

brr: Basic usage.

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The **brr** package performs Bayesian inference on the rate ratio $\phi = \frac{\lambda}{\mu}$ in the *two Poisson samples model* given by two independent observations

$$\begin{cases} x \sim \mathcal{P}(\lambda S) \\ y \sim \mathcal{P}(\mu T) \end{cases}$$

where λ and μ are the unknown incidence rates and S and T are the known observation-opportunity sizes, or, for short, the sample sizes. S and T are also called the times at risk when they represent some durations.

The **brr** package implements the *semi-conjugate family of prior distributions*. Precisely, for positive numbers a , b , c and d , to be set by the user, the following independent prior distributions are assigned on μ and $\phi = \lambda/\mu$:

$$\mu \sim \mathcal{G}(a, b) \quad \text{and} \quad \phi \sim \frac{T+b}{S} \times \mathcal{B}'(c, d),$$

Then the joint posterior on (μ, ϕ) is given by

$$(\mu \mid \phi, x, y) \sim \mathcal{G}(a + x + y, b + \phi S + T) \quad \text{and} \quad (\phi \mid x, y) \sim \frac{T+b}{S} \times \mathcal{B}'(c + x, a + d + y).$$

In particular :

- when $a = c = 0.5$ and $b = d = 0$, the prior is the reference prior, also called the *non-informative prior*;
- when $a, b > 0$, $c = 0.5$ and $d = 0$, the prior is the semi-reference prior, also called the *semi-informative prior*, that is to say the reference prior after the arbitrary Gamma prior distribution $\mathcal{G}(a, b)$ is assigned on μ .

Setting parameters with **brr**

Use the **Brr** function to set the prior parameters, the sample sizes, and the observed counts. One can proceed step by step, for example below we start by supplying the parameters a and b of the prior Gamma distribution on μ :

```
library(brr)
model <- Brr(a=2, b=3)
summary(model)
```

```
## Type of prior distribution: semi-informative prior
##
## *Prior distribution on  $\mu$ *: Gamma(a=2,b=3)
##
## +-----+-----+-----+-----+-----+-----+
## | mode | mean | sd | Q1 | Q2 | Q3 |
## +=====+=====+=====+=====+=====+=====+
## | 0.3333 | 0.6667 | 0.4714 | 0.3204 | 0.5594 | 0.8975 |
## +-----+-----+-----+-----+-----+-----+
##
## *Prior distribution on  $\phi$ *: Non-informative prior
##
## *Sample sizes*
##   S (treated group): not supplied yet
##   T (control group): not supplied yet
##
## *Observed counts*
##   x (treated group): not supplied yet
##   y (control group): not supplied yet
##
## *Posterior distribution on  $\phi$ *:
##   a, b, c, d, S, T, x and y must be supplied
```

Since c and d were not supplied, **brr** automatically considers the non-informative prior on ϕ . Equivalently the same **brr** object can be defined by **Brr(a=2, b=3, c=NULL, d=NULL)** or **Brr(a=2, b=3, c=0.5, d=0)**.

The **brr** object is a function which can be used to update itself with new parameters, for example:

```
model <- model(c=3, d=3, S=100, T=100)
summary(model)
```

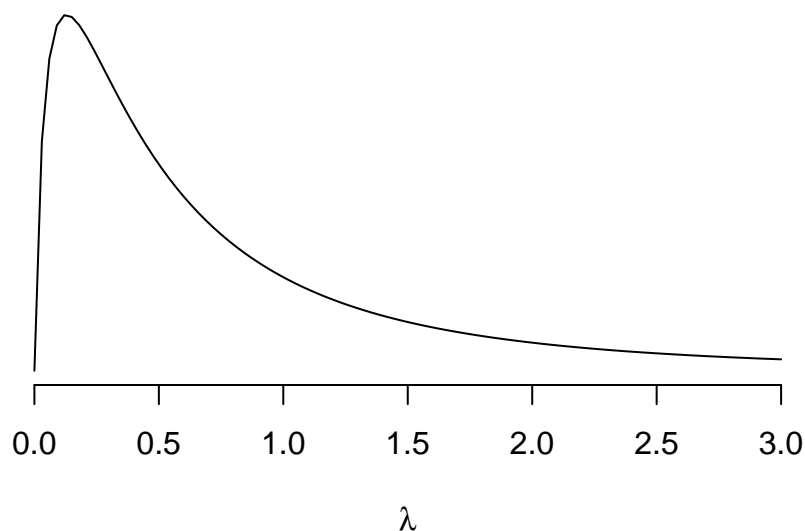
```
## Type of prior distribution: informative prior
##
## *Prior distribution on  $\mu$ *: Gamma(a=2,b=3)
##
## +-----+-----+-----+-----+-----+-----+
## | mode | mean | sd | Q1 | Q2 | Q3 |
## +=====+=====+=====+=====+=====+=====+
## | 0.3333 | 0.6667 | 0.4714 | 0.3204 | 0.5594 | 0.8975 |
## +-----+-----+-----+-----+-----+-----+
##
## *Prior distribution on  $\phi$ *: Beta2(c=3,d=3,scale=1.03)
##
## +-----+-----+-----+-----+-----+-----+
```

```
## | mode | mean | sd | Q1 | Q2 | Q3 |
## +-----+-----+-----+-----+-----+-----+
## | 0.515 | 1.545 | 1.995 | 0.578 | 1.03 | 1.836 |
## +-----+-----+-----+-----+-----+-----+
##
## *Sample sizes*
##   S (treated group): 100
##   T (control group): 100
##
## *Observed counts*
##   x (treated group): not supplied yet
##   y (control group): not supplied yet
##
## *Posterior distribution on  $\phi$ *:
##   a, b, c, d, S, T, x and y must be supplied
```

Now that a, b, c, d, S and T have been supplied, the user can play with all functions related to the prior distributions.

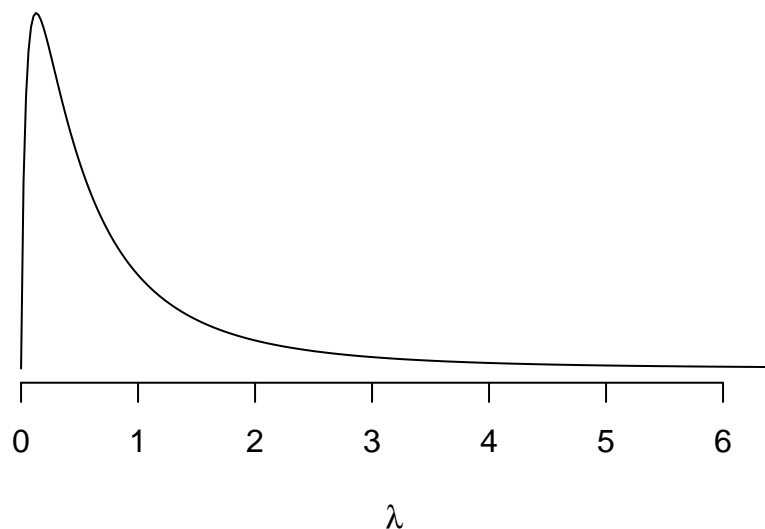
For example, `dprior(model, "lambda", .)` evaluates the density of the prior distribution of λ . This is convenient to generate a graphic with the `curve` function:

```
par(mar=c(4, 3, 1, 1))
curve(dprior(model, "lambda", x), from=0, to=3, axes=FALSE,
      xlab=expression(lambda), ylab=NA)
axis(1)
```



The `brr` package also provides a way to generate a plot with automatic aesthetics:

```
par(mar=c(4, 3, 1, 1))
plot(model, dprior(lambda))
```



If you are not pleased with the automatic bounds of the interval over which the curve is plotted, set your bounds in the **bounds** argument of the **plot** function. You can also set usual graphical parameters such as **lwd**, **lty**, ...

Posterior inference with **brr**

Posterior inference is available after, obviously, setting the observed counts x and y :

```
model <- model(x=14, y=20)
summary(model)
```

```
## Type of prior distribution: informative prior
##
## *Prior distribution on  $\mu$ *:  Gamma(a=2,b=3)
##
## +-----+-----+-----+-----+-----+-----+
## | mode   | mean   | sd     | Q1    | Q2    | Q3    |
## +=====+=====+=====+=====+=====+=====+
## | 0.3333 | 0.6667 | 0.4714 | 0.3204 | 0.5594 | 0.8975 |
## +-----+-----+-----+-----+-----+-----+
##
## *Prior distribution on  $\phi$ *:  Beta2(c=3,d=3,scale=1.03)
##
## +-----+-----+-----+-----+-----+-----+
## | mode   | mean   | sd     | Q1    | Q2    | Q3    |
## +=====+=====+=====+=====+=====+=====+
## | 0.515   | 1.545  | 1.995  | 0.578  | 1.03   | 1.836  |
## +-----+-----+-----+-----+-----+-----+
##
```

```
## *Sample sizes*
##   S (treated group): 100
##   T (control group): 100
##
## *Observed counts*
##   x (treated group): 14
##   y (control group): 20
##
## *Posterior distribution on  $\phi$ *: Beta2(17,25,scale=1.03)
##
## +-----+-----+-----+-----+-----+-----+
## | mode   | mean   | sd     | Q1     | Q2     | Q3     |
## +=====+=====+=====+=====+=====+=====+
## | 0.6338 | 0.7296 | 0.2363 | 0.5613 | 0.696  | 0.8605 |
## +-----+-----+-----+-----+-----+-----+
##
## Pr('relative risk is greater than 1') = 0.876132340775555
```

Estimates are provided by the **coef** function:

```
coef(model)
```

```
## Estimates of  $\phi$ 
##
## mode : 0.6338462
## mean : 0.7295833
## median : 0.6959761
## intrinsic : 0.6966658
## intrinsic2 : 0.7033363
```

Posterior credibility intervals are provided by the **confint** function:

```
confint(model)
```

```
## 95%-credibility intervals about  $\phi$ 
##
## +-----+-----+-----+
## | interval | lwr   | upr   |
## +=====+=====+=====+
## | equi-tailed | 0.3679 | 1.284 |
## +-----+-----+-----+
## | equi-tailed* | 0.3679 | 1.284 |
## +-----+-----+-----+
```

```
## |      hpd      | 0.3232 | 1.202 |
## +-----+-----+-----+
## | intrinsic    | 0.3624 | 1.268 |
## +-----+-----+-----+
## | intrinsic2    | 0.3522 | 1.244 |
## +-----+-----+-----+
```

Predictions are provided by the **predict** function after adding the sample sizes of the future experiment:

```
model <- model(Snew=500, Tnew=500)
predict(model)
```

```
## Predictions and 95%-credibility prediction intervals
##
## +-----+-----+-----+-----+-----+
## | obs   | size | median | lwr  | upr   |
## +=====+=====+=====+=====+=====+
## | xnew  | 500  | 71     | 38   | 117   |
## +-----+-----+-----+-----+-----+
## | ynew  | 500  | 102    | 62   | 156   |
## +-----+-----+-----+-----+-----+
```