This paper proposes scalar and representative observables for multi-dimensional POL-

SAR data, from which statistically consistent discrimination measures can be derived.

Specifically, the statistical behaviour of the POLSAR covariance matrix determinant

is used to derive a scalar and generic statistical model for multi-dimensional POLSAR

data, which is specifically applicable to the two and three dimensional versions of par-

tial and full monostatic polarimetric SAR data. As the POLSAR covariance matrix

determinant generalizes the SAR intensity towards multiple dimensions, the proposed

model is able to subsume the traditional SAR intensity model under the umbrella of a

unified model. Consequently, the main beneficial implication of the proposed approach

is that it provides a consistent –(model?) unifying the currently disconnected proposals

for SAR and POLSAR discrimination measures, and simplifies the adaptation of

existing SAR data processing techniques for POLSAR data.

1. Introduction

Relentless growth in computing power has allowed once computationally-

demanding Synthetic Aperture Radar (SAR) to become a feasible and preferred

technique for earth observation. Basic SAR has also been extended in a few di-

rections, one of which is polarimetric SAR (POLSAR). POLSAR exploits the natural

polarization property of Electro-Magnetic (EM) waves, encoded in multiple chan-

nels, compared to traditional one-channel SAR.

POLSAR data, like SAR, is stochastic but is also multi-dimensional, making it

even harder to interpret. It is therefore important to establish a simple and in-

tuitive understanding of the data. Statistical models are undoubtedly crucial in

understanding its stochastic nature. While several models have been proposed for

POLSAR data, they tend to be complex and unintuitive due to the multidimen-

sional nature of the data. Practical POLSAR data processing however, makes heavy

use of scalar discrimination measures, which should be based on statistically con-

sistent models of the multidimensional data. It is thus important to establish scalar

and representative observables for multi-dimensional POLSAR data.

A few scalar POLSAR observables with accompanying statistical models have

been proposal (Conradsen et al. 2003, Alberga et al. 2008, Joughin et al. 1994,

Lee et al. 1994b, Touzi and Lopes 1996, Lopez-Martinez and Fabregas 2003, Erten

2012), but none is able to provide meaningful scalar discrimination measures. As

such no observable has been widely accepted as being highly representative of

this multi-dimensional data, which severely limits their applicability in practical

data processing applications. Alternatively, a few POLSAR discrimination mea-

sures have been proposed[references?], but all are based on likelihood ratio statistics. These should

ideally be based on an exact and consistent distribution but so far only asymptotic

distributions have been demonstrated.

This paper presents scalar and representative observables, and one associated

generic statistical model, that describe multi-dimensional POLSAR data and pro-

vide a consistent foundation for the derivation of discrimination measures. POL-

SAR data have been statistically modelled as following the complex Wishart dis-

tribution. Consequently, the generic statistical model for the covariance matrix

determinant is presented as being just a scalar projection of the multidimensional

data. This model is then used to derive several scalar and consistent statistical

descriptions suggesting that their associated observables are capable of being used

as discrimination measures for POLSAR.

This paper will also show that the specific one dimensional (1-D) version of the pro-

posed model matches the traditional statistical model used for SAR intensity. This

effectively incorporates SAR theory under the umbrella of the proposed scalar ap-

proach for multi dimensional POLSAR. The different discrimination proposals for

SAR and POLSAR are reviewed in light of this and the proposed in this paper is

shown to provide a strong, unifying and consistent foundation. The applicability

of these theoretical models will be illustrated by experiments where the specific 1,

2 and 3-D versions of the proposed models are validated against practical data.

2. Related Work in Literature

Different target decomposition theorems have identified many possible scalar ob-

servables for complex POLSAR data. Alberga et al. (2008) evaluated the perfor-

mance of different scalar POLSAR observables for classification. While many were

presented, their corresponding statistical models and classifiers were not available.

Furthermore, the paper concluded that it is impossible to identify a single best rep-

resentation. Although, to be fair, the observables were identified for describing a

decomposed portion of the complex POLSAR data, rather than a unified represen-

tation. Using a different approach, given that the joint distribution for POLSAR is

known to be the multi-variate complex Wishart, it is possible to derive the scalar

statistical models for some univariate POLSAR observables. However, this is non-

trivial – so far, only a handful of such models have been proposed, including:

(i) cross-pol ratio rHV/HH = |SHV |2/|SHH|2 (Joughin et al. 1994),

(ii) co-pol ratio rV V/HH = |SV V |2/|SHH|2 (Joughin et al. 1994),

(iii) co-pol phase difference \_V V/HH = arg(SV V S\_HH) (Joughin et al. 1994) (Leeet al. 1994b),

(iv) magnitude g = |avg(SpqS\_rs)| (Lee et al. 1994b),

(v) normalized magnitude \_ = |avg(SpqS\_rs)| √avg(|Spq|2)avg(|Srs|2)(Lee et al. 1994b),

(vi) intensity ratio w = avg(|Spq|2)/avg(|Srs|2) (Lee et al. 1994b),

(vii) and the Stokes parameters Si, 0 ≤ i ≤ 3 (Touzi and Lopes 1996).

More recently, statistical models for each element of the POLSAR covariance ma-

trix, i.e. SpqS\_rs, (Lopez-Martinez and Fabregas 2003) as well as for the largest

eigen-value of the covariance matrix \_1 (Erten 2012) have been proposed. Although

useful, these have not been shown to result in statistically consistent discrimination

measures or be representative of the complex POLSAR data.

2.1 POLSAR Discrimination Measures

Euclidean or Manhattan distance measures for matrices are not widely used for

POLSAR due to the multiplicative nature of the noise. Instead, the Wishart dis-

tance is probably most common, as part of the well-known Wishart classifier (Lee

et al. 1999), defined (Lee et al. 1994a) as: d(Cx,Cy) = ln |Cy| + tr(CxC−1y ) where

tr(C) denotes the trace of the POLSAR covariance matrix C. As a measure of

distance, its main disadvantage is that d(Cy,Cy) = ln |Cy| 6= 0.

Recent works have suggested alternative dissimilarity measures including the

symmetric and asymmetric refined Wishart distance (Anfinsen et al. 2007),

d(Cx,Cy) =12tr(C−1x Cy + C−1y Cx) − d (1)

d(Cx,Cy) = ln |Cx| − ln |Cy| + tr(CxC−1y ) − d (2)

the Bartlett distance (Kersten et al. 2005),

d(Cx,Cy) = 2 ln |Cx+y| − ln |Cx| − ln |Cy| − 2d ln 2 (3)

the Bhattacharyya distance (Lee and Bretschneider 2011),

r(Cx,Cy) = |Cx|1/2|Cy|1/2|(Cx + Cy)/2| (4)

and the Wishart Statistical test distance (Cao et al. 2007),

d(Cx,Cy) = (Lx + Ly) ln |C| − Lx ln |Cx| − Ly ln |Cy| (5)

Closer examinations of these reveal that most are related: The Bhattacharyya

and Bartlett distances are easily shown to be related. At the same time, Barlett

can be considered a special case of the Wishart Statistical Test distance, when

the two data sets have the same number of looks, i.e. Lx = Ly. The close relation

among measures may be due to the fact that all of their publications

are based on the same statistical model in (Conradsen et al. 2003). In (Conradsen

et al. 2003), to determine if the two scaled multi-look POLSAR covariance matrixes

Zx and Zy, which have Lx and Ly as the corresponding number of looks, come from

the same underlying stochastic process, the likelihood ratio statistics for POLSAR

covariance matrix is considered:

… (6)

Taking the log-transformation of the above equation, and denoting Cvx = Zx/Lx,

Cvy = Zy/Ly and Cvxy = (Zx + Zy)/(Lx + Ly) then:

… (7)

… (8)

To detect changes, a test statistic is developed for this discrimination measure.

i.e. a distribution is derived for the dissimilarity measure. However, Conradsen

et al. (2003) only use an asymptotic distribution. By contrast, this paper proposes

a statistical model for the determinant of the POLSAR covariance matrix |Cv| which is capable of providing an exact distribution for the test.

3. The Generic Scalar Statistical Model for POLSAR

In this section, the generic scalar statistical model for POLSAR is presented. First,

let \_ = E[ss\_T ] denote the population expected value of the POLSAR covariance

matrix, where s\_T is the complex conjugate transpose of the POLSAR scattering

vector, s. If this is jointly circular complex Gaussian with expected covariance ma-

trix \_, then the PDF of s can be written as pdf(s;\_) = {1/\_d|\_|}exp{−s\_T\_−1s} where || denotes the matrix determinant. The sample POLSAR covariance ma-

trix is formed as the mean of Hermitian outer product of independent single-look

scattering vectors,

… (9)

where L is the number of looks and si denotes the partial or full POLSAR scattering

vector respectively. Complex Wishart distribution statistics are normally used for

the scaled covariance matrix Z = LCv, whose PDF is given as:

… (10)

with 􀀀d(L) = \_d(d−1)/2Qd−1i=0 􀀀(L − i) and d the dimension number of the POL-

SAR covariance matrix. The approach taken in this paper differs by applying a

homoskedastic log transformation on a less-than-well-known relationship. Good-

man (1963) found that the ratio between observable and expected values of the

sample covariance matrix determinants behaves like a product of d chi-squared

random variables with different degrees of freedom:

… (11)

We use this to develop a generic scalar statistical model. From Eqn. 11 we have:

… (12)

Over a homogeneous area, \_v, d and L are considered constant. Thus Eqn. 12

indicates that a multiplicative speckle noise pattern is present in the original POL-

SAR domain. Moreover, since the average and variance of these chi-squared dis-

tributions are constant, i.e. avg [\_(2L)] = 2L and var [\_(2L)] = 4L, their product

and summation also have fixed summary statistics. Specifically:

…………….

Combining these results with Eqn. 12 , we have:

… (13)

… (14)

For a real world captured image, while parameters d and L do not change for

the whole image, the underlying \_v is likely to differ from one region to the next.

Thus over a heterogeneous scene, the stochastic process for |Cv| and ln |Cv| vary

depending on the underlying signal \_v. Eqn. 14 implies that the variance of |Cv| will also differ depending on the underlying signal \_v (i.e. it is heteroskedastic).

Similar to the way intensity-ratio is proposed as the discrimination measure for

the multiplicative and heteroskedastic SAR intensity (Rignot and van Zyl 1993),

this paper proposes the determinant-ratio and the change-ratio as discrimination

measures for the POLSAR data.

If the true value of the underlying signal \_v is known a priori, then the

determinant-ratio of the signal random variable (R\_) is defined as: R\_ = |Cv|/|\_v| For POLSAR data from a homogeneous area, but when the true value of \_v

is unknown, then a random variable called the change-ratio RC) is defined,

RC = |C1|/|C2| where C1 and C2 are samples of the covariance matrix determinant

in an assumed homogeneous area. Using the results from Eqn. 12, we have

… (15)

Since each elementary component follows fixed distributions (i.e. \_2(2L)), this vari-

able naturally also follows fixed distributions. Moreover, it is independent of the

underlying \_v, indicating its statistically consistent properties (i.e. its applicability

as a POLSAR discrimination measure).

4. SAR as a one-dimensional case of POLSAR

This section shows that the proposed generic model is applicable to the 1-D case

(d = 1), physically equivalent to collapsing the multi-dimensional POLSAR dataset

into single dimensional SAR data. Mathematically, the sample covariance matrix

Cv is reduced to the sample variance while determinant |Cv| becomes the scalar

variance. As variance is equal to intensity I in SAR, our result is consistent with

previous results for SAR intensity. Thus the proposed generic model for POLSAR,

collapsed into 1-D will be shown to apply also to traditional SAR intensity.

The results for our models can be summarised using the following equations:

…….

Upon setting d = 1 into the above equations, the equations become:

… (16)

… (17)

Since the PDF of chi-squared distribution can be written as:

..

Applying variable change theorem into the above equations results in:

..

These equations match exactly with the following traditional model for multi-look

SAR intensity:

… (18)

… (19)

… (20)

considering that |Cv| 7→ I and |\_v| 7→ ¯I as multi-dimensional POLSAR collapses

into single-dimensional SAR.

5. Unifying different discrimination measures for SAR and POLSAR

Statistical models are the foundation for discrimination measures in both SAR and

POLSAR. For the mature SAR field, the statistical model for SAR intensity has

been used to derive the most widely used intensity-ratio discrimination measure

(Rignot and van Zyl 1993). For the less mature POLSAR field the same case should

apply, except that so far only asymptotic distributions have been derived for the

most common foundation, i.e. the likelihood test statistics (Conradsen et al. 2003).

With the insight gained from section 4., this section presents a few new results. First,

similar to the way that the statistical SAR intensity models have been used as

a foundation for SAR discrimination measures, e.g. intensity-ratio (Rignot and

van Zyl 1993), the proposed POLSAR covariance matrix determinant statistical

model can also be viewed as providing a foundation for POLSAR discrimination

measures, i.e. the likelihood test statistics. Secondly, new discrimination measures

for POLSAR may be derived by learning from the existing SAR discrimination

measures.

As for the first matter, in view of the models given in Eqn 12, the likelihood test

statistics presented in (Conradsen et al. 2003) and rewritten in Eqns 7 & 8 can be

expressed as lnQ ∼ k + Lx\_d

… (21)

where k = d [(Lx + Ly) ln(Lx + Ly) − Lx ln Lx − Ly ln Ly]. This, in essence, derives

an exact statistical distribution for the likelihood test statistics, as opposed to the

asymptotic distribution derived in (Conradsen et al. 2003).

As a by-product of this exact derivation, several discrimination measures for the

common case of Lx = Ly are further proposed. They are the determinant-ratio and

the change-ratio presented in Section 3.. Compared to existing discrimination mea-

sures for POLSAR reviewed in Section 2., the proposed dissimilarity measures are

simpler in both concept and computation. They are hence multi-dimensional extensions

of the widely used SAR intensity-ratio discrimination measure.

6. Model Validation

We now verify the models in Eqns. 12, ?? and 15 against real-life captured data. Each model

requires the estimation of two parameters from the data; the dimensional

number d, and the look number L. d is related to the type of (POL)SAR data

captured, with d = 1, 2, 3 corresponding to the cases of SAR, partial and full

POLSAR, respectively. L is nominally stated by the data provider (or estimated

using the technique proposed by (Anfinsen et al. 2009)).

To show the robustness of the proposed models, their validations are tested on

two different POLSAR sensors: (1) airborne four-look (L = 4) AIRSAR Flevoland

image and (2) fine-quad single-look (L = 1) complex RADARSAT2 image. Since

the determinant of the covariance matrix is only significant on multi-look data,

nine-look processing is first applied to the single-look data (L = 9).

6.1 The Traditional case of SAR (d = 1)

Fig. 1 presents the results of a test where the intensity of single-channel SAR data

(HH) for sample homogeneous areas is extracted. Histograms are then plotted for

both the intensity and intensity ratio against the theoretical PDF. In all cases,

the good visual match between the actual data and model distribution tends to

validate the proposed model.

6.2 The Multi-dimensional case of POLSAR (d = 2, 3)

The look-number is estimated for each dataset (Anfinsen et al. 2009). and the

actual distribution plotted against the model yields an obvious visual match in

Fig. 2 which shows the d = 2 plots for determinant, determinant ratio and change

ratio for both datasets.

:

Similarly, Fig. 3 explores the d = 3 case for the same data and model parameters.

Again, although the histogram is much tighter, the match is visually obvious in all

cases.

7. Conclusion

This paper has proposed a generic statistical model for the POLSAR covariance

matrix determinant based on the complex Wishart POLSAR target vector dis-

tribution.The model has been validated for specific d=2 and d=3 cases using

real-life partial and full POLSAR data. The paper also establishes two(?) POLSAR

discrimination measures: the determinant-ratio and the change-ratio, which essen-

tially are the generic version of the SAR intensity-ratio. We have also shown a

near-perfect match between the specific d = 1 model and that for traditional SAR

intensity, effectively bringing the existing SAR theories under the umbrella of this

new model.

The main emphasis of this paper is to consider the POLSAR covari-

ance matrix determinant as a scalar and representative observable for the multi-

dimensional POLSAR data. Compared to other published scalar observables, the

determinant is highly representative of the multi-dimensional data. It is also in-

variant to a change of polarization basis.

. Its repre-

sentative power is justified for the following reasons. Firstly, the covariance matrix

determinant, when collapsed into 1-D, transforms neatly into the representative

SAR intensity. Secondly, the statistical model for this observable is shown to be

the generic multi-dimensional extension of the traditional model for the one di-

mensional SAR intensity. Finally, this observable also leads to the establishment

of the determinant-ratio discrimination measure for POLSAR data, which should

increase the usability of this proposal.

There are several beneficial implications of the proposed approach presented in

this paper. Currently while the field of SAR is much more developed than the

field of POLSAR, the two fields remain quite separated. Hence many techniques

applicable to SAR cannot be directly extended to POLSAR. Since the scalar and

representative model for POLSAR generalizes the traditional model for SAR inten-

sity, its main benefit is that it enables the convenient adaptation of many existing

SAR data processing techniques for POLSAR data. One example has already been

presented in this paper, where existing SAR discrimination measures are extended

towards POLSAR. At the same time, it provides a consistent theory unifying the

seemingly disparate discrimination measure proposals for SAR and POLSAR data.

Of course, the models proposed in this article also have their own limitations.

Firstly, they are based on the complex Wishart distribution which is only guar-

anteed to work for homogeneous areas. Secondly, while it is desirable for a large

number of applications to reduce the multi-dimensional POLSAR data to a scalar

value, such a reduction is unlikely to be lossless in the general case.

In this paper, the theoretical model has only been validated for partial (d = 2)

and full (d = 3) monostatic POLSAR data, leaving the test of its validity for

datasets such as bistatic POLSAR data (d = 4) or interferrometric POLSAR data

(d = 6) for the future. Other possible extensions include investigating the applica-

bility of the observables into POLSAR heterogeneous areas, as well as exploring the

use of these generic models in conjunction with other target decomposition tech-

niques such as the Freeman-Durden decomposition (Freeman and Durden 1998) or

the entropy/anisotropy decomposition (Cloude and Pottier 1997).