Gravitational Radiation in Extra Dimensions

Alexander Migala,* Vincent Lee,†* Lee Lindblom,* Floor S. Broekgaarden,* George M. Fuller*

*University of California, San Diego
†University of California, Berkeley

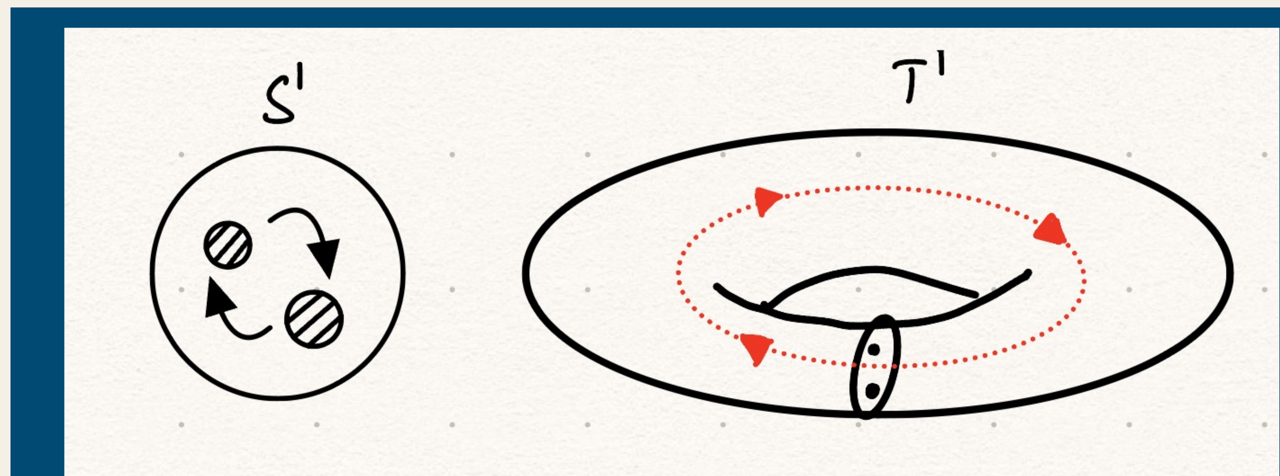
Abstract

Upcoming gravitational wave interferometry experiments such as LISA and Cosmic Explorer will allow for more opportunities to test theories of gravity. Compact object merger rates, which are the rates at which pairs of compact objects collide as a result of a gravitationally-bound orbit, are integral to our understanding of the evolution of the universe. When computing such rates, a key ingredient is the rate at which gravitational radiation emanates away from the orbit, as this determines how long the compact objects stay in mutual orbit. In Einstein's General Relativity, this is governed primarily by the calculation of a weak gravitational field that follows a $1/r$ law in the radiation zone. However, in alternative theories to gravitation, this is not always the case. In this work, we explore how alternative theories of gravitation may result in corrections to compact object merger rates and emitted radiation. This work develops a framework for analyzing how modified theories of gravity interact with the merger rate calculation. We demonstrate that a large correction from modified gravity would be necessary in order to make an appreciable change in merger rate.

Binary System and Gravitational Radiation

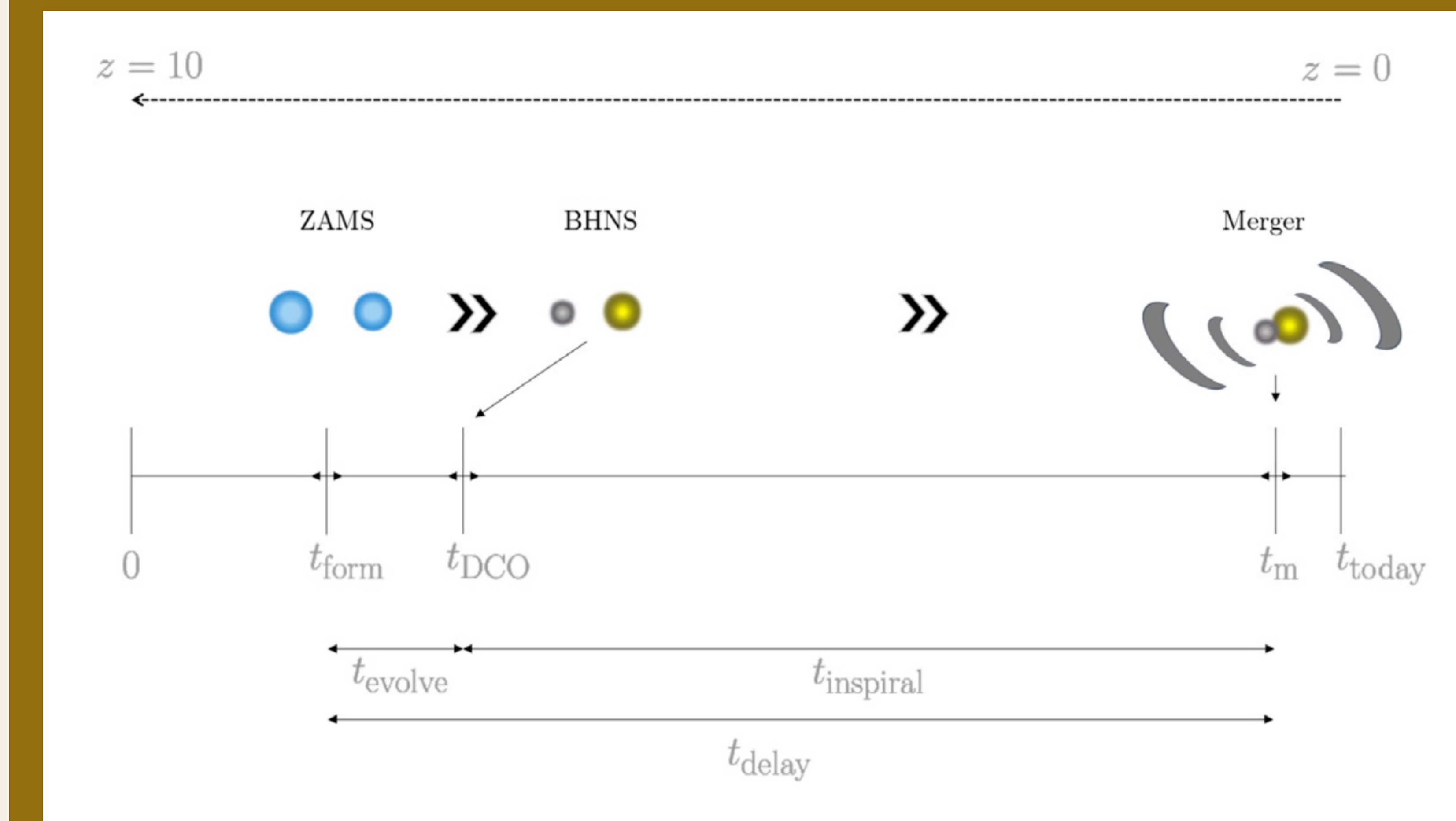
In the weak gravitational limit, a binary system bound in mutual orbit will lose energy at a rate that depends on the masses of the bodies, the eccentricity of the orbit (e), and the semimajor axis (a). This was originally derived by Peters [3,4] using Einstein gravity, and is true to leading order in the weak gravitational field

$$\left\langle \frac{dE}{dt} \right\rangle = -\frac{32}{5} \frac{\left(G^4 m_1^2 m_2^2 (m_1 + m_2)\right)}{c^5 a^5 (1 - e^2)^{\frac{7}{2}}} f(e)$$



From intuition, one might like to know whether extra dimensions enhance this flux or suppress this flux. Consider a binary system that is confined to live on a circle (S^1) and emanating radiation. Say that this world is attached to a small (compared to the radius of the binary) extra dimension where *only gravitons* can leak into this dimension. If this extra dimension is a circle, then the complete geometry is a torus. When radiation leaks into the extra dimension, it swings around and enhances the power; if it is not compact, it is suppressed.

Methods: We developed a framework for exploring flux corrections to Einstein gravity



A generic merger event (Figure from [1] with permission). Two stars are formed at ZAMS (Zero Age Main Sequence) at a source time t_f, then the stars evolve until at least one of them forms a compact object (ie Neutron Star and Black hole pair as shown above). Then, the binary system inspirals (with time coordinate t_c) while energy emanates away per Einstein gravity. The time between when the stars form and when they merge is t_d

Consider a generic correction to the flux, where we introduce a "power parameter," δ_P:

$$\left\langle \frac{dE}{dt} \right\rangle' = \delta_P \left\langle \frac{dE}{dt} \right\rangle$$

The merger rate calculation is as follows [1]:

$$R_m(t_m) = \int dZ \int dt_d \, S(Z, t_f) \frac{d^2 N_f(Z, t_d)}{dM_{SFR} dt_d}$$

Where S is a function that describes the distribution of metallicity (Z) across cosmic time. We consider how this calculation changes under the power-parameter description. Generally, this is:

$$R'_m(t_m, t_c)$$

$$= \int dZ \int_0^{t_m} dt'_d \, (\delta_P)^2 S\left(Z, t_m - \left(t'_d + t_c\left(1 - \frac{1}{\delta_P}\right)\right)\right) \frac{d^2 N_f\left(Z, t'_d + t_c\left(1 - \frac{1}{\delta_P}\right)\right)}{dM_{SFR} dt'_d}$$

If the correction is small ($\delta_P \approx 1$) we may expand $\delta_P = 1 + \epsilon,\ \epsilon \ll 1$ and find at leading order:

$$R'_m(t_m) \approx R_m(t_m) + 2\epsilon \int dZ \int_{t_m}^{t_m(1+\epsilon)} dx \, S(Z, t_m - x) \frac{d^2 N_f(Z, x)}{dM_{SFR} x} + \cdots$$

Where $x = \delta_P t'_d$

Results: Finding the Correction

Randall-Sundrum [3] (RS) Brane World Gravity

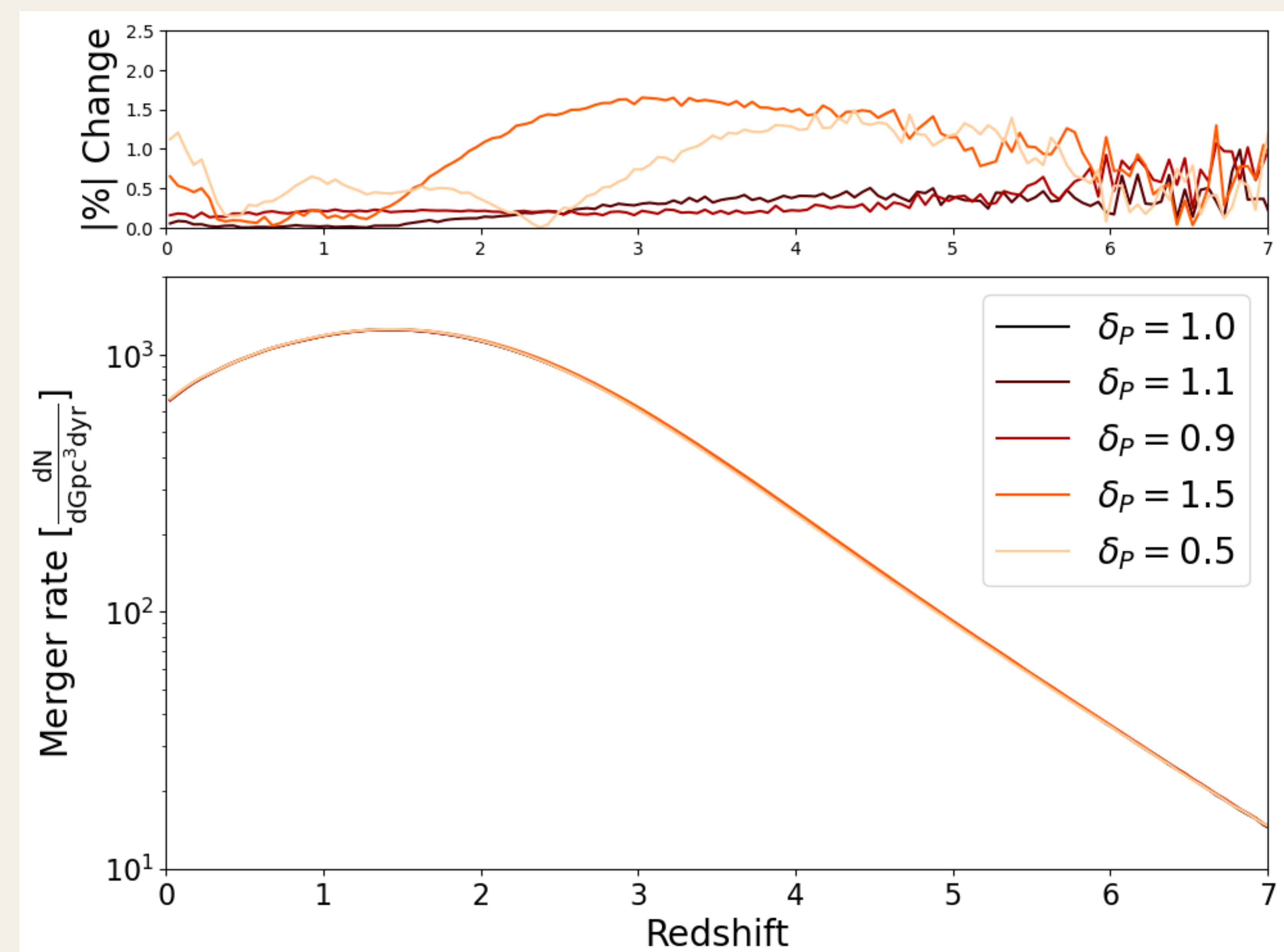
In RS gravity [3], we are theorized to live on a "brane" that is described by a 4-dimensional manifold (1 time, three spatial) as in the case of Einstein gravity; however, the graviton is allowed to propagate into extra dimensions that are assumed to be compact and small. In this picture, we find that the power correction, based on work from [6], is found to be

$$\left|\frac{L_{KK}}{L_{zero}}\right| \sim \frac{l^2 cT}{GMR^2} \rightarrow \delta_P \sim 1 + \frac{l^2 cT}{GMR^2}$$

Where l, T, M, R are the anti-deSitter curvature, characteristic time, total mass, and radius of the source. We find that this expansion is best described by the epsilon expansion approximation done above

Numerical Analysis: COMPAS

We consider a more general case where the power parameter may be tuned generically and observe the numerical results. To do so, we modified a population synthesis code, COMPAS [7, 8], to simulate populations of double compact object progenitors. Given a value of the power parameter, COMPAS also determined whether these remnants merged. From the outcome of this simulation, and after making an assumption on S, we can compare the merger rate numerically for different power parameter values



Conclusions

From these results, we see that compact extra dimensions do not produce appreciable corrections to the merger rate. However, the above results were both cases where the power parameter was independent of the source frame time. To find an appreciable correction, one might look for cosmological models where the size of the extra dimension scales with cosmic time, as in the case of Ref. [2]

[1] F. S. Broekgaarden *et al.*, "Impact of massive binary star...," Dec. 2021, doi: 10.1093/mnras/stab2716.
[2] N. Mohammedi, "Dynamical compactification...," May 2002, doi: 10.1103/physrevd.65.104018.
[3] L. Randall and R. Sundrum, "An Alternative to Compactification," Dec. 1999, doi: 10.1103/physrevlett.83.4690.
[4] P. C. Peters, "Gravitational Radiation and the Motion...," Nov. 1964, doi: 10.1103/PhysRev.136.B1224.
[5] P. C. Peters and J. Mathews, "Gravitational Radiation from...," July 1963, doi: 10.1103/PhysRev.131.435.
[6] S. Kinoshita *et al.*, "Quadrupole formula for...," Sept. 2005, doi: 10.1088/0264-9381/22/19/007.
[7]Team COMPAS *et al.*, "...COMPAS: Methods Paper II," Sept. 2025, doi: 10.3847/1538-4365/adf8d0.
[8] Team COMPAS *et al.*, "...COMPAS," Feb. 2022, doi: 10.3847/1538-4365/ac416c.
[9] Daniel Baumann and Liam Mcallister, *Inflation and String Theory*. 2015.