

# The Population Dynamics of Herring and Mackerel Stocks in Scottish Fisheries

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## 1 Introduction

This paper seeks to answer question "What effect will climate change have on Scottish fishery populations in the next 50 years?" Our answer, while not surprising, has some unexpected nuances and implications for Scottish fishing crews. To get our answer we created a simulation of fishing populations under the demands of climate change. We created a model that rated the survivability of fish stocks based on the temperature of the water they're in. The closer they are to their ideal temperature, the more likely they are to survive. As the temperature of the north sea rises, every species in the water will find that certain areas that were once ideally suited to their needs now poorly meet them, and waters that were once uninhabitable become much better habitats. Designing our study with the time and data constraints we had forced us to make many simplifying assumptions. We had to assume that certain affects were negligible enough they could be ignored, so that we could adequately understand the effect of what we believe are the greatest effects. We'll take care to point out the assumptions we make and the weaknesses of our model while trying to justify why the model we created is still adequate to make an assessment of how the ecosystem will change in the upcoming decades.

## 2 Assumptions

1. We are going to assume that the amount of fish that enter a region is equal to the number that leave, so that we don't have to worry about the migration of the fish.
2. Half of the fish population are female
3. Sea is a large heat reservoir.
4. People only fish in the abundant season (summer).
5. Fish reside near the sea floor.
6. Carrying capacity of a kind of fish is inversely proportional to its body mass.
7. Only 0.001% of the eggs could grow up to fish.

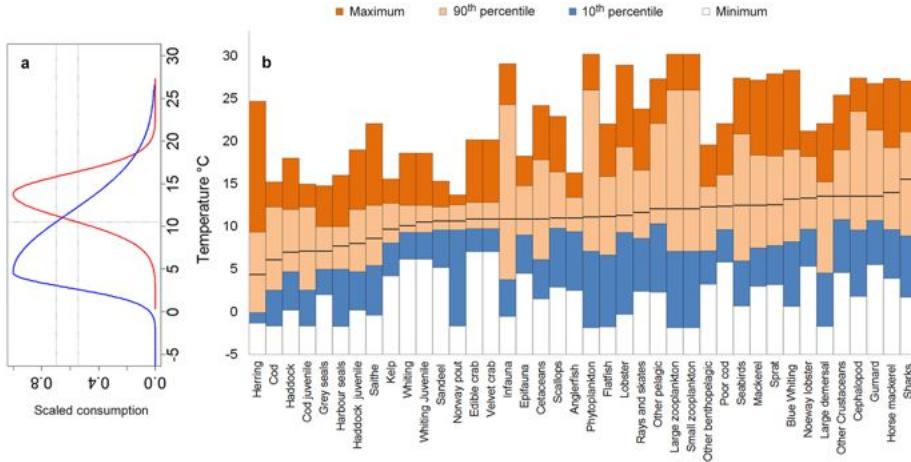


Figure 1: Temperature tolerance of different kinds of fish[1]

### 3 Relevant Data

#### 3.1 Herring and Mackerel

We summarize the basic information about herring and mackerel in the table below:

	Herring	Mackerel
Optimum Temperature $\mu$ (°C)	5 (estimation from Fig 1)	12 (estimation from Fig 1)
life span(yr)	12[7]	17[10]
sexually mature age(yr)	4[7]	2[10]
Carrying Capacity( $T/km^{-2}$ )	13.68[2]*	
Number of eggs laid by a female fish $N_e(\text{yr}^{-1})$	30000[7]	900000[8]
Average weight m(kg)	1.1[7]	1.8[11](p115)

\*This carrying capacity is a from a study of the carrying of capacity of Atlantic Herring in Georgia Straight.

Large Marine Ecosystem	SST Trend: ( $^{\circ}\text{C}$ Decade $^{-1}$ )		Std Dev ( $\sigma$ ) w/Trend	Std Dev ( $\sigma$ ) w/o Trend	
	20 <sup>th</sup> C	21 <sup>st</sup> C	20 <sup>th</sup> C	21 <sup>st</sup> C	20 <sup>th</sup> C
(22) North Sea	0.22	0.31	0.66	0.68	0.59

Figure 2: Temperature trend[12]

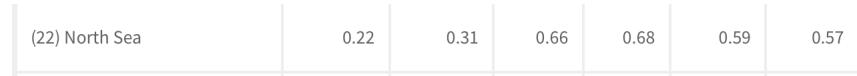


Figure 3: Temperature trend(cont.)[12]

### 3.2 The North Sea

1. Most of the herring and mackerel harvesting will be done on the east side of Scotland (North Sea) rather than west side, since west side is mainly for pelagic fish and lobster[14].
2. The main method of fishing in the North Sea is trawling[5], through which the web is pulling at the bottom or in the middle of the sea behind a boat[4].
3. We find a temperature map of the bottom of North Sea in 2010 summer (Fig 4a).
4. We find a depth map of the North Sea.(Fig 4b)
5. We find the North Sea temperature growth rate ( $dT$ ).According to Fig 3, it is most likely that  $dT = 0.31^{\circ}\text{C}/(10\text{yr}) = 0.031^{\circ}\text{C}/\text{yr}$ .

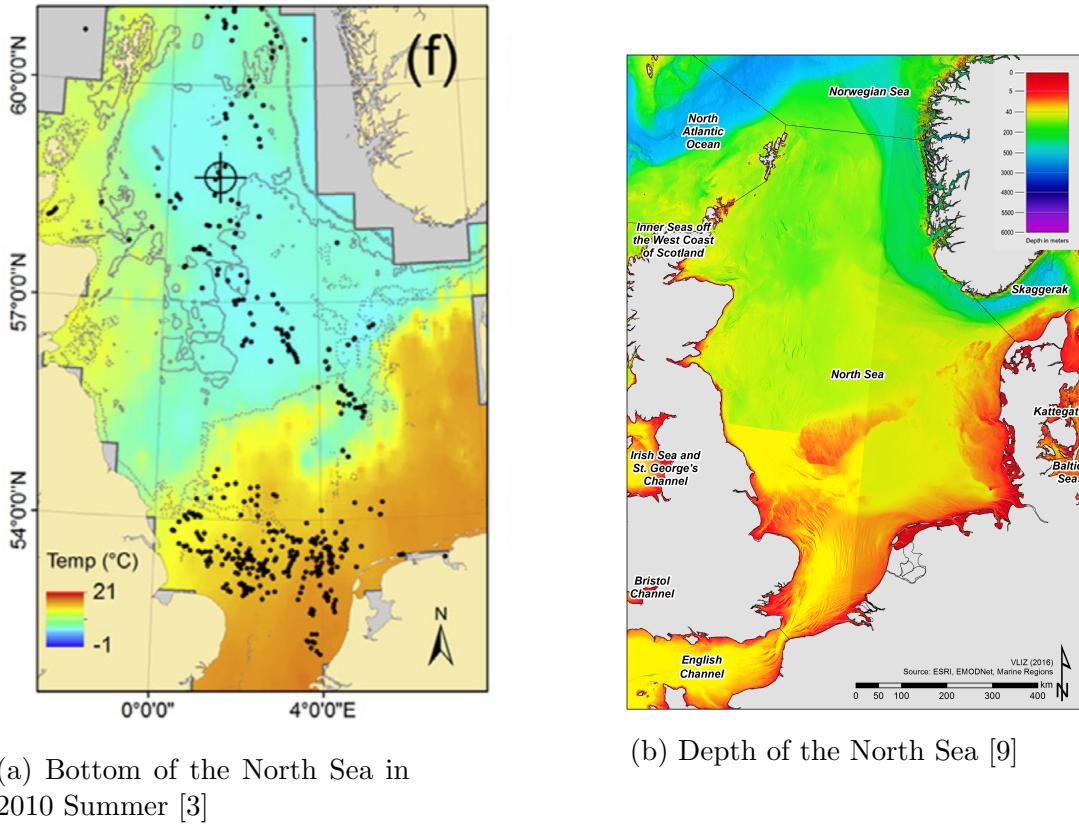


Figure 4: Temperature and Depth Map

### 3.3 Derivation of Some Basic Information & Some Other Preparation

#### 3.3.1 Some Other Preparation

1. In order to get the scale of the Fig 4a, we align it with Fig 4b. Then in order to quantify the temperature map (Fig 1), we divide the map into smaller pieces by a  $20 \times 27$  grid (See Fig 5).
2. We estimate the actual size of each block to be  $100\text{km}^2$  according to the scale.
3. Base on Fig 5, we estimate the temperature of each grid, using the temperature given us in the Fig 5. We write each grid in term of

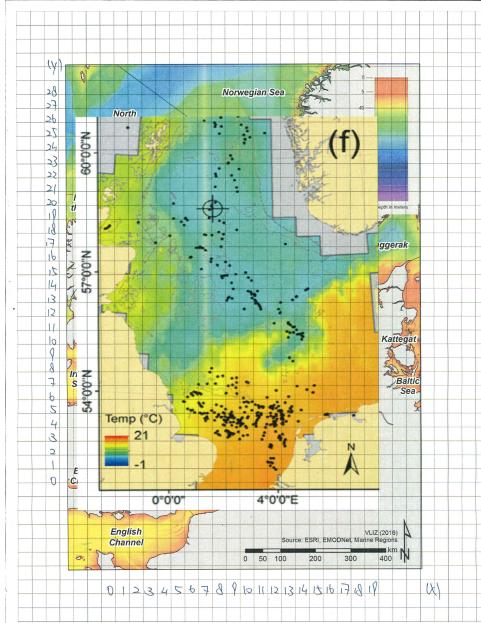


Figure 5: Devide the Bottom of the North Sea by  $20 \times 27$  Grid.

$\{x, y, Temp\}$ . For the grids, which stand for land, we mark the temperature to be 0, to avoid confusion.(See example as in Fig 6)

### 3.3.2 Basic information

1. We could calculate the carrying capacity of one block for herring:

$$K_h = \frac{\text{carrying capacity per } \text{km}^{-2} \cdot \text{area of a block}}{\text{mass of herring}}$$

```
In[144]:= NorthSea1 = {{0, 26, 0}, {1, 26, 0}, {2, 26, 0}, {3, 26, 9}, {4, 26, 9}, {5, 26, 9}, {6, 26, 9}, {7, 26, 10}, {8, 26, 11}, {9, 26, 11}, {10, 26, 9}, {11, 26, 9}, {12, 26, 0}, {13, 26, 0}, {14, 26, 0}, {15, 26, 0}, {16, 26, 0}, {17, 26, 0}, {18, 26, 0}, {19, 26, 0}, {0, 25, 0}, {1, 25, 0}, {2, 25, 0}, {3, 25, 9}, {4, 25, 10}, {5, 25, 11}, {6, 25, 11}, {7, 25, 11}, {8, 25, 11}, {9, 25, 11}, {10, 25, 11}, {11, 25, 11}, {12, 25, 0}, {13, 25, 0}, {14, 25, 0}, {15, 25, 0}, {16, 25, 0}, {17, 25, 0}, {18, 25, 0}, {19, 25, 0}},
```

Figure 6: Estimation of Temperature at each grid.

$$= \frac{13.68 \text{T} \cdot \text{km}^{-2} \cdot 1000 \text{km}^2}{1.1 \text{kg}} = 12436363$$

2. We could calculate the carrying capacity of one block for mackerel:

$$K_m = K_h \cdot \frac{\text{mass of herring}}{\text{mass of mackerel}} = 12436363 \cdot \frac{1.1 \text{kg}}{1.8 \text{kg}} = 7600000$$

3. We could calculate reproduction rate of herring in one block:

$$\begin{aligned} \lambda_h &= \# \text{ of eggs} \cdot \text{survival rate} \cdot \text{percentage of female} \cdot \text{percentage of mature fish} \\ &= 30000 \text{yr}^{-1} \cdot 0.001\% \cdot \frac{1}{2} \cdot \frac{2}{3} = 0.1 \text{yr}^{-1} \end{aligned}$$

4. We could calculate reproduction rate of mackerel in one block(by assumption):

$$\begin{aligned} \lambda_m &= \# \text{ of eggs} \cdot \text{survival rate} \cdot \text{percentage of female} \cdot \text{percentage of mature fish} \\ &= 900000 \text{yr}^{-1} \cdot 0.001\% \cdot \frac{1}{2} \cdot \frac{15}{17} = 4 \text{yr}^{-1} \end{aligned}$$

5. We could calculate natural death rate of herring at optimize temperature:

$$K_{2h} = \frac{1}{\text{life span of herring}} = \frac{1}{12} \text{yr}^{-1}$$

6. We could calculate natural death rate of mackerel at optimize temperature:

$$K_{2m} = \frac{1}{\text{life span of mackerel}} = \frac{1}{17} \text{yr}^{-1}$$

## 4 Population Dynamics

We combine a few ideas from different models to build the population model for both of the fish populations. The basic model for the rate of change in fish population is the compartment model:

$$\frac{dN}{dt} = \text{Born} + \text{Immigrated}_{\text{in}} - \text{Dead} - \text{Immigrated}_{\text{out}} \quad (1)$$

For a given region of the map, the rise in population is the sum of newly hatched fish and fish that immigrated into the area, while the lowering in the population is the amount of fish that die and the amount that immigrate out of the region By assumption 1. we know that there are an equal number

of fish swimming in and out of any grid space on the map over a year, so we can simplify our equation to

$$\frac{dN}{dt} = \text{Born} - \text{Dead}. \quad (2)$$

For the Born part, we decide to use the logistic growth model:

$$\dot{N} = \lambda \cdot N \left(1 - \frac{N}{K}\right), \quad (3)$$

where  $N$  is the population,  $\lambda$  is the natural growth rate, and  $K$  is the carrying capacity.

We need to take temperature into consideration we decide to use a Gaussian model

$$f(x) = \frac{1}{\sigma \sqrt{2 \cdot \pi}} e^{-\frac{1}{2} \left(\frac{x-\mu}{\sigma}\right)^2}, \quad (4)$$

(where  $\sigma$  is the standard deviation and  $\mu$  is the average) to adjust both the born term and death term.

With the Gaussian term in our model we can scale the *Born* term of the differential equation based on how far the temperature of the water is from ideal. The closer the temperature is to the optimize temperature, the closer to the natural growth rate will the actual growth rate be. When the actual temperature is different from the optimize temperature, the growth rate will be lower. Similarly, the closer the temperature is to the optimize temperature, the closer to the natural death rate will the actual death rate be. When the actual temperature is different from the optimize temperature, the death rate will be higher.

We didn't normalize the Gaussian so that when temperature equals  $\mu$  the value of the Gaussian term is 1 (when the temperature is ideal it has no effect on birth rates). This is what the Gaussian term looks like:

$$e^{-\frac{1}{2} \left(\frac{T-\mu}{\sigma}\right)^2}. \quad (5)$$

We eyeballed the graph in Figure 1 to estimate that a herring outside of  $(5 \pm 5)^\circ C$  has a 90% chance of being killed by the temperature. Our Gaussian is centered at the ideal temperature  $\mu = 5$  with a width parameter of  $\sigma_h = 3.9$ , corresponding to the fact that  $\mu + 5$  is the 90th percentile of

the distribution. We use the same method to calculate the  $\sigma$  for mackerel:  $\sigma_m = 4.3$ .

Now, we can put all these parts together and form the differential equation. We will use a C code to simulate the data, and therefore we will convert the number of individual to biomass in unit of tons.

We intentionally neglect the term caused by fishing industry because we want the research result can be considered by the policy makers before the fishing season starts. In this way, the policy makers can give accurate instruction about the fishing location and amount.

Our differential model is:

$$\dot{P}_h = [e^{-\frac{1}{2}(\frac{T-\mu_h}{\sigma_h})^2} \lambda_h \cdot P_h (1 - \frac{P_h}{K_h}) - (K_{2h} + 1 - e^{-\frac{1}{2}(\frac{T-\mu_h}{\sigma_h})^2})] \cdot \frac{m_h}{1000\text{tons}} \quad (6)$$

$$\dot{P}_m = [e^{-\frac{1}{2}(\frac{T-\mu_m}{\sigma_m})^2} \lambda_m \cdot P_m (1 - \frac{P_m}{K_m}) - (K_{2m} + 1 - e^{-\frac{1}{2}(\frac{T-\mu_m}{\sigma_m})^2})] \cdot \frac{m_m}{1000\text{tons}} \quad (7)$$

$$\dot{N} = dT \quad (8)$$

## 5 Numerical Simulation

To have the change of population density map, we need a numerical simulation. We used a C program to simulation the population change with respect to time according to the model mentioned before. In the simulation, we applied Eula's method because the model is discrete in years and thus does not require a extremely high accuracy. Here we provide the basic idea of Eula's method. The time derivative of population at each year is give by

$$\frac{dN}{dt} = \text{Born} + \text{Immigrated}_{in} - \text{Dead} - \text{Immigrated}_{out}. \quad (9)$$

Thus, we estimate the annual increment by

$$\Delta N(t) = \frac{dN(t)}{dt} \Delta t, \quad (10)$$

and the total population

$$N(t+1) = N(t) + \Delta N(t). \quad (11)$$

We run this process through  $t = 0$  to  $t = 49$  and compute  $N(50)$ . Since we lack a detailed study in the current population distribution of Herring and

Mackerel in North Sea, we find the initial condition by running the model without temperature change for 1000 years. According to the theory of logistic model, the initial condition should approach to a stable value according to the local temperature. Also, by changing the annual temperature increment, we simulate the best case scenario and worst case scenario.

## 6 Simulation Result

In the first simulation, we assume that the annual temperature increment of bottom of North Sea as  $0.031^{\circ}\text{C}$ . We first generate the temperature map of the sea bottom.

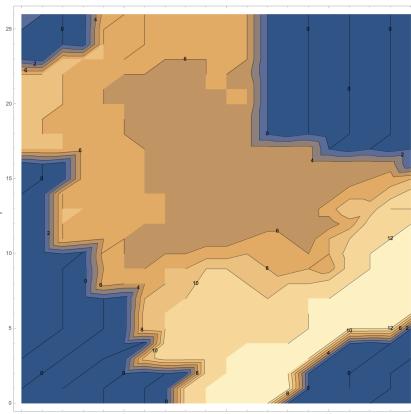
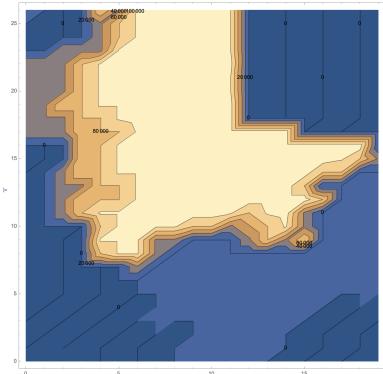


Figure 7: Initial temperature data(the lighter the color, the higher the temperature)

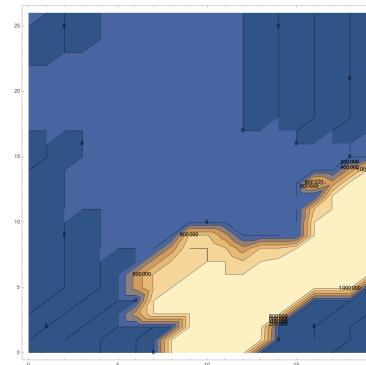
Therefore, with this input we generate the initial condition by following the population dynamics model to evolve 1000 years.

Notice that the lighter color in the graph represents the higher population density. Then we take the annual temperature increment as  $0.031^{\circ}\text{C}$ , and evolve the model for 50 years. We have the population density map.

We consider temperature increment to be  $0^{\circ}\text{C}$  in the best scenario and assume that  $0.031^{\circ}\text{C}$  increment to be most likely case, since we get that number from researchers' prediction according to the current situation. We also assume that the most likely case will be in the middle of best case and worst case. To estimate a worst case scenario, let the annual temperature



(a) Initial Herring Distribution

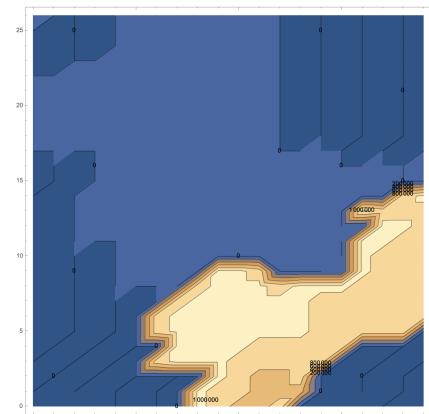


(b) Initial Mackerel Distribution

Figure 8: Initial Population distribution



(a) Herring Distribution after 50 years



(b) Mackerel Distribution after 50 years

Figure 9: Initial Population distribution after 50 years for annual temperature increment 0.031°C

increment be 0.062°C. We ended up with population after 50 years that is shown below.

Finally, we made a prediction based on worst scenario. The map shown below is the situation after 65 years with annual temperature increment 0.062°C.

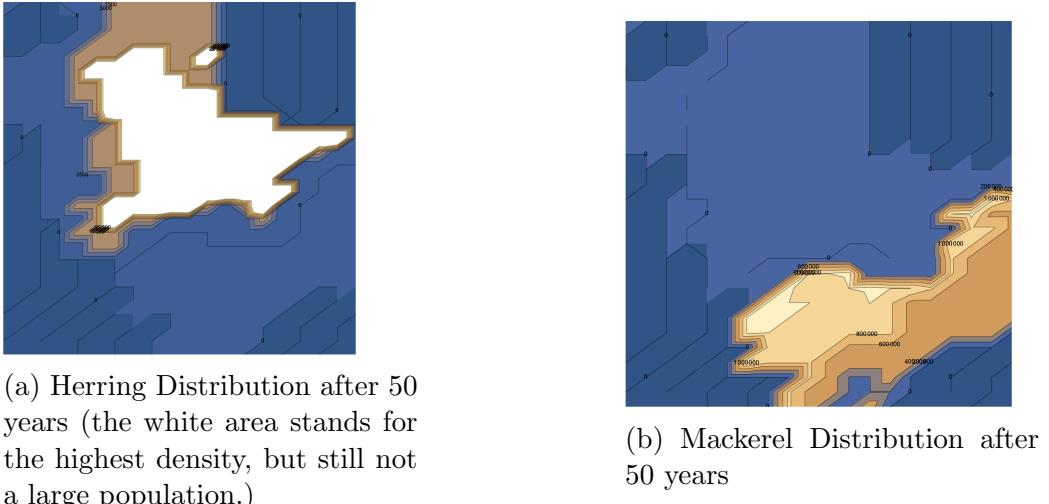


Figure 10: Population distribution after 50 years for annual temperature increment  $0.062^{\circ}\text{C}$

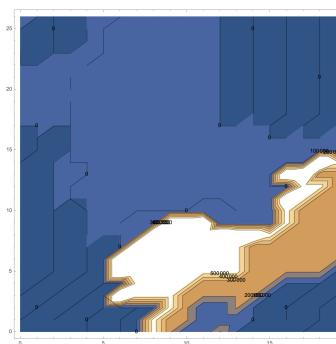


Figure 11: Mackerel Distribution after 65 years

## 7 Analysis and Conclusion

The population density map shows that as the climate changes, the population of herring moves farther from the Scottish coast and toward the south-eastern area of the North Sea. Although Mackerel has a better tolerance to warmer water, as the temperature rises, the water close to the shore soon becomes too warm for Mackerel to thrive. The majority moves to north west, which is also toward the center of the North Sea. We can still expect some

population of Mackerel from the shore until the temperature becomes totally unsuitable for Mackerel.

Under the best scenario, the North Sea stops getting warmer.  $\dot{T} = 0^{\circ}\text{C}/\text{yr}$ . In that case the fish distribution will remain unchanged, which means that the small fishing company will always be able to harvest.

Under the worst scenario where  $dT = 0.062^{\circ}\text{C}/\text{yr}$ , herring populations will decrease dramatically in the North Sea. The water soon becomes too warm for herring to reproduce and survive. After 50 years there is limited herring in the North Sea. Meanwhile Mackerel suffers a shrink of its habitat. As the temperature of ocean keeps rising, mackerel will suffer the same fate as herring. After 65 years, mackerel will have limited population in the North Sea. Therefore, the small fishing company will no longer be able to harvest the fish.

Based on the prediction made by our model, the fish population near the shore is going to decrease in the most likely and worst case scenario. Therefore, a short term suggestion for the small fishing companies in Scotland is to move south east where populations of herring and Mackerel are concentrating. In the long term, fish close to the shore are going to extinct while the only fish habitat is the center of North Sea. Our suggestion is to organize and unite the small fishing companies forming a fishing fleet. In the fleet some ships are in charge of catching while some are in charge of logistic and transportation. In this way a small catching can stay in the center of sea for the whole catching season while the transportation ships equipped with refrigerators take all the caught fish back to the land. There should be some ship in charge of providing the off shore ships with food, fuel, and medical supplies. Luckily, fishers do not need to worry about entering the territorial water of other countries. The Scottish Exclusive Economy Zone covers a very large portion of Central North Sea. Meanwhile Scotland is allowed to share the fisheries with other EU countries to harvest mackerel[13]. In the end, if the temperature change in the North Sea is not averted or at least slowed considerably, the fate of the life in these waters will remain in peril.

## 8 Strength and Weakness

In our model, we applied logistic growth to simulate population change. By splitting the sea into a grid, we were able to discuss the local population change, and thus create mackerel and herring population density maps. This

type of result is easy to analyze and constructive in further research and policy making.

However, the model is very simplified as we did not consider the changing ocean current, ecology of predators, and other factors of the changing ocean environment such as PH value, salt concentration, etc. We used the bottom temperature map in our simulation but we did not consider pressure and depth.

In simulating the worst case scenario, we assumed the annual increment of ocean temperature is twice the predicted value. In fact, the trend of global warming is not fully understood. It is potentially non-linear. Furthermore, the ocean is a heat reservoir that may react to climate change in a more complicated way. Our model can be tested as we compare our prediction with data taken from North Sea in the future.

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## 10 Magazine Article

### The Future of Herring and Mackerel

I work for a consulting firm that uses mathematical modeling to predict future trends in population data. My team was hired by a Scottish North Atlantic fishery management consortium to predict trends in mackerel and herring populations throughout Scottish fisheries. We wrote this article to report our findings and to give advice to small fishing companies throughout Scotland in the upcoming decades.

The most up to date data and models show that currently the oceans and seas of the Earth are warming at a rate of around 0.031 degrees Celsius per year. While this effect may not be noticeable on short time scales, it will become more and more apparent in only a few decades. Like humans every fish species has a biologically optimum temperature. Herring, for instance, are best suited to cold waters around 5 degrees Celsius. While they can survive in warmer and colder environments, their survivability diminishes. We found that a  $1\frac{1}{2}$  degree change in the temperature of the North Sea will have a considerable affect on herring stocks. While there's currently large populations of the fish close to the British coast, these coastal waters will soon become too warm for herring. Fishermen, especially from smaller fishing companies, should invest in refrigeration units for their vessels. This is because it will be necessary for crews to trek further out into the sea, and they need to keep their catch from going bad while making the journey to and from the shore.

Unlike herring, mackerel prefer warmer temperatures, around 12 degrees celsius. We found that while some waters will become less tolerable for the fish, others may actual become more suited to mackerel in the upcoming decades. Unfortunate for Scotsmen, however, is that most of these stocks will be far from the Scottish homeland. We found that particularly populous will be the area around the Dogger Bank and waters that within the territorial boundaries of The Netherlands. Mackerel near the Scottish shore will be negatively impacted by climate change - much like herring. In order to prosper in the era of climate change, Scottish fishing crews will need to refocus their efforts to either the far north, where herring stocks will be affected the least, or the far south, will mackerel stocks will begin to migrate.