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COUPLING VARIATIONAL DATA ASSIMILATION AND OPERATOR LEARNING FOR EFFECTIVE STATE ESTIMATION ON COMPLEX SYSTEMS

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ABSTRACT

We introduce a hybrid twin approach to solve physical systems modeled by parametrized Partial Differential Equations (PDE). Our approach, combines the variational data assimlation(PBDW) [1] with operator learning(Deeponets)[2] to reconstruct the state of an imperfect physical system given many observations. The PBDW method is a non-intrusive, reduced basis, in-situ data assimilation framework, that estimates the solution of a biased model given many observations. This method provide a state estimate framework for parameterized model that estimate both the anticipated and the non anticipated physics while ensuring orthogonality between both parts. The anticipated physics is approximated by a background space constructed with reduced basis while the non-anticipated physics is infered by a Deeponets. The hybrid twin was performed using two approaches to ensure orthogonality. A first approach with a weak constrain, that regularizes the loss function of the neural operator with the dot product between the un-anticipated physics and the background space, in this case the trunk net learn from the whole space but the constrain is only garanteed in the training stage. And a second approach with strong constrain where the trunk is replaced by a set of basis function, each orthogonal to the background space by construction, that garantee orthogonaly in both the training and inference stage of the un-anticipated physics. The hybrid twin was applied on a 2D Helmoltz equation on a homogeneous and bounded domain with a biased source, to showcase its potential for solving complex physical systems.

[1] Maday, Y., Patera, A. T., Penn, J. D. and Yano, M. A parameterized-background data-weak approach to variational
data assimilation : formulation, analysis, and application to acoustics International Journal for Numerical Methods
in Engineering 2015

[2] DeepONet: Learning nonlinear operators for identifying differential
equations based on the universal approximation theorem of
operators