

ETG Turbulence Isotropization

Stefan Tirkas

CIPS

University of Colorado, Boulder

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Outline

- 1 Drift Wave Instabilities
- 2 Hasegawa-Mima Fluid Model
- 3 Zonal Flow Excitation

Drift Wave Instabilities

- Drift waves are most simply characterized as density, temperature and electrostatic potential fluctuations in low- β plasmas.
- Modes relevant to tokamak physics include ion-temperature-gradient modes (ITG), electron-temperature-gradient modes (ETG), and collisionally-trapped electron modes (CTEM).
- Low-frequency drift wave turbulence is largely responsible for the anomalous transport of plasma particles across magnetic field lines.



Ion-Temperature-Gradient Mode

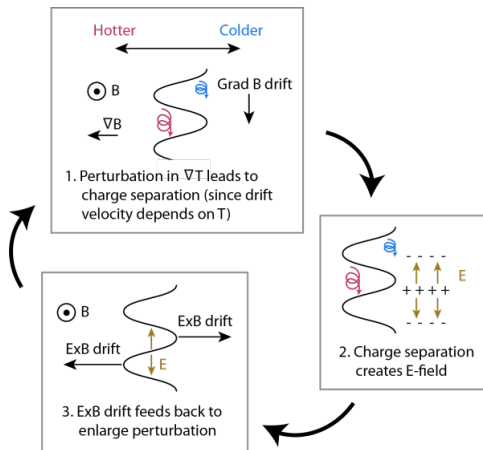
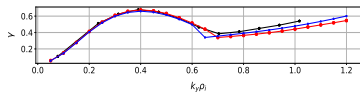
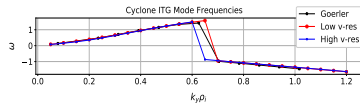
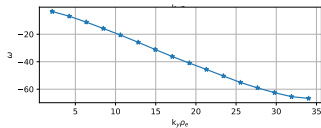
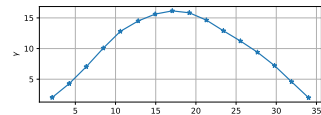


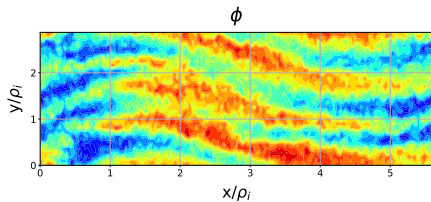
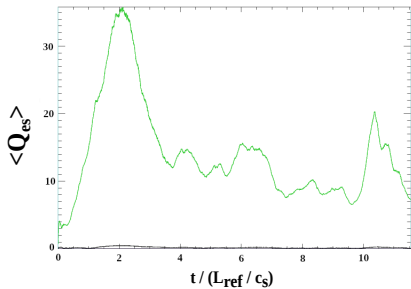
Figure: Simple picture of ITG instability.

ETG Simulation in GENE



ETG Mode Frequencies





Hasegawa-Mima Fluid ETG Model

- Partial differential equation derived from fluid continuity and momentum equations.
- Approximations made that are useful to describing turbulence in tokamak plasmas.
 - Cyclotron motion periods much smaller than time scales that quantities of interest change on (B, Φ, n) .
 - Long length scales along \hat{b} -direction - k_{\parallel} ignorable.
 - Quasi-neutrality of particle densities is enforced.
 - Isothermal equation of state, with adiabatic ions that have negligible temperatures.
- Shown to cause isotropic behavior for long wavelength modes as well as an inverse energy-cascade.

Hasegawa-Mima Equations

We start with the fluid continuity and momentum equations, where we have already taken the ion approximations discussed on the previous slide:

$$\frac{\partial n_e}{\partial t} + \nabla \cdot (n_e \vec{v}_e) = 0, \quad (1)$$

$$m_e \frac{d\vec{v}_e}{dt} = (1 + \tau) e \nabla \delta \Phi - \frac{e}{c} \vec{v}_e \times \vec{B} - \frac{\nabla P_e}{n_e}. \quad (2)$$

We break equation (2) up into parallel and perpendicular components, and break up \vec{v}_e in terms of higher and lower order terms to find,

$$\begin{aligned} \vec{v}_{e,0} &= \vec{v}_{\parallel} + \vec{v}_{\perp,0} = \vec{v}_{\parallel} + (1 + \tau) \vec{v}_E + \vec{v}_D \\ \vec{v}_{e,1} &= \vec{v}_{\perp,1} \end{aligned} \quad (3)$$

Hasegawa-Mima Equations

Taking the standard electron dynamic normalization,

$$\Phi = \frac{e\delta\Phi}{T_i}, \quad -\frac{1}{r_n} = \frac{\partial_x n_e}{n_e}, \quad -\frac{1}{r_t} = \frac{\partial_x T_e}{T_e}, \quad \eta_e = \frac{r_n}{r_t},$$
$$\rho_e = \sqrt{\frac{\tau m_e}{m_i}} \rho_i, \quad \vec{x} = \frac{\vec{X}}{\rho_e}, \quad t = \frac{\rho_e}{r_n} \omega_{ce} t$$
(4)

and plugging into equation () gives the form of the H-M ETG model,

$$\begin{aligned} & -\left(1 - \frac{1+\tau}{2\tau} \nabla_{\perp}^2\right) \partial_t \Phi + \frac{1+\tau}{2\tau} \frac{r_n^2}{\rho_e^2} \partial_t^{-1} \nabla_{\parallel}^2 \Phi + \frac{(1+\tau)(1+\eta_e)}{4\tau} \partial_y \nabla_{\perp}^2 \Phi \\ & + \frac{1+\eta_e}{2\tau} \partial_y \Phi + \frac{(1+\tau)^2}{\tau^2} \frac{r_n}{4\rho_e} (\hat{b} \times \nabla_{\perp} \Phi \cdot \nabla_{\perp}) \nabla_{\perp}^2 \Phi = 0. \end{aligned}$$
(5)

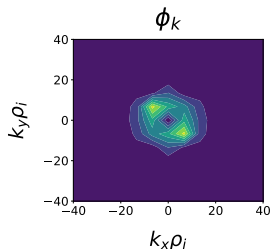
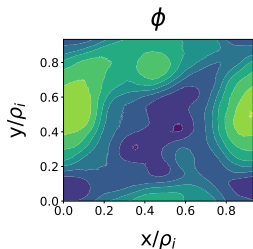
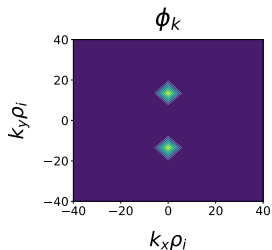
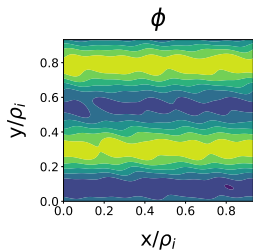
Hasegawa-Mima Equations

Finally we drop the parallel gradient term since $k_{\parallel}^2/k_{\perp}^2 \sim \epsilon^2$, and simplify the bracketed expression to find the final form of our model,

Pseudo-Spectral Solver

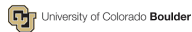
Equation () is solved numerically using the pseudo-spectral method with a 4th-order Runge-Kutta time advancement. We take the Fourier transform of the equation, but leave the non-linear terms in real space. We can solve for $\zeta_{x,y}$ and $\Phi_{x,y}$ in Fourier space, inverse transform to real space, take the necessary non-linear products, and transform the final result back to Fourier space to carry out the time advancement.

ETG H-M Results



GENE ETG Streamer Test

Effects of Isotropization



Zonal Flow Excitation

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References

