Programming Assignment 2: Decomposition of Graphs

Revision: July 27, 2020

Introduction

Welcome to your second programming assignment of the Algorithms on Graphs class! In this assignment, we focus on directed graphs and their parts.

In this programming assignment, the grader will show you the input and output data if your solution fails on any of the tests. This is done to help you to get used to the algorithmic problems in general and get some experience debugging your programs while knowing exactly on which tests they fail. However, for all the following programming assignments, the grader will show the input data only in case your solution fails on one of the first few tests (please review the questions **??** and **??** in the FAQ section for a more detailed explanation of this behavior of the grader).

Learning Outcomes

Upon completing this programming assignment you will be able to:

1. check consistency of Computer Science curriculum;
2. find an order of courses that is consistent with prerequisite dependencies;
3. check whether any intersection of a city is reachable from any other intersection.

Passing Criteria: 2 out of 3

Passing this programming assignment requires passing at least 2 out of 3 programming challenges from this assignment. In turn, passing a programming challenge requires implementing a solution that passes all the tests for this problem in the grader and does so under the time and memory limits specified in the problem statement.

Contents

Graph Representation in Programming Assignments

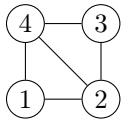
In programming assignments, graphs are given as follows. The first line contains non-negative integers n and m — the number of vertices and the number of edges respectively. The vertices are always numbered from 1 to n. Each of the following m lines defines an edge in the format `u v` where $1 \le u, v \le n$ are endpoints of the edge. If the problem deals with an undirected graph this defines an undirected edge between u and v. In case of a directed graph this defines a directed edge from u to v. If the problem deals with a weighted graph then each edge is given as `u v w` where u and v are vertices and w is a weight.

It is guaranteed that a given graph is simple. That is, it does not contain self-loops (edges going from a vertex to itself) and parallel edges.

Examples:

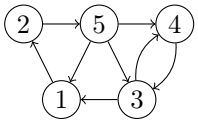
- An undirected graph with four vertices and five edges:

```
4 5
2 1
4 3
1 4
2 4
3 2
```



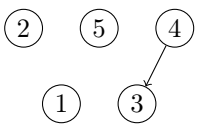
- A directed graph with five vertices and eight edges.

```
5 8
4 3
1 2
3 1
3 4
2 5
5 1
5 4
5 3
```



- A directed graph with five vertices and one edge.

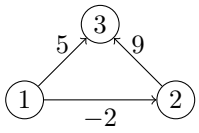
```
5 1
4 3
```



Note that the vertices 1, 2, and 5 are isolated (have no adjacent edges), but they are still present in the graph.

- A weighted directed graph with three vertices and three edges.

```
3 3
2 3 9
1 3 5
1 2 -2
```



1 Checking Consistency of CS Curriculum

Problem Introduction

A Computer Science curriculum specifies the prerequisites for each course as a list of courses that should be taken before taking this course. You would like to perform a consistency check of the curriculum, that is, to check that there are no cyclic dependencies. For this, you construct the following directed graph: vertices correspond to courses, there is a directed edge (u, v) is the course u should be taken before the course v. Then, it is enough to check whether the resulting graph contains a cycle.

Problem Description

Task. Check whether a given directed graph with n vertices and m edges contains a cycle.

Input Format. A graph is given in the standard format.

Constraints. $1 \leq n \leq 10^3$, $0 \leq m \leq 10^3$.

Output Format. Output 1 if the graph contains a cycle and 0 otherwise.

Time Limits.

language	`C`	`C++`	`Java`	`Python`	`C#`	`Haskell`	`JavaScript`	`Ruby`	`Scala`
time (sec)	1	1	1.5	5	1.5	2	5	5	3

Memory Limit. 512MB.

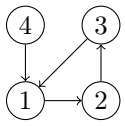
Sample 1.

Input:

```
4 4
1 2
4 1
2 3
3 1
```

Output:

```
1
```



This graph contains a cycle: $3 \to 1 \to 2 \to 3$.

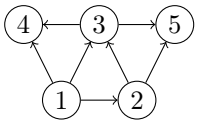
Sample 2.

Input:

```
5 7
1 2
2 3
1 3
3 4
1 4
2 5
3 5
```

Output:

```
0
```



There is no cycle in this graph. This can be seen, for example, by noting that all edges in this graph go from a vertex with a smaller number to a vertex with a larger number.

2 Determining an Order of Courses

Problem Introduction

Now, when you are sure that there are no cyclic dependencies in the given CS curriculum, you would like to find an order of all courses that is consistent with all dependencies. For this, you find a topological ordering of the corresponding directed graph.

Problem Description

Task. Compute a topological ordering of a given directed acyclic graph (DAG) with n vertices and m edges.

Input Format. A graph is given in the standard format.

Constraints. $1 \leq n \leq 10^5$, $0 \leq m \leq 10^5$. The given graph is guaranteed to be acyclic.

Output Format. Output *any* topological ordering of its vertices. (Many DAGs have more than just one topological ordering. You may output any of them.)

Time Limits.

language	`C`	`C++`	`Java`	`Python`	`C#`	`Haskell`	`JavaScript`	`Ruby`	`Scala`
time (sec)	2	2	3	10	3	4	10	10	6

Memory Limit. 512MB.

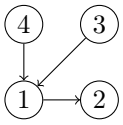
Sample 1.

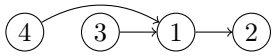
Input:

```
4 3
1 2
4 1
3 1
```

Output:

```
4 3 1 2
```





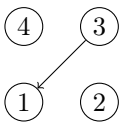
Sample 2.

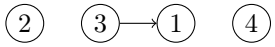
Input:

```
4 1
3 1
```

Output:

```
2 3 1 4
```





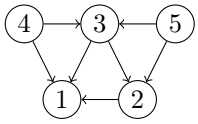
Sample 3.

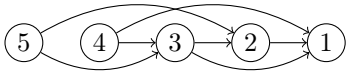
Input:

```
5 7
2 1
3 2
3 1
4 3
4 1
5 2
5 3
```

Output:

```
5 4 3 2 1
```





3 Checking Whether Any Intersection in a City is Reachable from Any Other

Problem Introduction

The police department of a city has made all streets one-way. You would like to check whether it is still possible to drive legally from any intersection to any other intersection. For this, you construct a directed graph: vertices are intersections, there is an edge (u, v) whenever there is a (one-way) street from u to v in the city. Then, it suffices to check whether all the vertices in the graph lie in the same strongly connected component.

Problem Description

Task. Compute the number of strongly connected components of a given directed graph with n vertices and m edges.

Input Format. A graph is given in the standard format.

Constraints. $1 \le n \le 10^4$, $0 \le m \le 10^4$.

Output Format. Output the number of strongly connected components.

Time Limits.

language	C	C++	Java	Python	C#	Haskell	JavaScript	Ruby	Scala
time (sec)	1	1	1.5	5	1.5	2	5	5	3

Memory Limit. 512MB.

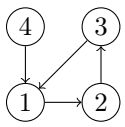
Sample 1.

Input:

```
4 4
1 2
4 1
2 3
3 1
```

Output:

```
2
```



This graph has two strongly connected components: $\{1, 3, 2\}$, $\{4\}$.

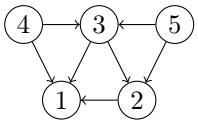
Sample 2.

Input:

```
5 7
2 1
3 2
3 1
4 3
4 1
5 2
5 3
```

Output:

```
5
```



This graph has five strongly connected components: {1}, {2}, {3}, {4}, {5}.

4 Appendix

4.1 Compiler Flags

C (`gcc 7.4.0`). File extensions: `.c`. Flags:

```
gcc -pipe -O2 -std=c11 <filename> -lm
```

C++ (`g++ 7.4.0`). File extensions: `.cc`, `.cpp`. Flags:

```
g++ -pipe -O2 -std=c++14 <filename> -lm
```

If your `C/C++` compiler does not recognize `-std=c++14` flag, try replacing it with `-std=c++0x` flag or compiling without this flag at all (all starter solutions can be compiled without it). On Linux and MacOS, you most probably have the required compiler. On Windows, you may use your favorite compiler or install, e.g., `cygwin`.

C# (`mono 4.6.2`). File extensions: `.cs`. Flags:

```
mcs
```

Go (`golang 1.13.4`). File extensions: `.go`. Flags

```
go
```

Haskell (`ghc 8.0.2`). File extensions: `.hs`. Flags:

```
ghc -O2
```

Java (`OpenJDK 1.8.0_232`). File extensions: `.java`. Flags:

```
javac -encoding UTF-8
java -Xmx1024m
```

JavaScript (`NodeJS 12.14.0`). File extensions: `.js`. No flags:

```
nodejs
```

Kotlin (`Kotlin 1.3.50`). File extensions: `.kt`. Flags:

```
kotlinc
java -Xmx1024m
```

Python (`CPython 3.6.9`). File extensions: `.py`. No flags:

```
python3
```

Ruby (`Ruby 2.5.1p57`). File extensions: `.rb`.

```
ruby
```

Rust (`Rust 1.37.0`). File extensions: `.rs`.

```
rustc
```

Scala (`Scala 2.12.10`). File extensions: `.scala`.

```
scalac
```

4.2 Frequently Asked Questions

Why My Submission Is Not Graded?

You need to create a submission and upload the *source file* (rather than the executable file) of your solution. Make sure that after uploading the file with your solution you press the blue "Submit" button at the bottom. After that, the grading starts, and the submission being graded is enclosed in an orange rectangle. After the testing is finished, the rectangle disappears, and the results of the testing of all problems are shown.

What Are the Possible Grading Outcomes?

There are only two outcomes: "pass" or "no pass." To pass, your program must return a correct answer on all the test cases we prepared for you, and do so under the time and memory constraints specified in the problem statement. If your solution passes, you get the corresponding feedback "Good job!" and get a point for the problem. Your solution fails if it either crashes, returns an incorrect answer, works for too long, or uses too much memory for some test case. The feedback will contain the index of the first test case on which your solution failed and the total number of test cases in the system. The tests for the problem are numbered from 1 to the total number of test cases for the problem, and the program is always tested on all the tests in the order from the first test to the test with the largest number.

Here are the possible outcomes:

- `Good job!` Hurrah! Your solution passed, and you get a point!
- `Wrong answer.` Your solution outputs incorrect answer for some test case. Check that you consider all the cases correctly, avoid integer overflow, output the required white spaces, output the floating point numbers with the required precision, don't output anything in addition to what you are asked to output in the output specification of the problem statement.
- `Time limit exceeded.` Your solution worked longer than the allowed time limit for some test case. Check again the running time of your implementation. Test your program locally on the test of maximum size specified in the problem statement and check how long it works. Check that your program doesn't wait for some input from the user which makes it to wait forever.
- `Memory limit exceeded.` Your solution used more than the allowed memory limit for some test case. Estimate the amount of memory that your program is going to use in the worst case and check that it does not exceed the memory limit. Check that your data structures fit into the memory limit. Check that you don't create large arrays or lists or vectors consisting of empty arrays or empty strings, since those in some cases still eat up memory. Test your program locally on the tests of maximum size specified in the problem statement and look at its memory consumption in the system.
- `Cannot check answer. Perhaps the output format is wrong.` This happens when you output something different than expected. For example, when you are required to output either "Yes" or "No", but instead output 1 or 0. Or your program has empty output. Or your program outputs not only the correct answer, but also some additional information (please follow the exact output format specified in the problem statement). Maybe your program doesn't output anything, because it crashes.
- `Unknown signal 6 (or 7, or 8, or 11, or some other).` This happens when your program crashes. It can be because of a division by zero, accessing memory outside of the array bounds, using uninitialized variables, overly deep recursion that triggers a stack overflow, sorting with a contradictory comparator, removing elements from an empty data structure, trying to allocate too much memory, and many other reasons. Look at your code and think about all those possibilities. Make sure that you use the same compiler and the same compiler flags as we do.
- `Internal error: exception...` Most probably, you submitted a compiled program instead of a source code.

- `Grading failed.` Something wrong happened with the system. Report this through Coursera or edX Help Center.

May I Post My Solution at the Forum?

Please do not post any solutions at the forum or anywhere on the web, even if a solution does not pass the tests (as in this case you are still revealing parts of a correct solution). Our students follow the Honor Code: "I will not make solutions to homework, quizzes, exams, projects, and other assignments available to anyone else (except to the extent an assignment explicitly permits sharing solutions)."

Do I Learn by Trying to Fix My Solution?

My implementation always fails in the grader, though I already tested and stress tested it a lot. Would not it be better if you gave me a solution to this problem or at least the test cases that you use? I will then be able to fix my code and will learn how to avoid making mistakes. Otherwise, I do not feel that I learn anything from solving this problem. I am just stuck.

First of all, learning from your mistakes is one of the best ways to learn.

The process of trying to invent new test cases that might fail your program is difficult but is often enlightening. Thinking about properties of your program makes you understand what happens inside your program and in the general algorithm you're studying much more.

Also, it is important to be able to find a bug in your implementation without knowing a test case and without having a reference solution, just like in real life. Assume that you designed an application and an annoyed user reports that it crashed. Most probably, the user will not tell you the exact sequence of operations that led to a crash. Moreover, there will be no reference application. Hence, it is important to learn how to find a bug in your implementation yourself, without a magic oracle giving you either a test case that your program fails or a reference solution. We encourage you to use programming assignments in this class as a way of practicing this important skill.

If you have already tested your program on all corner cases you can imagine, constructed a set of manual test cases, applied stress testing, etc, but your program still fails, try to ask for help on the forum. We encourage you to do this by first explaining what kind of corner cases you have already considered (it may happen that by writing such a post you will realize that you missed some corner cases!), and only afterwards asking other learners to give you more ideas for tests cases.