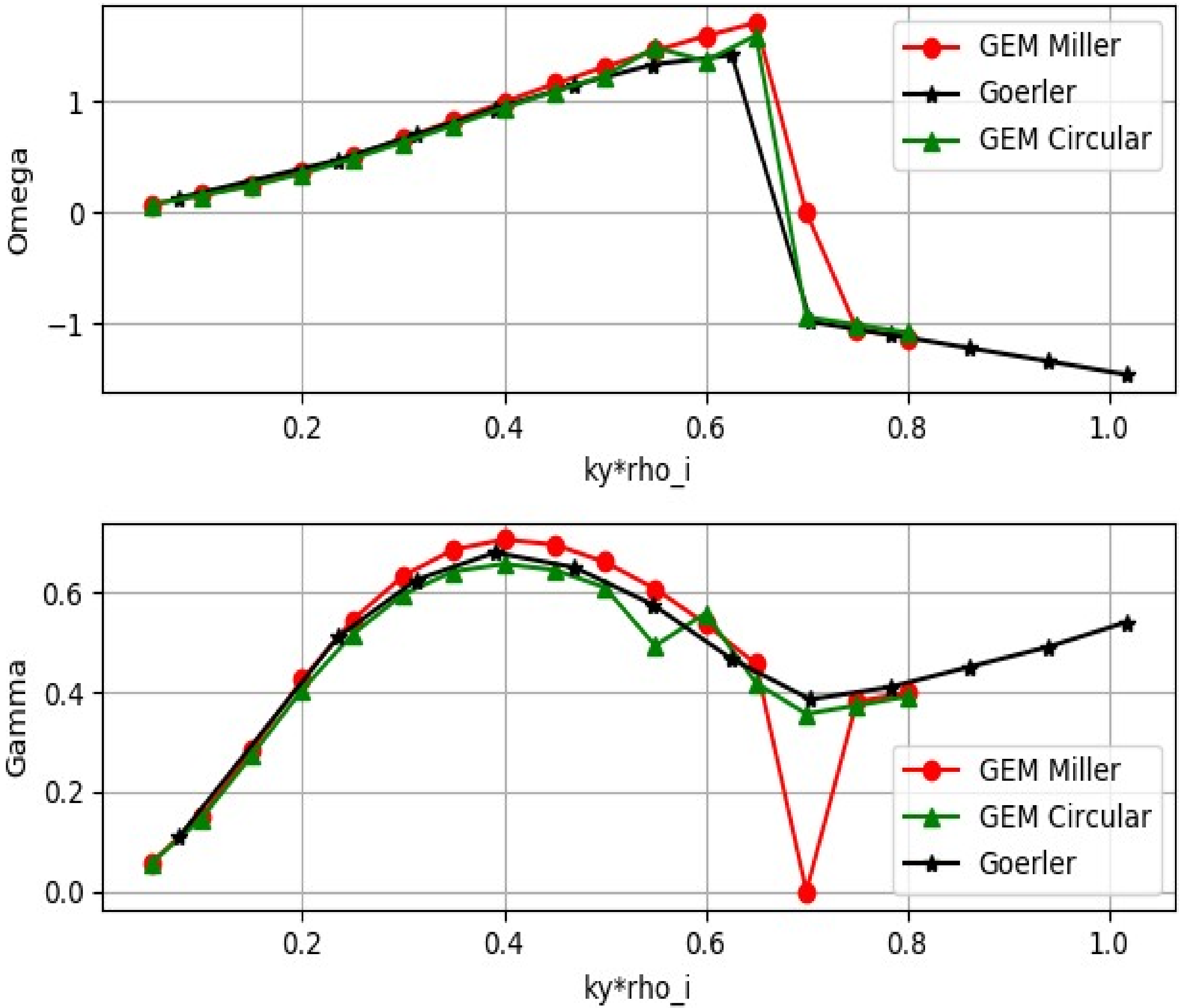
FIG. 6. Linear (a) growth rate and (b) real frequency spectra from GKW, ORB5, and GENE for the electrostatic case. The local (flux tube) results with linear growth rates maximized over the radius and ballooning angle are plotted for reference, as well.

Scanscript Results

ITG case w/ Kin El

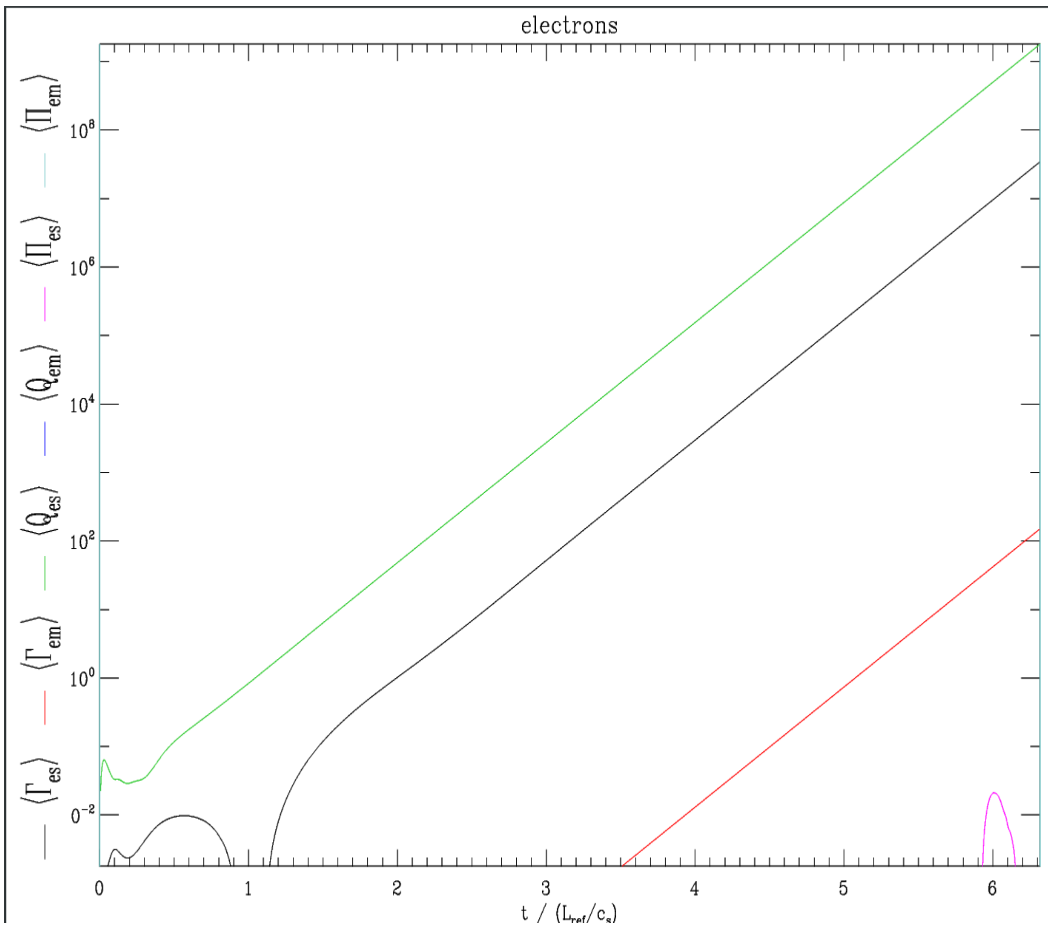
<u>Differences</u>

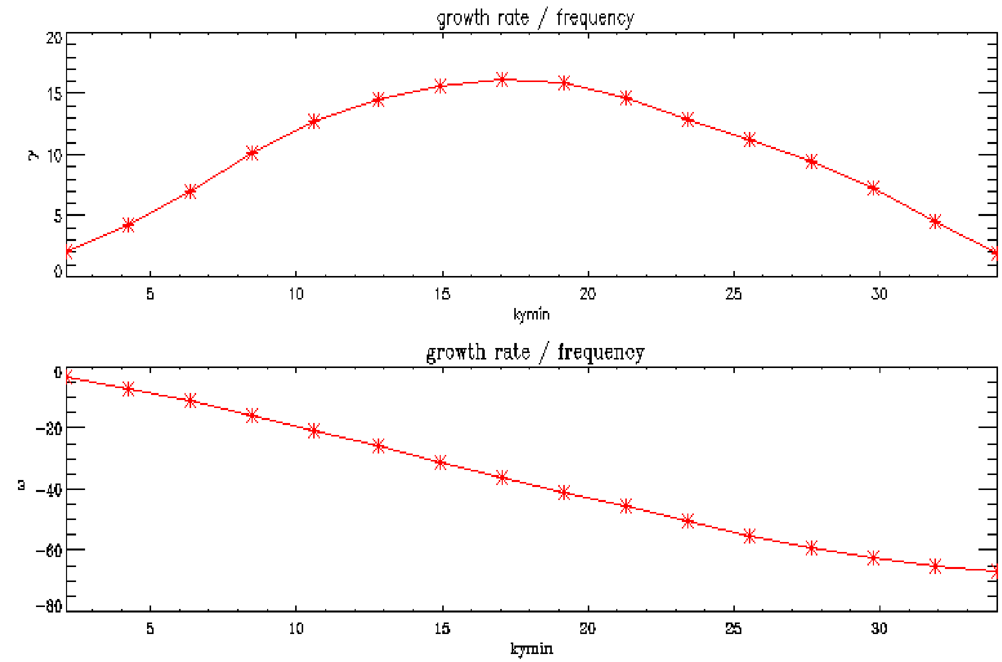
- β = 0 (Goerler), 1e-4 (GENE)
- m_e/m_i = 5.556e-4 (GENE) 5.44617e-4 (Goerler)



ETG Results

- Circular
- $\nabla T_i = 0$

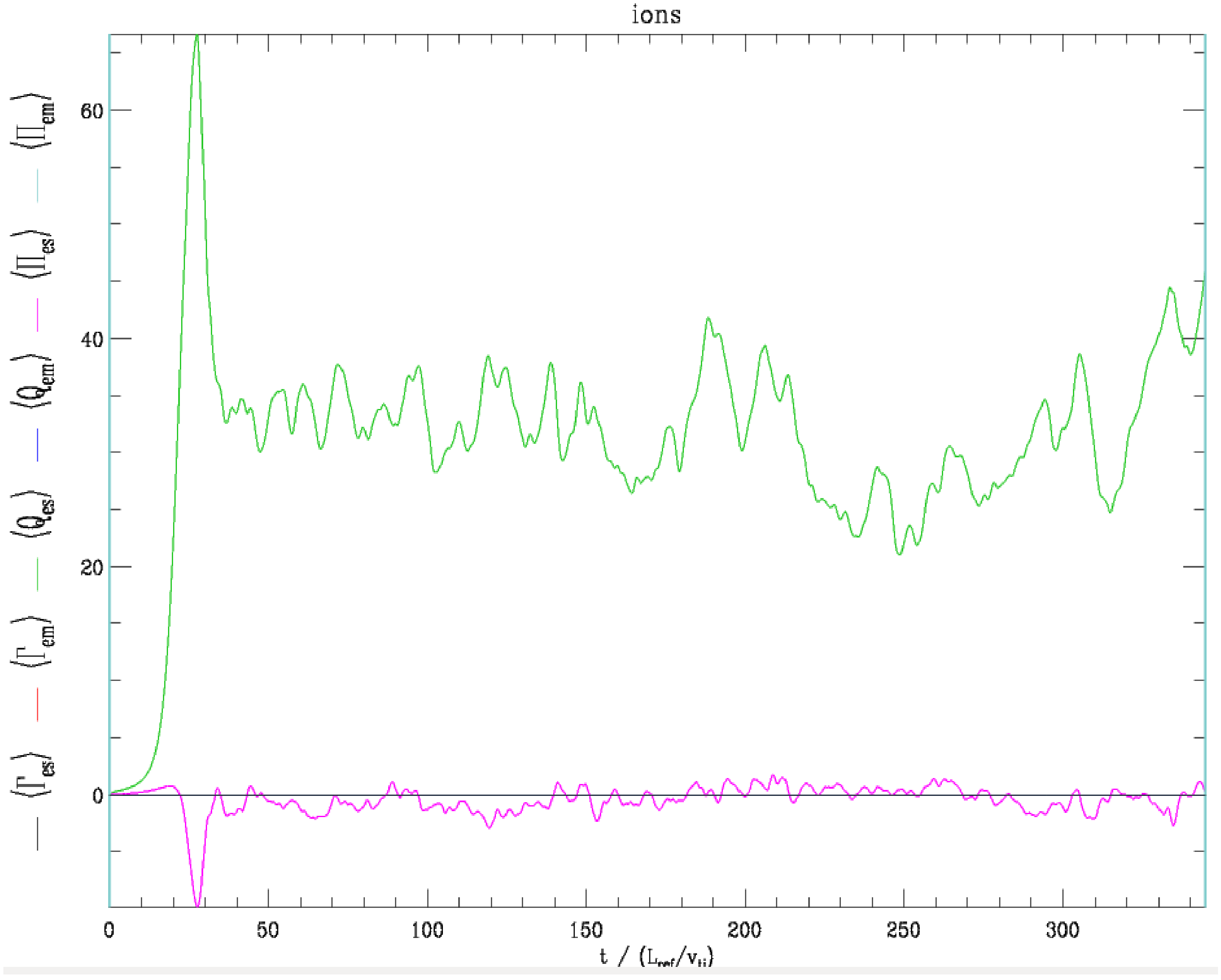




A.2.1 derived reference quantities

p_{ref}	reference pressure $p_{\mathrm{ref}} = n_{\mathrm{ref}} T_{\mathrm{ref}}$
c_{ref}	reference velocity $c_{\mathrm{ref}} = \sqrt{T_{\mathrm{ref}}/m_{\mathrm{ref}}}$ (without $\sqrt{2}$!)
Ω_{ref}	reference gyrofrequency $\Omega_{\mathrm{ref}} = q_{\mathrm{ref}} B_{\mathrm{ref}}/(m_{\mathrm{ref}} c)$
ρ_{ref}	reference gyroradius $\rho_{\mathrm{ref}} = c_{\mathrm{ref}}/\Omega_{\mathrm{ref}}$
ρ^*_{ref}	reference gyroradius-to-machine-size ratio $\rho^*_{\mathrm{ref}} = \rho_{\mathrm{ref}}/L_{\mathrm{ref}}$
Γ_{gb}	particle flux GyroBohm units $\Gamma_{\mathrm{gb}} = c_{\mathrm{ref}} n_{\mathrm{ref}} (\rho^*_{\mathrm{ref}})^2$
Q_{gb}	heat flux GyroBohm units $Q_{\mathrm{gb}} = c_{\mathrm{ref}} p_{\mathrm{ref}} (\rho^*_{\mathrm{ref}})^2$
Π_{gb}	momentum flux GyroBohm units $\Pi_{\mathrm{gb}} = c^2_{\mathrm{ref}} m_{\mathrm{ref}} n_{\mathrm{ref}} (\rho^*_{\mathrm{ref}})^2$

Nonlinear Cyclone Base Case



Next Time?

- How to verify that your results are realistic/expected using the IDL diagnostic tool.
- Nonlinear ETG simulations? (flux/phi/etc.)
- Compare ETG results to GEM?
- More info on expected gamma/omega plots?
- More info on different geometries?