# Answer to JSTAR reviewers' comment

ID: JSTARS-2014-00192

Title: Scalar and Representative Observables, and Their Associated Statistical Models, for POLSAR data.

## Reviewer 1:

**Comment:**

**Overall it is an interesting paper but some elements are missing in order to ensure a good repeatability of the results.**

Answer:

The paper has been written with an aim to ensure good repeatability. The proposed PDF can be tested with both Monte Carlo simulations and real-life captured data. Several repeatable and falsifiable test experiments are demonstrated. In fact, readers are welcomed to retry these experiments on different datasets. The authors would be delighted if the reviewer can be specific on what elements can be added to increase the paper's repeatability or if he had any specific difficulty in retrying these experiments.

**Comment:**

**I am a little bit sceptical about the derivation of the PDF for the ratio (21) in order to get (23). Are you just showing a ratio of chi-squared distributed random variables without actually computing the ratio PDF? Usually the derivation of the PDF of the ratio of independent random variables implies an integration using a Jacobian, therefore some details or reference would be appreciated.**

Answer:

The PDF derivation was originally prepared in an appendix, but was removed to keep the paper concise. Please find the appendix attached. It should be noted however: that the PDF derivation, as commented by the reviewer, is complex and was not done for every cases of dimension (d) and look number (L). For simple cases, e.g. d=1 the PDFs are derived in the appendix and are presented in the paper in Eqns. (24) to (26).

**Comment:**

**The pdf[] notations could be removed on page 4, since we know that we are dealing with PDFs.**

Answer:

The pdf[] notation is used to differentiate the notation in Eqns. (21) to (23) also in page 4 where, as the Reviewer commented earlier, are not written in PDF format. For example, Eqn. (16) illustrates an observable which behaves like product of multiples random variables.

**Comment:**

**P4 L55, define the notation $\Lambda$ in ln(Q)! Is it the Wilk's lambda distribution?**

Answer:

The $\mathbf{\Lambda}$ notation in this paper stands for our so-called log-chi-squared distribution. It is defined as $\Lambda \approx \ln \chi$ where $\chi$ is a random variable following the chi-squared distribution. Given that the PDF of chi-squared distribution is available $pdf(\chi; L) = \frac{\chi^{L-1} e^{-\frac{\chi}{2}}}{2^L \Gamma(L)}$, the PDF of log-chi-squared distribution can be derived as: $pdf(\Lambda; L) = \frac{e^{L\Lambda - \frac{e^{\Lambda}}{2}}}{2^L \Gamma(L)}$.

It is not the same with Wilk's distribution. Even though they are related in the sense that:

1. If both random variables A&B follow Wishart distribution and can be considered as independent of each other, then

The derived observable $\lambda = \frac{\det(A)}{\det(A+B)} = \frac{1}{\det(I+A^{-1}B)}$ follows Wilk's distribution

2. We assert that $\mathbb{R}_C = \frac{\det(A)}{\det(B)}$ (in this paper) and its log-transformed version (not presented here) $\mathbb{L}_C = \ln|A| - \ln|B|$ follows fixed distributions.
3. Not shown in the submitted paper, but elsewhere, i.e. the first author's thesis, it is shown that $\mathbb{L}_C = \ln|A| - \ln|B| \propto \sum_{i=0}^{d-1}[\Lambda(2L-2i) - \Lambda(2L-2i)]$ where $\mathbf{\Lambda}$ stands for our so-called log-chi-squared distribution.

To avoid confusion the equation for ln(Q) is removed from the paper.

**Comment:**

**I am a little surprised that the authors chose to look immediately at real data, I would have expected some Monte-Carlo simulations where the parameter values for various pdfs are perfectly known (in particular L which is estimated here) and uncertainty about sample homogeneity are also absent.**

Answer:

Monte Carlo simulations were carried out, in various different ways, in our detailed work. However we do not feel that showing a stochastic Monte-Carlo simulation matching its theoretical statistics model warrants enough value to be included in this paper. It probably would be nicer to show that the model can be validated against some simulator. Unfortunately, we do not have access to a decent physical POLSAR simulator. Furthermore, we felt it is more important that this paper shows the validation of the model against real-life practical data. Thus, all the others are kept out of this paper in the interest of brevity / space constraint.

**Comment:**

**The various histograms may be easier to display in a log-probability axis especially for the tail behaviour.**

Answer:

Our work does include log-transformed versions of the proposed models. They are found to be not only consistent, i.e. independent of the underlying signal, but also additive and homoscedastic. However, they were not included in the paper due to the space constraint. We have shown them in the Appendix, and will include it in the final paper if the reviewer deems it is necessary.

**Comment:**

**The various sample sizes should be given as well as the estimated L values.**

Answer:

The sample sizes for homogeneous patches of the AIRSAR and RADARSAT2 datasets are 50x50 and 300x300 respectively. The computed L values are: 3.2752 and 3.4241 respectively. This information is now added in the revised article.

**Comment:**

**Section IV, in the multi-dimensional case, it is not clear what analytical relation similar to (26) was used to compute the model PDFs for (22) and (23).**

Answer:

The paper did not use (26) to compute (22) and (23). Rather, it shows that Eqns. (24) to (26) is a special case of Eqns. (22) and (23), where d=1. Thus, the proposed models for POLSAR encompass the traditional models for SAR intensity as its special case.

## Reviewer 2:

**Comment:**

**The authors do not show that this parameter is any better than others, such as span, except in the sense that its pdf is defined completely, not just asymptotically.**

Reply:

There are multiple advantages of the proposed models that have been indicated in various part of the paper. The paper suggests that its proposed models for multi-dimensional POLSAR also encompass the traditional model for SAR as its special case (i.e. d=1). Moreover, its scalar observable leads to consistent measures of distance, which other scalar observables normally used, such as span, cannot. Even better are the properties of these proposed consistent measures of distance. First, compared to existing measures of distance (section II.B) the proposed pdf is, as the Reviewer noted, defined completely and not just asymptotically. Second, extending from the widely used intensity-ratio in SAR, the determinant-ratio can be considered as its natural extension in the multi-dimensional case!

We have further emphasized the above by summarizing them in the Conclusion section of the paper.

**Comment:**

**They also do not acknowledge that much of the useful information in a polarimetric image is in the relationship between the terms of the scattering matrix.**

Answer:

The paper does acknowledge that the proposed models are NOT lossless (P7 L27), since they are scalar representation of a multidimensional dataset. For example, what may be lost include the intra-relationship among the terms of the multidimensional data. We believe, however, that the lost is minimal in the class of possible scalar observables for POLSAR. There is a couple of evidence for that. First the proposed scalar observables are representative, i.e. they lead to consistent discrimination measures. Second, when the multidimensional dataset itself is collapsed into single dimension, the proposed model degrades smoothly into the widely accepted model for the single-channel SAR. The revised text is now rewritten to give further emphasis to the points above.

**Comment:**

**Other parameters they present … are ill-defined in how to put them to practical use.**

Answer:

The paper suggests that the proposed determinant, determinant-ratio or change-ratio models for POLSAR also include the traditional models for SAR intensity, SAR intensity-ratio and SAR change-ratio. Thus their usage pattern can be learned from the practical use of these SAR models. Still a new paragraph (in page 4) is inserted illustrating different possible usage of the proposed models.

**Comment:**

**No comparisons with established procedures are made.**

Answer:

The paper focuses on proposing new statistical models for several scalar and representative observables for POLSAR. The topic is important, we feel, as many established techniques have been shown to be derived from similar discrimination measures. Since the focus of this paper is not to propose new procedures / application, the authors feel that the usual ‘’comparison with established procedures” is not essential in this specific case.

Instead, this paper focuses on comparing the proposed models with existing models for both SAR and POLSAR, and their advantages are shown. For practical application, the paper also includes a portion to illustrate how the proposed models can be useful. Even for this purpose, instead of normal “comparison with established procedures”, a higher-level approach is pursued. Since the proposed determinant, determinant-ratio or change-ratio models for POLSAR also includes the traditional models for SAR intensity, SAR intensity-ratio and SAR change-ratio, the paper shows how we can adapt existing SAR data processing techniques for the POLSAR data.

**Detail Comment:**

**P1/Col1/Para2: The authors state without citation that existing models are "complex and unintuitive". This statement needs validation … justification.**

Answer

POLSAR is multidimensional and stochastic. There have already been several attempts to model all elements of the multidimensional POLSAR data [LopezMartinez_2003_TGRS, Lee_1994_TGRS]. In comparison with the equations in our proposed model, their mathematical equations are evidently complex. These citations have already been included in the related work section. They are now added into the specified location in the revised paper.

**Detail Comment**

**P2/Col1/L36-52: text is misleading. It says that 1-7 are statistical models, not parameters for which models have been proposed. p,q,r,s needs to be defined.**

Answer:

1-7 list out different univariate POLSAR observables, for which statistical models have been proposed in the cited publications. The text may be a bit hard to decode but we feel that it is correct. The paper is revised, however, to make it easier to understand.

p,q,r,s are notation indicating any of the commonly used polarization combination (i.e. hh,vv,hv) . The explanation is added in the revised text.

**Detail Comment:**

**P2/Col1/L56-60: have been shown by whom? Citation required, or is this the authors' opinion?**

Answer:

The text reads: "… none of the underlying observables have been shown to meet the dual criteria of (i) resulting in statistically consistent discrimination measures and thus (ii) being representative of the complex POLSAR data". This is shown in the very next section.

The next section reviews all, to the best of our knowledge, widely used discrimination measures for POLSAR. Evidently none of the commonly used discrimination measures are based on the reviewed observables. Most of them actually are based on the determinant of the covariance matrix, whose model is among those proposed in this paper.

**Detail Comment:**

**P3/Col1/L10-15: While it may be nice for mathematical purity to have an exact distribution instead of an asymptotic one, it should be demonstrated that the asymptotic assumption is invalid for**

**POLSAR data. Ultimately, it needs to be shown that better separation of regions may be obtained using the proposed distribution than with existing methods.**

Answer:

First, we feel that having an exact distribution instead of an asymptotic one is an obvious theoretical contribution. And while we show that the exact distribution is valid, that does not necessarily lead to the conclusion that the asymptotic assumption is invalid for POLSAR data.

Second, while we believe that better separation can be achieved with the proposed model, we feel that it is a relatively different topic. Due to space constraints, each paper should focus on a single topic we assume. Evidently, this theoretical results need to be established first, before we can propose the techniques.

Third, it should also be noted that the paper does include a paragraph (page 7) discussing the application and the advantages of the proposed models. One advantage, in terms of easier classifying POLSAR data, is, for example, the use of the change ratio, which evidently is much cheaper in computation than existing measures, such as the Bhattacharyya ratio.

**Detail Comment:**

**P3/Col1/L25: Incorrect nomenclature. Single pol transmit, dual-pol receive is “compact polarimetry”. Partially polarized signals contain both polarized and unpolarised power.**

Answer:

The term we used is “partial polarimetry” and it is totally different from “partially polarized signals”. We are aware of the term “compact polarimetry” which were used by Souyris [Souyris_2005_TGRS]. In the cited paper, the proposed mode is 45 degree in Transmit. This fact is important because there is also another proposal by Raney [Raney_2006_IGARSS], termed “hybrid polarimetry”, where circularly polarized signals are transmitted. By “partial polarimetry”, we try to indicate that our model works not only on “full polarimetric SAR” or “traditional SAR”, but also covers both the above mentioned case, and more. In fact, our paper validates the case where either the horizontally or the vertically polarized signal is transmitted.

**Detail Comment:**

**P3/Col2/Eqns. 16, 17: Confusing notation or a typo? These are $\chi^2$ distributions. Is the exponent missing in the equations?**

Answer:

Yes, these are $\chi^2$ distributions. The exponent notation was used in this paper to denote the dimensionality number (d). In the revised paper, the squared-exponent notation is re-used for the chi-squared distribution to avoid the confusion.

**Detail Comment:**

**P3/Col2/L43-48: This paragraph is a circular argument. SAR speckle noise is multiplicative. You use Goodman's result to capture this, and Eqn. 19 … also does. It is not an implication. Note the Eqns. 18 and 19 break down for L<d, i.e. for single look imagery.**

Answer:

To clarify, our intention is to show that the determinant of POLSAR covariance matrix is not only multiplicative but also heteroskedastic. Subsequently we show that these properties are also similar to SAR intensity. Apparently, that the idea has not been fully conveyed to the reviewer. As such, in the revised paper, the text is heavily updated to emphasize the point.

Concerning Eqns. 18, 19, it is true that they are broken when L<d. It should be noted that L stands for Number-of-Looks and d is the dimension number. Thus when L<d, the determinant is also ill-defined, as is the Complex Wishart distribution (Eqn. 13).

**Detail Comment:**

**P3/Col2/Eqn. 20: why would the underlying covariance ever be known a priori? It is what we are trying to estimate.**

Answer:

The underlying covariance matrix may not be known with absolute accuracy. It however, may be considered as known to some degree of confidence, for example, in detecting ship out of a "homogeneous" background of sea water applications. Hence, in that sense Eqn. 20 may be useful,

From another perspective, this can also be considered as a purely logical advancement. In fact, in the very next sentence, the case of unknown underlying covariance matrix is covered.

**Detail Comment:**

**P4/Col1/Eqn. 21: for this equation to work, regions must be known to be homogeneous. Almost no natural regions are …. Consequently, this parameter, while formally satisfying, may be useless in practice.**

Answer:

The paper shows that when the two underlying covariance are the same, Eqn. 21 is theoretically satisfying. Its use in practice, however, is dependent on our imagination. One application, for example, is given two observable covariance matrixes; Eqn. 21 can be used to test the null hypothesis of same underlying covariance. With some minor calibration, this technique can be applied in, for example, change detection applications. In essence, that means the comment of: "for Eqn. 21 to work, regions must be known to be homogeneous" are not strictly true in all possible cases.

**Detail Comment:**

**P4/Col1/Eqn. 23: is wrong. With numerator and denominator the same, it has fixed values of Rc=1,2,6 for d=1,2,3**

Answer:

It should be noted that the Eqn. 23 indicates a stochastic process. The division of two stochastic variables having the same underlying distribution, in the general case, does not lead to a fixed number. It only leads to fixed distribution. This is similar to the subtraction of two independent random variables having the same, say, Gaussian distribution. The latter case does not lead to a fixed value of zero, but a fixed distribution with expected value of zero. What the reviewer probably meant is: when d=1,2,3, Rc follows fixed distributions.

**Detail Comment:**

**P4/Col1/L28-38: 1D SAR is not 3D SAR collapsed. Single pol is a single component of compact or full pol SAR.**

Answer:

We do not wish to say that 3D SAR can be physically collapsed into 1D SAR and we do not intend to mean that. Instead, what the paper meant is that the proposed generic mathematical models for multidimensional SAR (i.e. d=3 for full-pol SAR, d=2 for part-pol SAR) when collapsed into single-dimension (i.e. setting d=1 into these mathematical formula) results in the traditional model for SAR intensity! That means: the multidimensional models proposed in this paper are generic and include the traditional SAR intensity models as one of its special case!

**Detail Comment:**

**P4/Col1/last Eqn.: you did not set d=1 here**

Answer:

Thanks for pointing out the glaring mistake. It is rectified in the revised paper.

**Detail Comment:**

**P4/Col2/top Eqn.: Without clarification, I cannot figure out how the succeeding results are calculated.**

Answer:

A separate appendix is now attached to show the mathematical derivation.

**Detail Comment:**

**P5/Col1/L40: A Radarsat2 image of what?**

Answer:

Updated in the revised paper: A Radarsat2 image of Murda Merbok (Malaysia)

**Detail Comment:**

**Anfinsen's ENL is, in general, non-integer. How did you deal with that? Why did you not use the formal L values from the imagery (1,4 or 9).**

Answer:

The computed ENL did result in non-integer. There is no problem with that as Matlab can simulate Chi-Squared distribution with non-integer degrees of freedom. Thus the model plot can be plotted using Matlab simulations. The paper shows that the Matlab simulation matches nicely with the observed data.

We did not use the formal L values from the imagery because the estimated ENL gives a better match. We decided not to discuss too much about why the estimated ENL gives a better match, or why the two differs since the paper is already too long. Interested reader can refer to my thesis for further details.

**Detail Comment:**

**How did you determine that the regions you choose were truly homogeneous? How did you determine the theoretical determinant for the ratio test? Which other region was selected for the change test?**

Answer:

The determination of homogeneity is admittedly by experience. We choose the regions that are calm water surfaces as homogeneous. These, for example, are large lake or calm sea area.

Assuming the area is homogeneous, the theoretical determinant is the determinant of the region's ensemble average. The change test makes use of the same assumed homogeneous region, shifted by a few pixels.

**Detail Comment:**

**P6, 7/Section VII: You used Anfinsen's ENL to compute the L value required to make the data appear homogeneous. It should result in an ENL image that is instructive. The ENL value should vary around the image, although the actual number of looks does not. Consequently, your noise images show no structure, because all the variance structure has been extracted sin the ENL computation.**

Answer:

There are a few points to clarify, we believe. First, ENL estimation is not applied in this section's experiment. Second, we do not wish to show that the noise images show no structure. In fact, quite contrary, we wish to show that the residual image of 5x5 boxcar filter shows more recognizable structures than the 3x3 boxcar filter. The point in this section is to show as an example that existing techniques in SAR can be readily extended to POLSAR using our proposed models.

## References

[LopezMartinez_2003_TGRS] C. Lopez-Martinez and X. Fabregas. **Polarimetric SAR speckle noise model**. *Geoscience and Remote Sensing, IEEE Transactions on*, 41(10):2232–2242, 2003.

[Lee_1994_TGRS] Jong-Sen Lee, K.W. Hoppel, S.A. Mango, and A.R. Miller. **Intensity and phase statistics of multilook polarimetric and interferometric SAR imagery**. *IEEE Transactions on Geoscience and Remote Sensing*, 32(5):1017–1028, Sep 1994.

[Souyris_2005_TGRS] J.C. Souyris, P. Imbo, R. Fjortoft, Sandra Mingot, and Jong-Sen Lee. **Compact polarimetry based on symmetry properties of geophysical media: the pi/4 mode**. *Geoscience and Remote Sensing, IEEE Transactions on*, 43(3):634 – 646, Mar. 2005.

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