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Scalar and Representative Observables, and Their Associated Statistial Model, for Polarimetric SAR Data

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This paper proposes novel scalar and representative observables, including several statistically consistent discrimination measures, that are particularly useful in the interpretation and processing of multi-dimensional POLSAR data. Specifically, the statistical behaviour of the POLSAR covariance matrix determinant is used to derive a scalar and generic statistical model for multi-dimensional POLSAR data, which is shown applicable to both two and three dimensional versions of partial and full mono-static POLSAR data. As the POLSAR covariance matrix determinant generalizes the SAR intensity towards multiple dimensions, the proposed model is able to subsume the traditional SAR intensity model under the umbrella of a unified model. Consequently, the main beneficial implications of the proposed approach are that it provides a consistent theory unifying the currently disconnected proposals for SAR and POLSAR discrimination measures, and it simplifies the adaptation of existing SAR data processing techniques for POLSAR data.

1. Introduction

Relentless growth in computing power has allowed the once computationally-demanding Synthetic Aperture Radar (SAR) to become now a feasible and preferred technique for earth observation. Basic SAR has also been extended in a few directions, one of which is polarimetric SAR (POLSAR). POLSAR exploits the natural polarization property of Electro-Magnetic (EM) waves, encoded in multiple channels, compared to traditional one-channel SAR.

POLSAR data, like SAR, is stochastic. Moreover it is also multi-dimensional, making it even harder to interpret. It is therefore important to establish a simple and intuitive understanding of the data. Statistical models are undoubtedly crucial in understanding its stochastic nature. While several models have been proposed for POLSAR data, they tend to be complex and unintuitive due to the multidimensional nature of the data. Practical POLSAR data processing however, makes heavy use of scalar discrimination measures, which should be based on statistically consistent models of the multidimensional data. It is thus important to establish scalar and representative observables for multi-dimensional POLSAR data.

A few scalar POLSAR observables with accompanying statistical models have been proposed (Conradsen *et al.* 2003, Alberga *et al.* 2008, Joughin *et al.* 1994, Lee *et al.* 1994b, Touzi and Lopes 1996, Lopez-Martinez and Fabregas 2003, Erten 2012), but none is able to provide meaningful scalar discrimination measures. As such no observable has been widely accepted as being highly representative of

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this multi-dimensional data, which severely limits their applicability in practical data processing applications. Alternatively, a few POLSAR discrimination measures have been proposed (Lee *et al.* 1994a, Anfinsen *et al.* 2007, Kersten *et al.* 2005, Lee and Bretschneider 2011, Cao *et al.* 2007, Conradsen *et al.* 2003), but all are based on likelihood ratio statistics. These should ideally be based on an exact and consistent distribution but so far only asymptotic distributions have been demonstrated (Conradsen *et al.* 2003).

This paper presents scalar and representative observables, and one associated generic statistical model, that describe multi-dimensional POLSAR data and provide a consistent foundation for the derivation of discrimination measures. The development is based on the common assumption that POLSAR data follows the complex Wishart distribution. Subsquently, the generic statistical model for the covariance matrix determinant is presented as being just a scalar projection of the multidimensional data. This model is then used to derive several scalar and consistent statistical distributions suggesting that their associated observables are capable of being used as discrimination measures for POLSAR.

This paper also shows that the specific one dimensional (1-D) version of the proposed model matches the traditional statistical model used for SAR intensity. This effectively incorporates the existing SAR theory under the umbrella of the proposed scalar approach for multi dimensional POLSAR. The different discrimination proposals for SAR and POLSAR are reviewed in light of this and the proposal in this paper is shown to provide a strong, unifying and consistent foundation. The applicability of these theoretical models will be illustrated by experiments where the specific 1, 2 and 3-D versions of the proposed models are validated against real-life captured data.

2. Related Work in Literature

2.1 *Scalar Observables for POLSAR Data*

Different target decomposition theorems have identified many possible scalar observables for complex POLSAR data. Alberga *et al.* (2008) evaluated the performance of different scalar POLSAR observables for classification. While many observables were presented, their corresponding statistical models and classifiers were not available. Furthermore, the paper concluded that it is impossible to identify a single best representation, although, to be fair, the observables were identified for describing a decomposed portion of the complex POLSAR data, rather than a unified representation.

Using a different approach, given that the joint distribution for POLSAR is known to be the multi-variate complex Wishart, it is possible to derive the scalar statistical models for some univariate POLSAR observables. However, this is non-trivial – and so far, only a handful of such models have been proposed, including:

(i) cross-pol ratio $r_{HV/HH} = |S_{HV}|^2/|S_{HH}|^2$ (Joughin *et al.* 1994),
(ii) co-pol ratio $r_{VV/HH} = |S_{VV}|^2/|S_{HH}|^2$ (Joughin *et al.* 1994),
(iii) co-pol phase difference $\phi_{VV/HH} = arg(S_{VV}S^*_{HH})$ (Joughin *et al.* 1994) (Lee *et al.* 1994b),
(iv) magnitude $g = |avg(S_{pq}S^*_{rs})|$ (Lee *et al.* 1994b),
(v) normalized magnitude $\xi = \frac{|avg(S_{pq}S^*_{rs})|}{\sqrt{avg(|S_{pq}|^2)avg(|S_{rs}|^2)}}$ (Lee *et al.* 1994b),
(vi) intensity ratio $w = avg(|S_{pq}|^2)/avg(|S_{rs}|^2)$ (Lee *et al.* 1994b),
(vii) and the Stokes parameters $S_i, 0 \leq i \leq 3$ (Touzi and Lopes 1996).

More recently, statistical models for each element of the POLSAR covariance matrix, i.e. $S_{pq}S^*_{rs}$, (Lopez-Martinez and Fabregas 2003) as well as for the largest eigen-value of the covariance matrix λ_1 (Erten 2012) have been proposed. Although useful, these have not been shown to result in statistically consistent discrimination measures or to be representative of the complex POLSAR data.

2.2 *POLSAR Discrimination Measures*

Euclidean or Manhattan distance measures for matrices are not widely used for POLSAR due to the multiplicative nature of the noise. Instead, the Wishart distance is better known, as part of the Wishart classifier (Lee *et al.* 1999), defined as (Lee *et al.* 1994a): $d(C_x, C_y) = \ln|C_y| + tr(C_x C_y^{-1})$ where $tr(C)$ denotes the trace of the POLSAR covariance matrix C. As a measure of distance, its main disadvantage is that $d(C_y, C_y) = \ln|C_y| \neq 0$.

Recent works have suggested alternative dissimilarity measures including the symmetric and asymmetric refined Wishart distance (Anfinsen *et al.* 2007),

$$d(C_x, C_y) = \frac{1}{2} tr(C_x^{-1} C_y + C_y^{-1} C_x) - d \tag{1}$$

$$d(C_x, C_y) = \ln|C_x| - \ln|C_y| + tr(C_x C_y^{-1}) - d \tag{2}$$

the Bartlett distance (Kersten *et al.* 2005),

$$d(C_x, C_y) = 2\ln|(C_x + C_y)/2| - \ln|C_x| - \ln|C_y| - 2d\ln 2 \tag{3}$$

the Bhattacharyya distance (Lee and Bretschneider 2011),

$$r(C_x, C_y) = \frac{|C_x|^{1/2}|C_y|^{1/2}}{|(C_x + C_y)/2|} \tag{4}$$

and the Wishart Statistical test distance (Cao *et al.* 2007),

$$d(C_x, C_y) = (L_x + L_y)\ln|(L_x C_x + L_y C_y)/(L_x + L_y)| - L_x \ln|C_x| - L_y \ln|C_y| \tag{5}$$

Closer examinations reveal that most are related: The most obvious are the Bhattacharyya and Bartlett distances. At the same time, the Barlett distance can be considered a special case of the Wishart Statistical Test distance, when the two data sets have the same number of looks, i.e. $L_x = L_y$. The close relation among measures may be due to the fact that all of their publications are based on the same statistical model in Conradsen *et al.* (2003). In Conradsen *et al.* (2003), to determine if the two scaled multi-look POLSAR covariance matrixes Z_x and Z_y, which have L_x and L_y as the corresponding number of looks, come from the same underlying stochastic process, the likelihood ratio statistics for POLSAR covariance matrix is considered:

$$Q = \frac{(L_x + L_y)^{d\cdot(L_x+L_y)}}{L_x^{d\cdot L_x} L_y^{d\cdot L_y}} \frac{|Z_x|^{L_x}|Z_y|^{L_y}}{|Z_x + Z_y|^{(L_x+L_y)}} \tag{6}$$

Taking the log-transformation of the above equation, and denoting $C_{vx} = Z_x/L_x$, $C_{vy} = Z_y/L_y$ and $C_{vxy} = (Z_x + Z_y)/(L_x + L_y)$ then:

$$Q = \frac{|C_{vx}|^{L_x} \cdot |C_{vy}|^{L_y}}{|C_{vxy}|^{L_x+L_y}} \tag{7}$$

$$\ln Q = L_x \ln|C_{vx}| + L_y \ln|C_{vy}| - (L_x + L_y)\ln|C_{vxy}| \tag{8}$$

To detect changes, a test statistic is developed for this discrimination measure. i.e. a distribution is derived for the dissimilarity measure. However, Conradsen *et al.* (2003) only used an asymptotic distribution. By contrast, this paper proposes a statistical model for the determinant of the POLSAR covariance matrix $|C_v|$ which is capable of providing an exact distribution for the test.

3. The Generic Scalar Statistical Model for POLSAR

In this section, the generic scalar statistical model for POLSAR is presented. First, let $\Sigma = E[ss^{*T}]$ denote the population expected value of the POLSAR covariance matrix, where s^{*T} is the complex conjugate transpose of the POLSAR scattering vector, s. If this is jointly circular complex Gaussian with expected covariance matrix Σ, then the PDF of s can be written as $pdf(s;\Sigma) = \{1/\pi^d|\Sigma|\}exp\{-s^{*T}\Sigma^{-1}s\}$ where $||$ denotes the matrix determinant. The sample POLSAR covariance matrix is formed as the mean of Hermitian outer product of independent single-look scattering vectors,

$$C_v = \langle ss^{*T} \rangle = \frac{1}{L}\sum_{i=1}^{L} s_i s_i^{*T} \tag{9}$$

where L is the number of looks and s_i denotes the partial or full POLSAR scattering vector respectively. Complex Wishart distribution statistics are normally used for the scaled covariance matrix $Z = LC_v$, whose PDF is given as:

$$pdf(Z;d,\Sigma,L) = \frac{|Z|^{L-d}}{|\Sigma^L|\Gamma_d(L)} e^{-tr(\Sigma^{-1}Z)} \tag{10}$$

with $\Gamma_d(L) = \pi^{d(d-1)/2}\prod_{i=0}^{d-1}\Gamma(L-i)$ and d the dimension number of the POLSAR covariance matrix. The approach taken in this paper differs by making use of applying a less-than-well-known relationship. Goodman (1963) found that the ratio between observable and expected values of the sample covariance matrix determinants behaves like a product of d chi-squared random variables with different degrees of freedom:

$$\chi_L^d = (2L)^d \frac{|C_v|}{|\Sigma_v|} \sim \prod_{i=0}^{d-1}\chi(2L-2i) \tag{11}$$

This is used to develop a generic scalar statistical model. From Eqn. 11 we have:

$$|C_v| \sim |\Sigma_v| \cdot \frac{1}{(2L)^d} \cdot \prod_{i=0}^{d-1}\chi(2L-2i) \tag{12}$$

Over a homogeneous area, Σ_v, d and L are considered constant. Thus Eqn. 12 indicates that a multiplicative speckle noise pattern is present in the original POLSAR domain. Moreover, since the average and variance of these chi-squared distributions are constant, i.e. $avg\left[\chi(2L)\right] = 2L$ and $var\left[\chi(2L)\right] = 4L$, their product and summation also have fixed summary statistics. Specifically Eqn. 12 , gives us:

$$avg\left[|C_v|\right] = \frac{|\Sigma_v|}{L^d}\prod_{i=0}^{d-1}(L-i) \tag{13}$$

$$var\left[|C_v|\right] = \frac{|\Sigma_v|^2}{L^{2d}}\left[\prod_{i=0}^{d-1}(L-i)(L-i+1) - \prod_{i=0}^{d-1}(L-i)^2\right] \tag{14}$$

For a real life captured image, while parameters d and L do not change for the whole image, the underlying Σ_v is likely to differ from one region to the next. Thus over a heterogeneous scene, the stochastic process for $|C_v|$ and $\ln|C_v|$ vary depending on the underlying signal Σ_v. Eqn. 14 implies that the variance of $|C_v|$ will also differ depending on the underlying signal Σ_v (i.e. it is heteroskedastic). Similar to the way intensity-ratio is proposed as the discrimination measure for the multiplicative and heteroskedastic SAR intensity (Rignot and van Zyl 1993), this paper proposes the determinant-ratio and the change-ratio as discrimination measures for the POLSAR data.

If the true value of the underlying signal Σ_v is *known a priori*, then the determinant-ratio of the signal random variable ($\mathbb{R}_\Sigma$) is defined as: $\mathbb{R}_\Sigma = |C_v|/|\Sigma_v|$ For POLSAR data from a homogeneous area, but the true value of Σ_v is *unknown*, then a random variable, called the change-ratio $\mathbb{R}_C$), is defined as: $\mathbb{R}_C = |C_1|/|C_2|$ where C_1 and C_2 are samples of the covariance matrix determinant in an assumed homogeneous area. Using the results from Eqn. 12, we have

$$\mathbb{R}_\Sigma \sim \frac{1}{(2L)^d} \cdot \prod_{i=0}^{d-1} \chi(2L-2i) \quad and \quad \mathbb{R}_C \sim \prod_{i=0}^{d-1} \frac{\chi(2L-2i)}{\chi(2L-2i)} \tag{15}$$

Since each elementary component follows fixed distributions (i.e. $\chi(2L)$), this variable naturally also follows fixed distributions. Moreover, it is independent of the underlying Σ_v, indicating its statistically consistent properties (i.e. its applicability as a POLSAR discrimination measure).

4. SAR as a one-dimensional case of POLSAR

This section shows that the proposed generic model is applicable to the 1-D case ($d = 1$), physically equivalent to collapsing the multi-dimensional POLSAR dataset into single dimensional SAR data. Mathematically, the sample covariance matrix C_v is reduced to the sample variance while its determinant $|C_v|$ becomes the scalar variance. As variance is equal to intensity I in SAR, this section shows that our proposed models are consistent with the traditional models for SAR intensity.

The results for our models can be summarised using the following equations:

$$|C_v| \sim |\Sigma_v| \cdot \frac{1}{(2L)^d} \cdot \prod_{i=0}^{d-1} \chi(2L-2i)$$

$$\mathbb{R}_\Sigma = \frac{|C_v|}{|\Sigma_v|} \sim \frac{1}{(2L)^d} \prod_{i=0}^{d-1} \chi(2L-2i) \quad and \quad \mathbb{R}_C = \frac{C_1}{C_2} \sim \prod_{i=0}^{d-1} \frac{\chi(2L-2i)}{\chi(2L-2i)}$$

Upon setting $d = 1$ into the above equations, the equations become:

$$|C_v| \sim \frac{|\Sigma_v|}{(2L)} \cdot \chi(2L) \tag{16}$$

$$\mathbb{R}_\Sigma = \frac{|C_v|}{|\Sigma_v|} \sim \frac{1}{(2L)} \cdot \chi(2L) \quad and \quad \mathbb{R}_C = \frac{C_1}{C_2} \sim \prod_{i=0}^{d-1} \frac{\chi(2L-2i)}{\chi(2L-2i)} \tag{17}$$

Since the PDF of chi-squared distribution can be written as:

$$\chi(2L) \sim pdf\left[\frac{\chi^{L-1}e^{-\chi/2}}{2^L\Gamma(L)}\right]$$

Applying variable change theorem into the above equations results in:

$$|C_v| \sim pdf\left[\frac{L^L x^{L-1}e^{-Lx/|\Sigma_v|}}{\Gamma(L)|\Sigma_v|^L}\right]$$

$$\mathbb{R}_\Sigma = \frac{|C_v|}{|\Sigma_v|} \sim pdf\left[\frac{L^L x^{L-1}e^{-Lx}}{\Gamma(L)}\right] \quad and \quad \mathbb{R}_C = \frac{C_1}{C_2} \sim pdf\left[\frac{\Gamma(2L-1)x^{L-1}}{\Gamma^2(L-1)(1+x)^{2L}}\right]$$

These equations match exactly with the following traditional model for multi-look SAR intensity:

$$I \sim pdf\left[\frac{L^L x^{L-1}e^{-Lx/\bar{I}}}{\Gamma(L)\bar{I}^L}\right] \tag{18}$$

$$\mathbb{R}_{\bar{I}} = \frac{I}{\bar{I}} \sim pdf\left[\frac{L^L x^{L-1}e^{-Lx}}{\Gamma(L)}\right] \tag{19}$$

$$\mathbb{R}_I = \frac{I_1}{I_2} \sim pdf\left[\frac{\Gamma(2L-1)x^{L-1}}{\Gamma^2(L-1)(1+x)^{2L}}\right] \tag{20}$$

considering that $|C_v| \mapsto I$ and $|\Sigma_v| \mapsto \bar{I}$ as multi-dimensional POLSAR collapses into single-dimensional SAR.

5. Unifying different discrimination measures for SAR and POLSAR

Statistical models are the foundation for discrimination measures in both SAR and POLSAR. For the more matured SAR field, the statistical model for SAR intensity has been used to derive the most widely used intensity-ratio discrimination measure (Rignot and van Zyl 1993). For the less matured POLSAR field the same case should apply, except that so far only asymptotic distributions have been derived for the most common foundation: likelihood test statistics (Conradsen *et al.* 2003).

With the insight gained from section 4., this section presents a few new results. First, similar to the way that the statistical SAR intensity models have been used as a foundation for SAR discrimination measures, e.g. intensity-ratio (Rignot and van Zyl 1993), the proposed statistical model for POLSAR covariance matrix determinant can also be viewed as providing a foundation for POLSAR discrimination measures, i.e. the likelihood test statistics. Secondly, new discrimination measures for POLSAR may be derived by learning from the existing SAR discrimination measures.

As for the first matter, in view of the models given in Eqn 12, the likelihood test statistics presented in Conradsen *et al.* (2003) and rewritten in Eqns 7 & 8 can be expressed as $\ln Q \sim k + L_x\Lambda^d_{L_x} + L_y\Lambda^d_{L_y} - (L_x+L_y)\Lambda^d_{(L_x+L_y)}$, and $Q \sim e^k \frac{(\chi^d_{L_x})^{L_x}\cdot(\chi^d_{L_y})^{L_y}}{(\chi^d_{L_x+L_y})^{L_x+L_y}}$ where $k = d\left[(L_x+L_y)\ln(L_x+L_y) - L_x\ln L_x - L_y\ln L_y\right]$. This, in essence, derives an exact statistical distribution for the likelihood test statistics, as opposed to the asymptotic distribution derived in Conradsen *et al.* (2003).

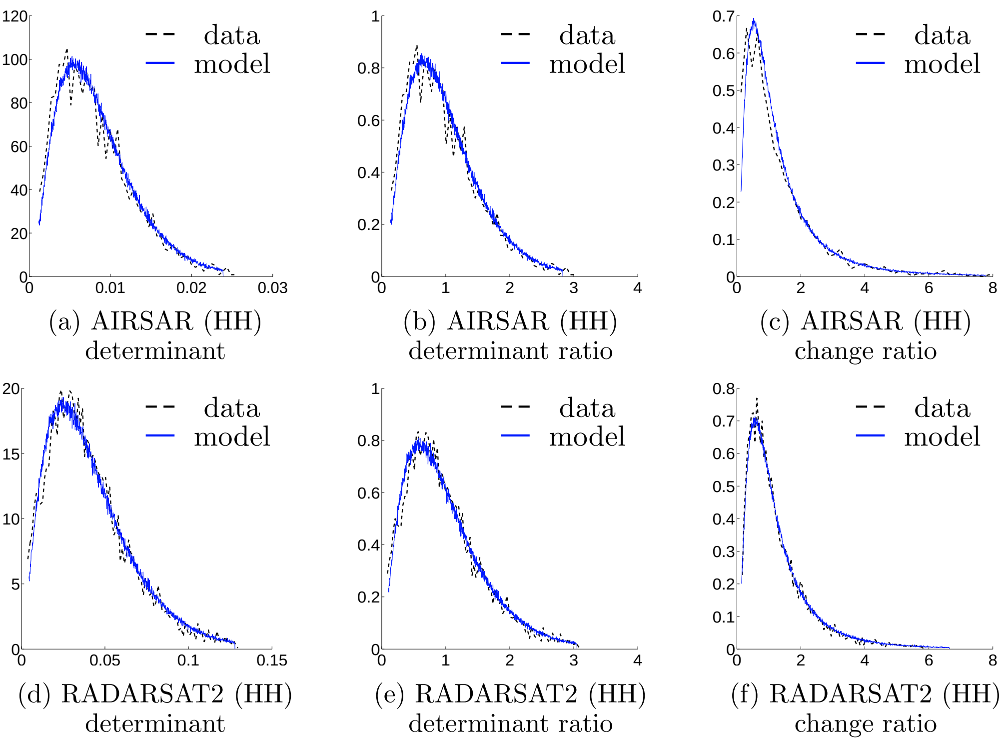
As a by-product of this exact derivation, several discrimination measures for the common case of $L_x = L_y$ are further proposed. They are the determinant-ratio and the change-ratio presented in Section 3. Compared to existing discrimination measures for POLSAR reviewed in Section 2., the proposed dissimilarity measures are simpler in both concept and computation. They are hence multi-dimensional extensions of the widely used SAR intensity-ratio discrimination measure.

6. Model Validation

The models in Eqns. 12 and 15 are now validated against real-life captured data. Each model requires the estimation of two parameters from the data; the dimensional number d, and the look number L. d is related to the type of (POL)SAR data captured, with $d = 1, 2, 3$ corresponding to the cases of SAR, partial and full POLSAR, respectively. L is normally stated by the data provider, but in this section it is estimated using the technique proposed by Anfinsen *et al.* (2009). To show the robustness of the proposed models, their validations are carried out on two different POLSAR sensors: (1) airborne four-look ($L = 4$) AIRSAR Flevoland image and (2) fine-quad single-look ($L = 1$) complex RADARSAT2 image. Since the determinant of the covariance matrix is only significant on multi-look data, nine-look processing is first applied to the single-look data ($L = 9$).

6.1 *The Traditional case of SAR* ($d = 1$)

Fig. 1 presents the results of a test where the intensity of single-channel SAR data (HH) for sample homogeneous areas are extracted. Histograms are then plotted for all of the proposed observables against their theoretical models. In all cases, the good visual match between the actual data and the respective model apparently validates the proposed model.



(a) AIRSAR (HH) determinant

(b) AIRSAR (HH) determinant ratio

(c) AIRSAR (HH) change ratio

(d) RADARSAT2 (HH) determinant

(e) RADARSAT2 (HH) determinant ratio

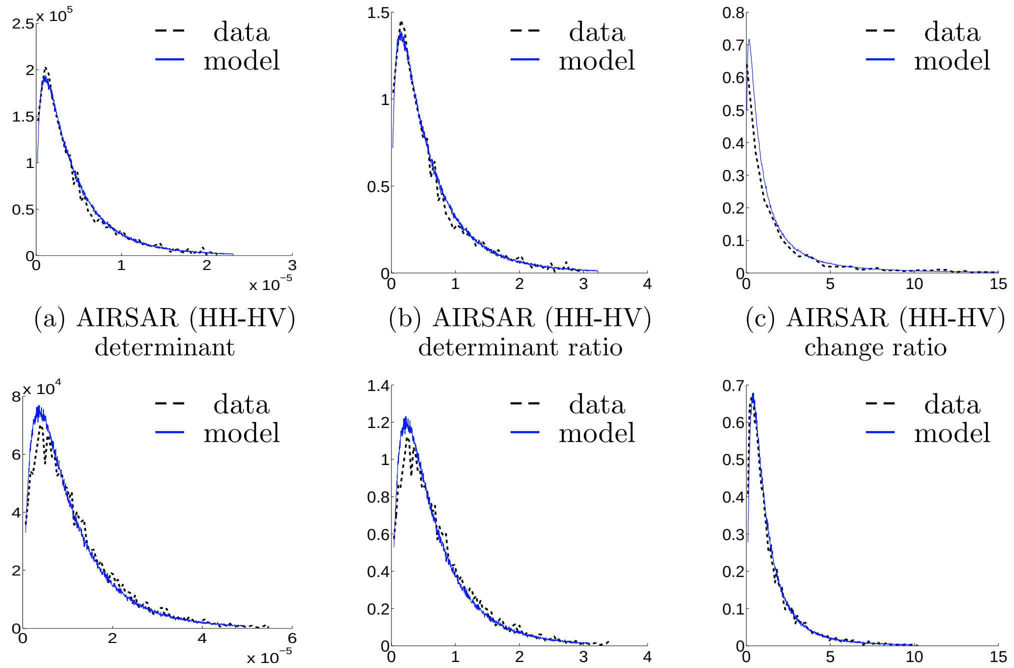
(f) RADARSAT2 (HH) change ratio

Figure 1.: The specific d=1 models are validated for both RADARSAT2 and AIRSAR datasets. Each histogram plots the signal under investigation (along the x-axis) against its probabilistic distribution across the patch (on the y-axis).

6.2 *The Multi-dimensional case of POLSAR ($d = 2, 3$)*

The look-number is estimated for each dataset (Anfinsen *et al.* 2009). And the actual distribution is plotted against the model yielding an obvious visual match in Fig. 2 which shows the $d = 2$ plots for determinant, determinant ratio and change ratio for both datasets.

Similarly, Fig. 3 explores the $d = 3$ case for the same data and model parameter estimation. Again, although the histogram is much tighter, the match is visually obvious in all cases.



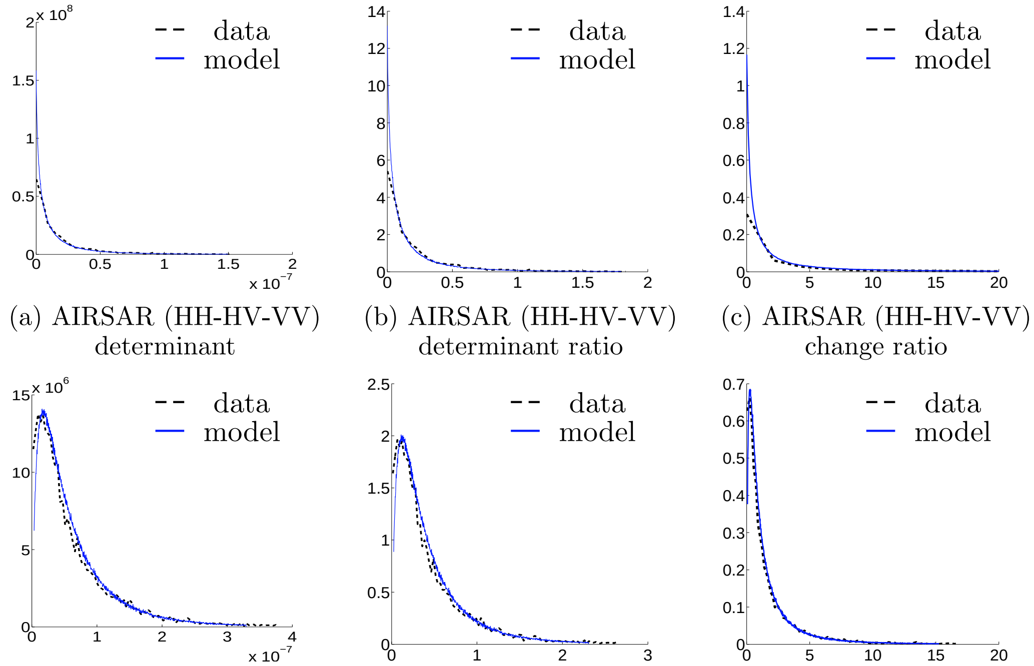
(a) AIRSAR (HH-HV) determinant
(b) AIRSAR (HH-HV) determinant ratio
(c) AIRSAR (HH-HV) change ratio
(d) RADARSAT2 (HH-HV) determinant
(e) RADARSAT2 (HH-HV) determinant ratio
(f) RADARSAT2 (HH-HV) change ratio

Figure 2.: The specific $d = 2$ models are validated on both RADARSAT2 and AIRSAR datasets. Each histogram plots the signal under investigation (along the x-axis) against its probabilistic distribution across the patch (on the y-axis).

7. Conclusion

This paper has proposed a generic statistical model for the POLSAR covariance matrix determinant based on the complex Wishart POLSAR target vector distribution.The model has been validated for specific $d = 2$ and $d = 3$ cases using real-life partial and full POLSAR data. The paper also establishes two new POLSAR discrimination measures: the determinant-ratio and the change-ratio, which essentially are the generic version of the SAR intensity-ratio. We have also shown a near-perfect match between the specific $d = 1$ model and that for traditional SAR intensity, effectively bringing the existing SAR theories under the umbrella of this new model.

The main emphasis of this paper is to consider the POLSAR covariance matrix determinant as a scalar and representative observable for the multi-dimensional POLSAR data. Compared to other published scalar observables, the determinant is highly representative of the multi-dimensional data. Its representative power is justified for the following reasons. Firstly, the covariance matrix determinant,



(a) AIRSAR (HH-HV-VV) determinant
(b) AIRSAR (HH-HV-VV) determinant ratio
(c) AIRSAR (HH-HV-VV) change ratio
(d) RADARSAT2 (HH-HV-VV) determinant
(e) RADARSAT2 (HH-HV-VV) det-ratio
(f) RADARSAT2 (HH-HV-VV) change ratio

Figure 3.: The specific $d = 3$ models are validated on both RADARSAT2 and AIRSAR datasets. Each histogram plots the signal under investigation (along the x-axis) against its probabilistic distribution across the patch (on the y-axis).

when collapsed into 1-D, transforms neatly into the representative SAR intensity. Secondly, the statistical model for this observable is shown to be the generic multi-dimensional extension of the traditional model for the one dimensional SAR intensity. Thirdly, this observable also leads to the establishment of new discrimination measures for POLSAR data, which should increase the usability of this proposal. Finally, similar to the eigen-values, the determinant of the POLSAR covariance matrix is invariant to a change of polarization basis.

Currently while the field of SAR is much more developed than the field of POLSAR, the two fields remain quite separated. Hence many techniques applicable to SAR cannot be directly extended to POLSAR. There lies the different beneficial implications of the proposed approach presented in this paper. Since the scalar and representative model for POLSAR generalizes the traditional model for SAR intensity, its main benefit is that it enables the convenient adaptation of many existing SAR data processing techniques for POLSAR data. One example has already been presented in this paper, where existing SAR discrimination measures are extended towards POLSAR. At the same time, it provides a consistent theory unifying the seemingly disparate discrimination measure proposals for SAR and POLSAR data.

Of course, the models proposed in this article also have their own limitations. Firstly, they are based on the complex Wishart distribution which is only guaranteed to work for homogeneous areas. Secondly, while it is desirable for a large number of applications to reduce the multi-dimensional POLSAR data to a scalar value, such a reduction is unlikely to be lossless in the general case.

In this paper, the theoretical model has only been validated for partial ($d = 2$) and full ($d = 3$) monostatic POLSAR data, leaving the test of its validity for datasets such as bistatic POLSAR data ($d = 4$) or interferrometric POLSAR data ($d = 6$) for the future. Other possible extensions include investigating the applicability of the observables into POLSAR heterogeneous areas, as well as exploring the use of these generic models in conjunction with other target decomposition techniques such as the Freeman-Durden decomposition (Freeman and Durden 1998) or the entropy/anisotropy decomposition (Cloude and Pottier 1997).

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