PHY493 F22 Summary

Alexander Migala with Dr. K. Scholberg as Mentor

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Abstract

In this analysis of the Neutrino Mass Hierarchy Problem, we use ternary plots to visualize various supernova events from the SNEWPY software package. While this project is currently unable to provide a robust discrimination between the Normal and Inverted mass orderings of neutrinos, we document our current methodology, findings, and next steps. The code used in our analysis may be found at `https://github.com/stlgolfer/snewpyternary`.

Contents

1 Guiding Research Question

The project currently aims to answer the following question:

Is there a statistical difference between the NMO and IMO orderings of neutrino masses? If so, is this difference detectable on Earth?

In its current form, the project has not yielded a robust model for discriminating between the Normal and Inverted orderings, although we have noted some notable correlation, as will be discussed later.

2 Summary of Relevant Physics

Here we summarize relevant physics concepts that are used throughout this project and explain why some concepts were implemented over others.

2.1 The Neutrino Mass Hierarchy Problem

In our current model for neutrinos, we have determined three weak eigenstates: ν_e, ν_τ, and ν_μ and three mass eigenstates: m_1, m_2, and m_3 [1]. What is not currently understood is whether

$$m_1 < m_2 < m_3$$

referred to as the **Normal Hierarchy/Ordering**, or

$$m_3 < m_1 < m_2$$

referred to as the **Inverted Hierarchy/Ordering**
is true. Formally, the sign of the following is currently not known [1]:

$$\Delta M^2 \coloneqq {m_3}^2 - \frac{m_2^2 + m_1^2}{2}$$

2.2 Parameter Selection

Fortunately, our current models for describing the survival probability of a neutrino traveling to a detector on Earth is well understood. However, there are still several factors to consider, such as flavor oscillation[1], the conditions surrounding its creation, and mixing angles–as described by the PMNS[2] mixing matrix. For our analysis, we consider only the two transformation prescriptions: AdiabaticMSW IMO and AdiabaticMSW NMO. While we could have chosen other

[1]Oscillation refers to a neutrino starting with one flavor at the start of its journey and changing in flavor as it travels
[2]Pontecorvo-Maki-Nakagawa-Sakata

prescriptions such as Inverse beta decay, we opted for the adiabatic initial condition, as this most accurately encompasses the physics surrounding the creation of neutrinos at a supernova progenitor.

3 Methodology

3.1 Nomenclature

Because our analysis was inventive in nature, we needed a way to denote different parts of the algorithm and the ternary plots themselves:

- Model[3]. A model can refer to two things: an overall SNEWPY model (such as one that is listed in the next section) or a model within a model. Nearly all of the SNEWPY models contain submodels that were all made by the same author. The parameters that these submodels vary are contingent on the modeler and what the original study was trying to accomplish [3].
- Author/Modeler. These terms describe who created the model and its submodels. Some of these are listed in the next section.
- Track. As each time bin is created in the ternary space, it forms a line with the previous bins. This line is called a "track." A single ternary plot can contain several tracks. A track is typically shown as a color and it varies in brightness according to the bin's time range (discussed further in the next section).

3.2 Assumptions

Throughout our analysis, we operated under a few key assumptions:

1. Because of our current state of particle detectors, accurately measuring the individual event rates for a given neutrino flavor on Earth is extremely difficult. Instead, it is much easier to reorganize them as ν_e, $\overline{\nu_e}$, and ν_x. ν_x is the combined event rates/events for the τ, μ, and their antiparticles[4].
2. The distance from Earth to the progenitor source is 10 kPc.
3. We organize the measured event rates under assumptions described by a "Proxy Configuration," which is described later.

[3] In our code, this is sometimes referred to as a "setno," which is not correct. We are aware that the code and our nomenclature do not currently line up, but we are working to correct this

[4] An in-depth discussion as to why these specific particles are combined as the ν_x amalgamated particle is discussed in this summary. However, we assume that the physics of these particles are similar enough that we can represent them as being the same

4. In our current implementation, no two models will share the same time bin size (even in log), as each model has a different time window. Because a method for standardizing the time bin size across models has yet to be found, we currently choose a bin size that yields a plot with reasonable error[5]

3.3 Process

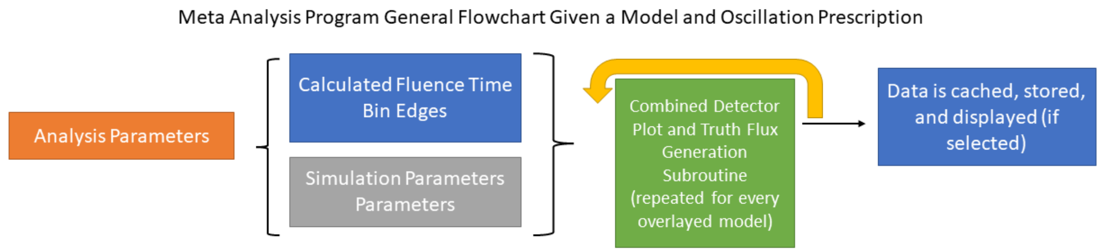
Because we wished to base our analysis around supernovae, we elected to make use of the SNEWPY [3] and SNOwGLoBES Python software packages [2]. SNEWPY provided us with software wrappers for several supernovae models, and SNOwGLoBES simulated the event rates and fluxes. However, not every model was relevant to our analyses. Of the 15 models that SNEWPY provides, we currently support 10 models:

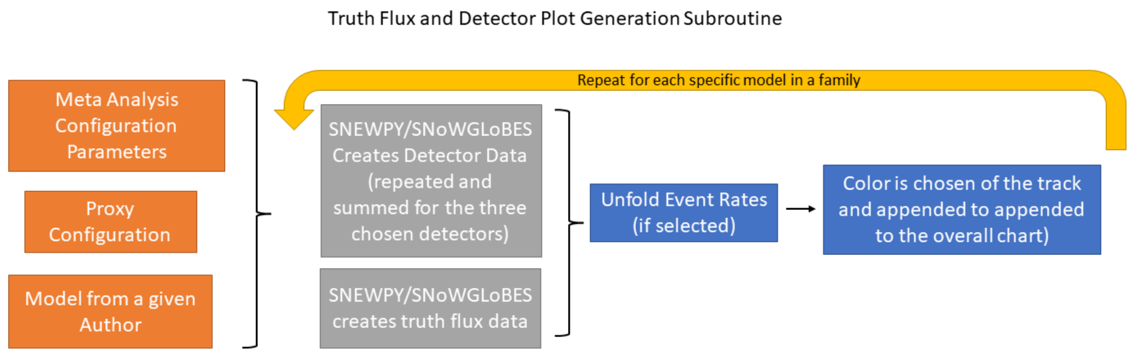
1. Bollig 2016
2. Fornax 2021 (though our current implementation has some known bugs)
3. Kuroda 2020
4. Nakazato 2013
5. Sukhbold 2015
6. Tamborra 2014
7. Walk 2018
8. Walk 2019
9. Warren 2020
10. Zha 2021

For these models, our overarching analysis process is to compute the truth flux and detector ternary plots, given a model and an oscillation prescription. For a truth flux, each point in ternary space represents a "fluence" (as defined by [2], and has units of neutrinos per unit area. For a detector plot, the computed event rates across all detectors are calculated using a similar process; however, the units here are in terms of reaction rates. In either case, the ratio between the three groups of neutrinos is taken and added to the ternary diagram. For each model within a family, we use different colors. We currently support three models per family on a single detector ternary plot. Because each time bin corresponds to a time range, we color each point in the ternary space according to its time relative to the model's time range. That is, time bins closer to the model's start will be darker; as bins move away from the model's start time,

[5]For example, a time bin with neutrino counts less than one would not give reasonable detector statistics

the bins become brighter. This process is linear and scaled to the model's time range. To help illustrate the overall process, we include the following diagrams:





3.4 Analysis Parameters

In the early stages of our analysis, we found that we needed a way to vary several of analyses' parameters. To aid this, we designed our code as a Command Line Interface (CLI). This means that our analysis code is mostly generic, and an in-depth analysis can be done with a handful of parameters, such as:

```
python meta_analysis.py Tamborra_2014 -p AdiabaticMSW_NMO...

...-p AdiabaticMSW_IMO --detproxy=BstChnl
```

A description of the exposed parameters are shown below:

```
Usage: meta_analysis.py [OPTIONS] MODELS...

Options:
  --showc BOOLEAN     Whether to show generated plots or not. Will always save
                      and cache
  -p TEXT             Prescriptions to use
  --distance INTEGER  The distance (in kPc) to the progenitor source
  --uselog BOOLEAN    Use logarithmically spaced (and shifted) time bins
  --setno INTEGER     Model set index. See model_wrappers.py
  --cache BOOLEAN     If true, use cache
  --presn BOOLEAN     If true, compute time bins from t<=0
  --tflux BOOLEAN     If true, only calculate the truth flux. set numbers are
                      not superimposed
  --detproxy TEXT     Detector proxy configuration. Options: AgDet or BstChnl
  --help              Show this message and exit.
```

3.5 Logarithmically Sized Time Bins/Fluences

Throughout our analysis, we use logarithmically sized time bins for each fluence calculated. This is not the same as simply taking the log of the time axis of a given model, and calculating the log bin edges from that; the reason this was avoided was because many of the models have important information right before core bounce (t=0), and would otherwise not be able to be log'd. To account for this, we instead shift the time axis of the original model such that all data occurs after t=0 and compute logarithmically sized bins from this.

3.6 Proxy Configurations

As was introduced in the "Assumptions" section, it is very difficult for us to measure exact rates of individual neutrinos here on Earth. This is why many detectors on Earth measure the rate using the rates from a chemical reaction with a material. For our analysis, we measured the rates from a 100kt water Cherenkov detector, a 40kt liquid argon detector, and a 20kt scintillator. The chemical reactions specifically are noted in [2]. This means that we were free to choose a configuration of these detector channels that reasonably estimated the event rates on Earth. We first tried an "Aggregate Detector" configuration, which was chosen based on minimizing each channels' detector error. However, we were only able to deduce relational conclusions. This led us to trying a "Best Channel" configuration, which only selected the channels with the least statistical error. Though we were unable to deduce a robust model, we were able to identify a few notable patterns; these patterns are discussed later in 4.1. See below for the specific proxy configurations[6]:

Aggregate Channel Proxy Configuration

Detector	ν_x Proxy	ν_e Proxy	$\overline{\nu_e}$ Proxy
Scint	NC	$\nu_{e,C_{12}} + \nu_{e,C_{13}} + e^-$	IBD
Argon	NC	$\nu_{e,Ar40} + e^-$	$\overline{\nu}_{e,Ar40}$
Water	NC	$\nu_{e,O16} + e^-$	IBD

Best Channel Proxy Configuration

Detector	ν_x Proxy	ν_e Proxy	$\overline{\nu_e}$ Proxy
Scint	NC	-	-
Argon	-	$\nu_{e,Ar40} + e^-$	-
Water	-	-	IBD

4 Current Findings

Though we cannot yet robustly discriminate between the two orderings, we have thus far identified two interesting patterns.

4.1 90% Coincidence

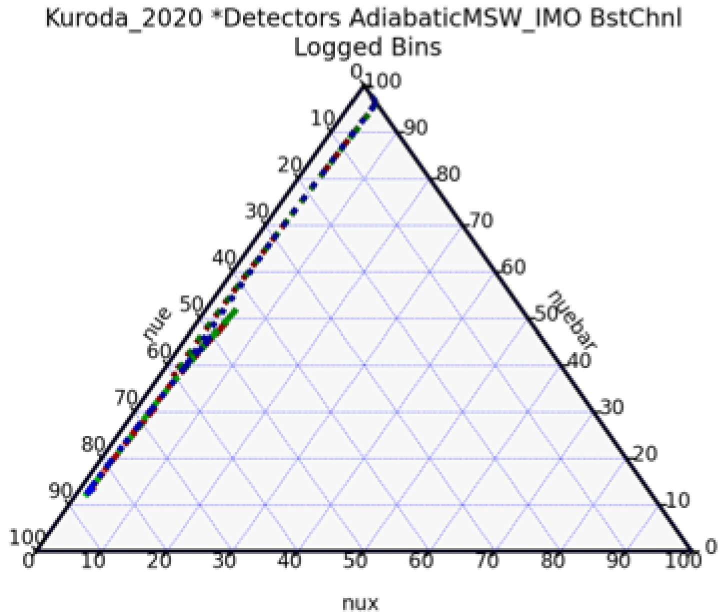
When we consider the Best Channel proxy configuration across several models (shown below), we observe that the NMO ternary plots apparently trace the 90%

[6]Note that IBD refers to Inverse Beta Decay and that NC refers to Neutral Current

line of the $\overline{\nu_e}$ ratio towards the beginning of its evolution before rapidly deviating towards the other two flavors[7]. Unfortunately, however, this observation is relevant for the Kuroda 2020, select Nakazato 2013, Sukhbold 2015, and Zha 2021 plots. For the Tamborra 2014, Walk 2018, and Walk 2019 models (data from simulations), this is less apparent, though the NMO plots tend to start more towards the $\overline{\nu_e}$ ratio and bend towards the other two flavors. Of course, these observations are relational and therefore not robust. Below are select models to illustrate:

[7]Upon inspection of the time-domain energy spectrum of these ternary plots, it was confirmed that this 90% apparent asymptote correlated with the core bounce event of the supernova

Kuroda 2020


Kuroda_2020 *Detectors AdiabaticMSW_IMO BstChnl
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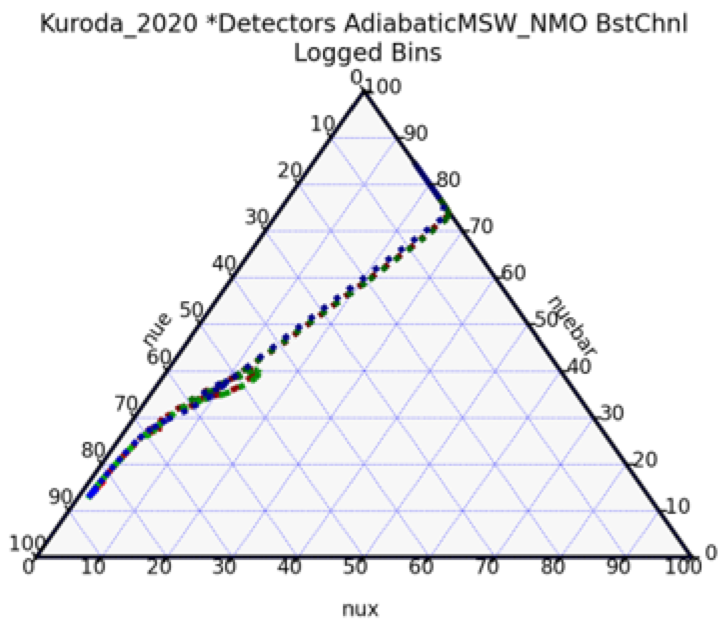
Kuroda_2020 *Detectors AdiabaticMSW_NMO BstChnl
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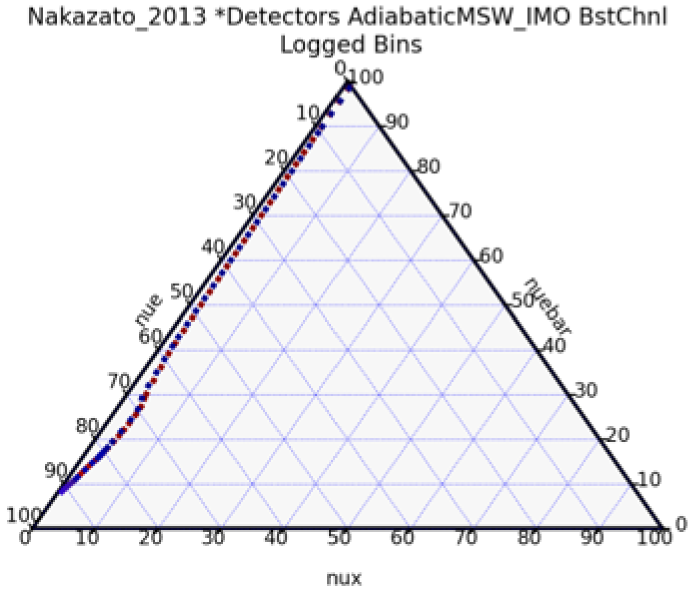
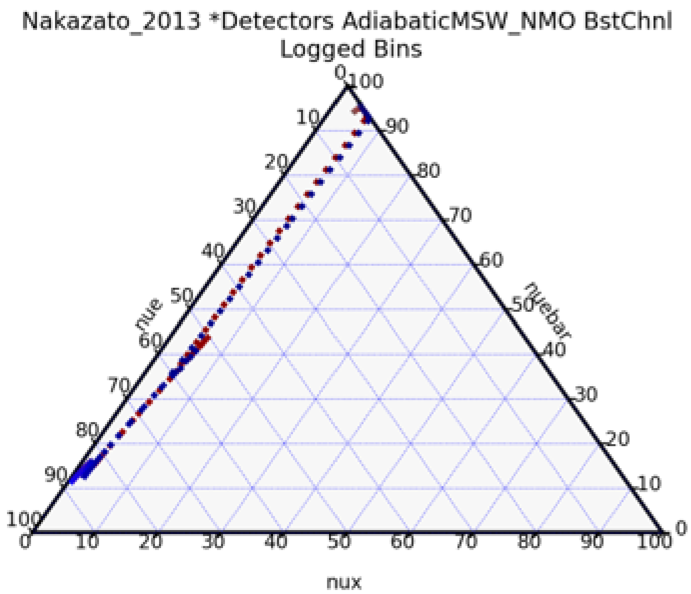
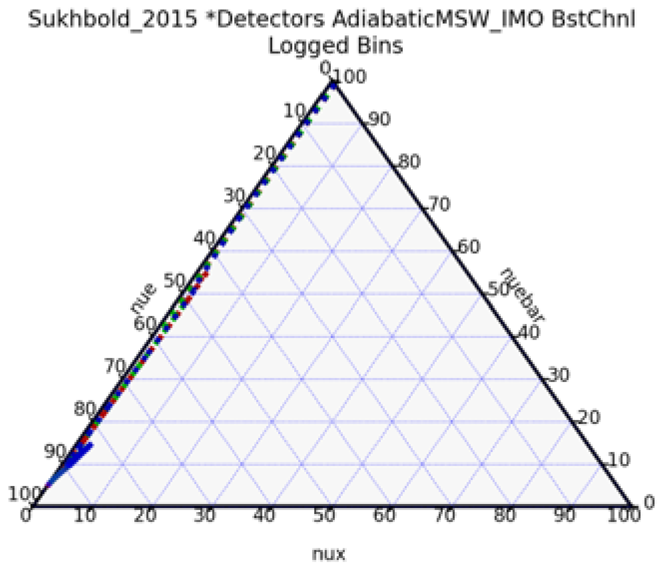
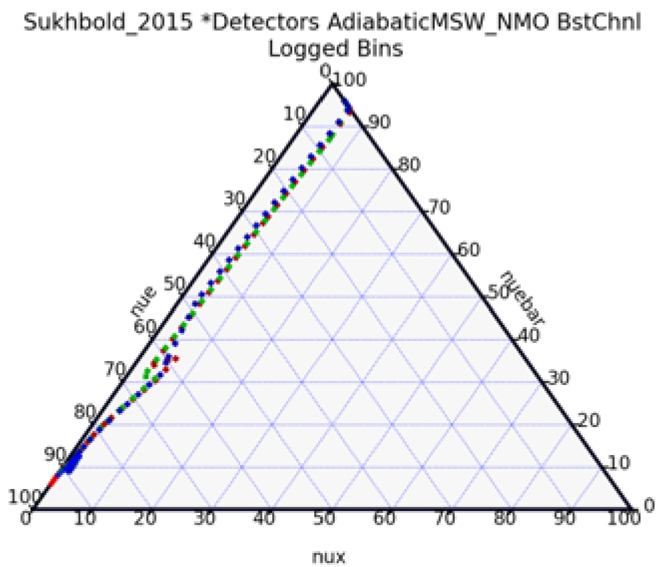
Nakazato 2013



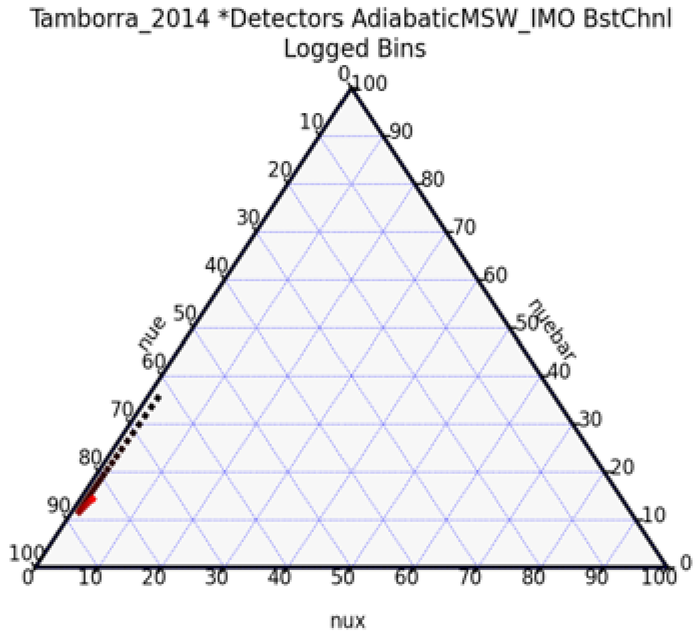


Sukhbold 2015





Tamborra 2014



Tamborra_2014 *Detectors AdiabaticMSW_IMO BstChnl
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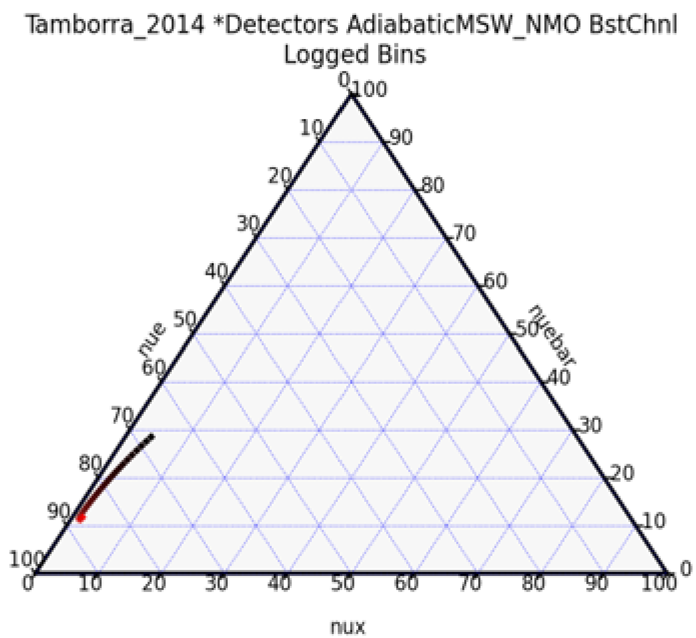

Tamborra_2014 *Detectors AdiabaticMSW_NMO BstChnl
Logged Bins
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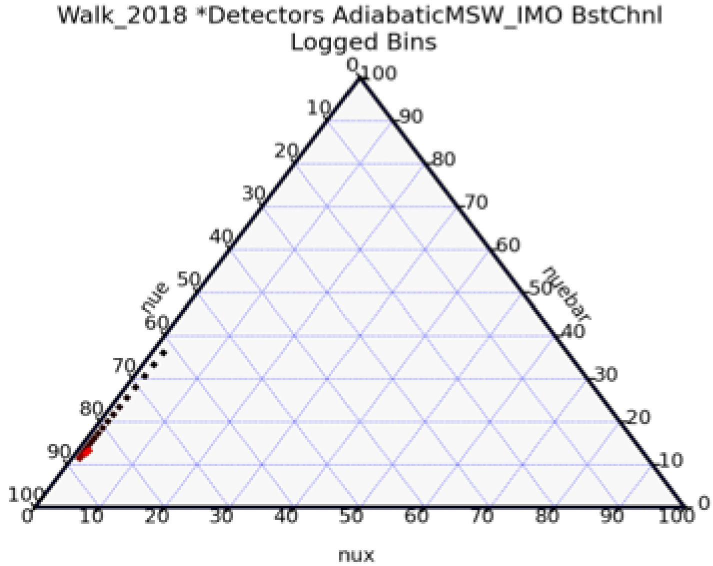
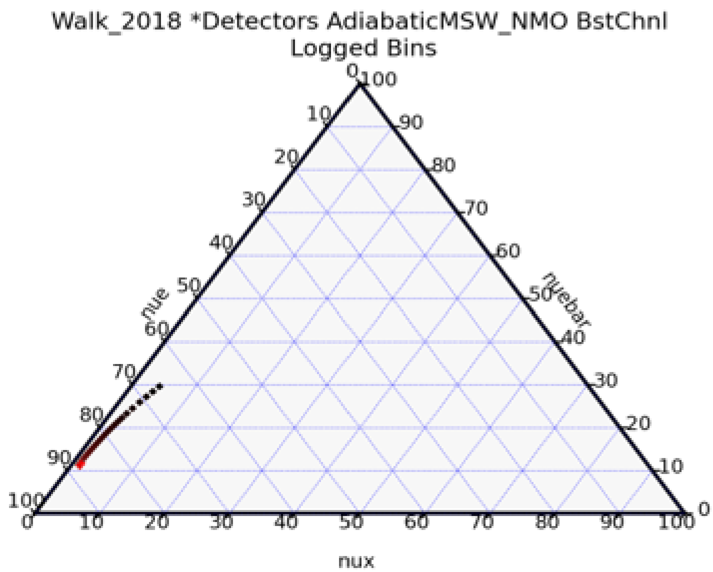
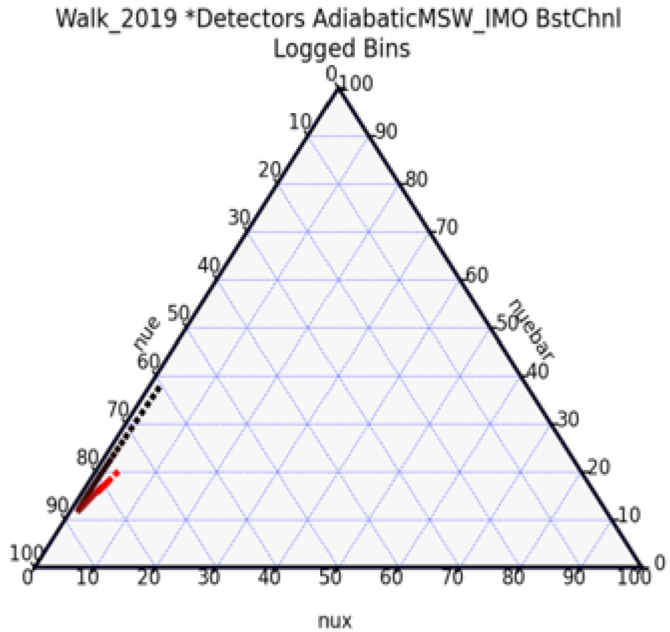
Walk 2018





Walk 2019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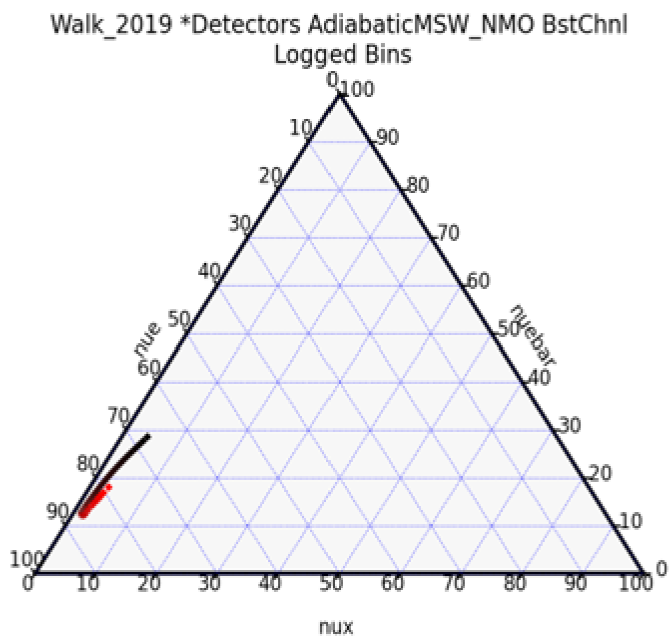
Walk_2019 *Detectors AdiabaticMSW_NMO BstChnl
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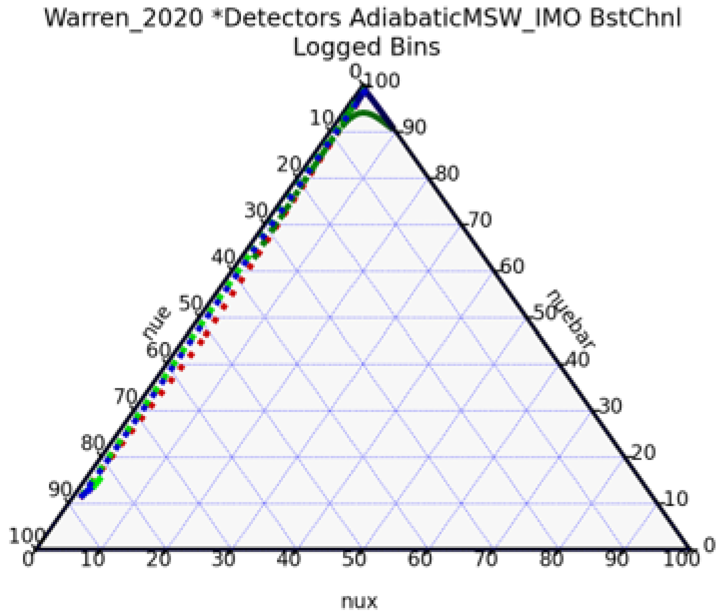
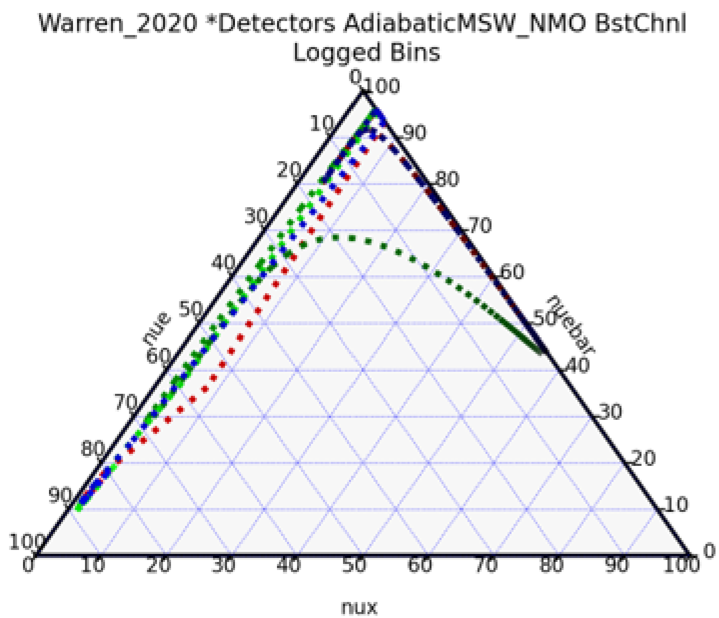
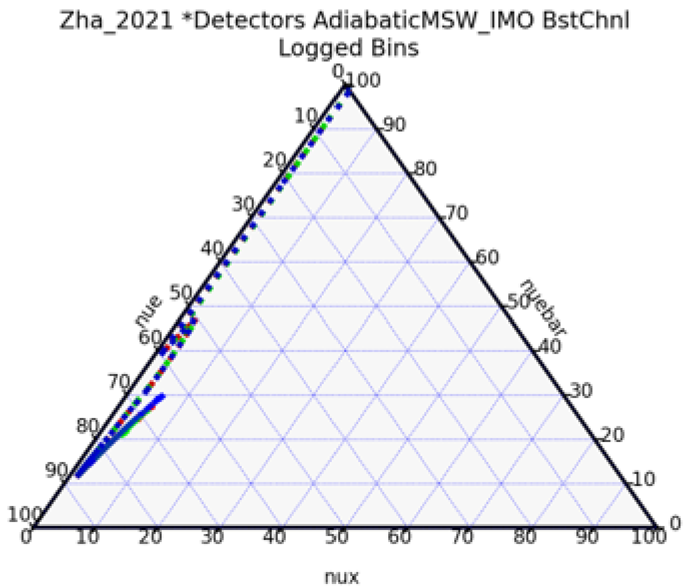
Warren 2020





Zha 2021


Zha_2021 *Detectors AdiabaticMSW_IMO BstChnl
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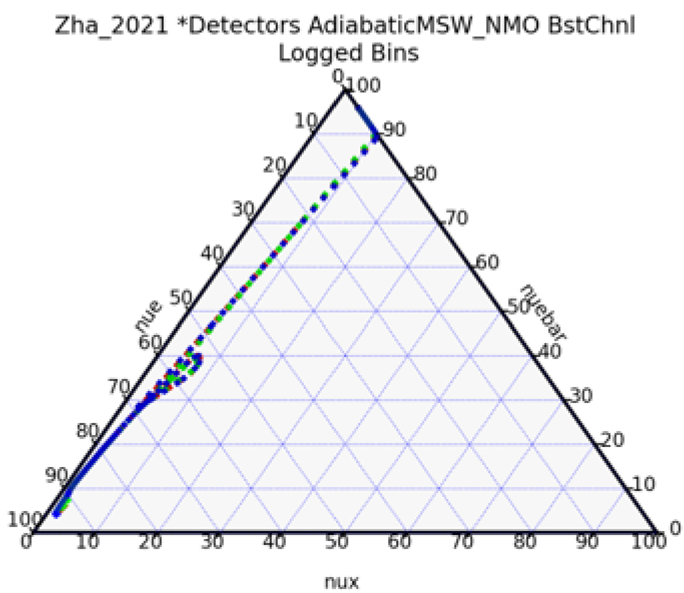
Zha_2021 *Detectors AdiabaticMSW_NMO BstChnl
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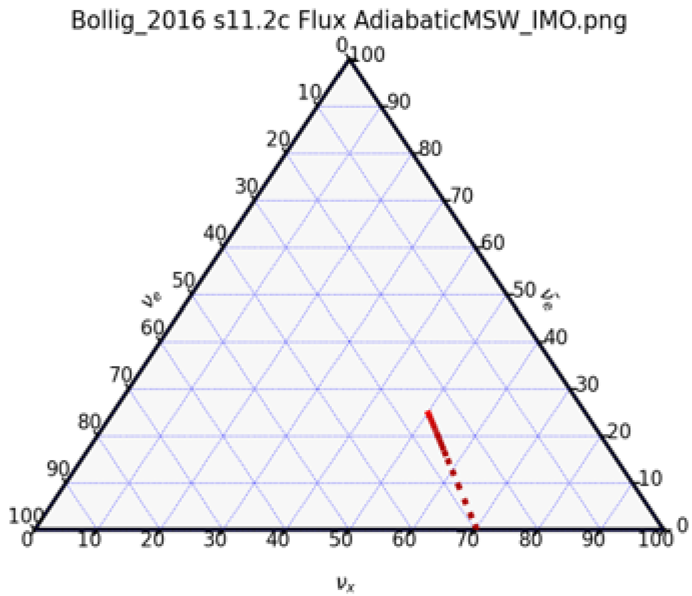
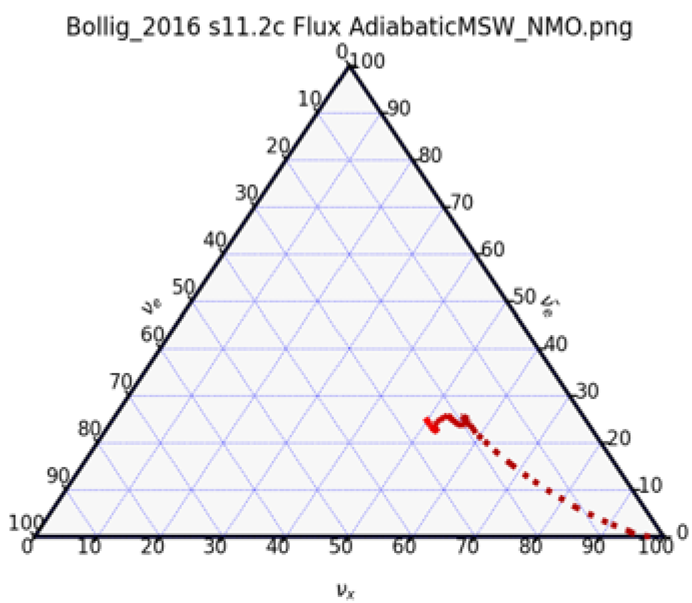
4.2 IMO Tell-Tale

Fortunately, there was an interesting observation made when analyzing the truth fluxes of several models: the IMO of every model analyzed occupied almost the same space on the ternary plot. Specifically, the IMO ordering tracks started at approximately $\nu_x = 70\%$, and worked their way up with the same slope towards approximately $(\overline{\nu_e} = 50\%, \nu_e = 75\%)$. Because the NMO track appeared significantly different than the IMO track, we concluded that it should be possible for us to uniquely determine an inverted ordering using detectors on Earth; the caveat being that we would need to understand a sufficient number of the parameters[8] involved in determining the convolved event rate observed on Earth. To illustrate this, select models are shown below:

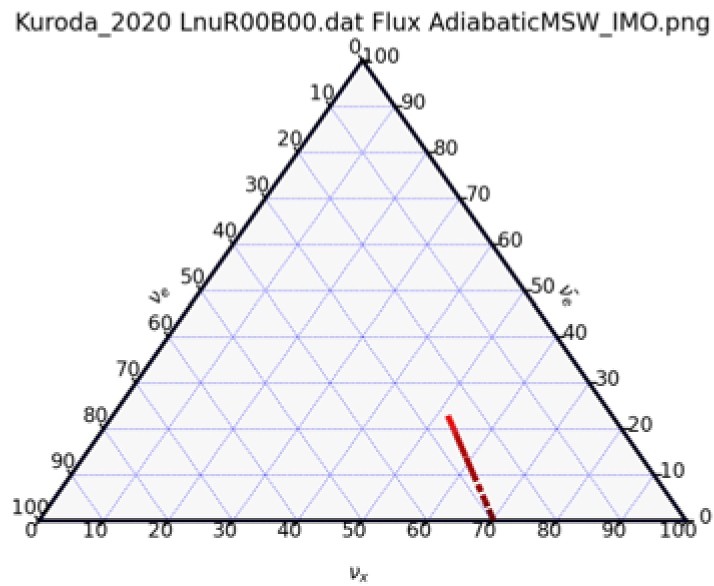
[8]The number of parameters and which parameters specifically are currently being investigated as part of the next stage of this research

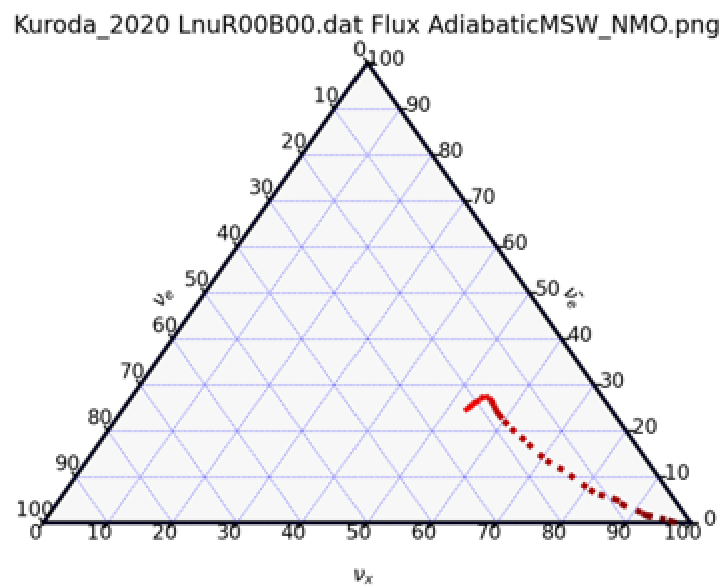
Bollig 2016


Bollig_2016 s11.2c Flux AdiabaticMSW_IMO.png
ν_e
$\bar{\nu}_e$
ν_x

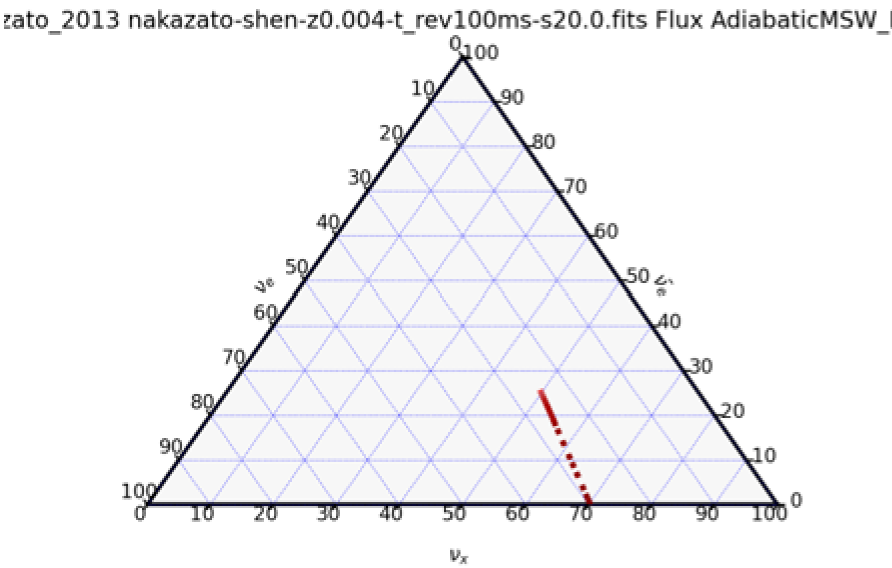

Bollig_2016 s11.2c Flux AdiabaticMSW_NMO.png
ν_e
$\bar{\nu}_e$
ν_x

Kuroda 2020

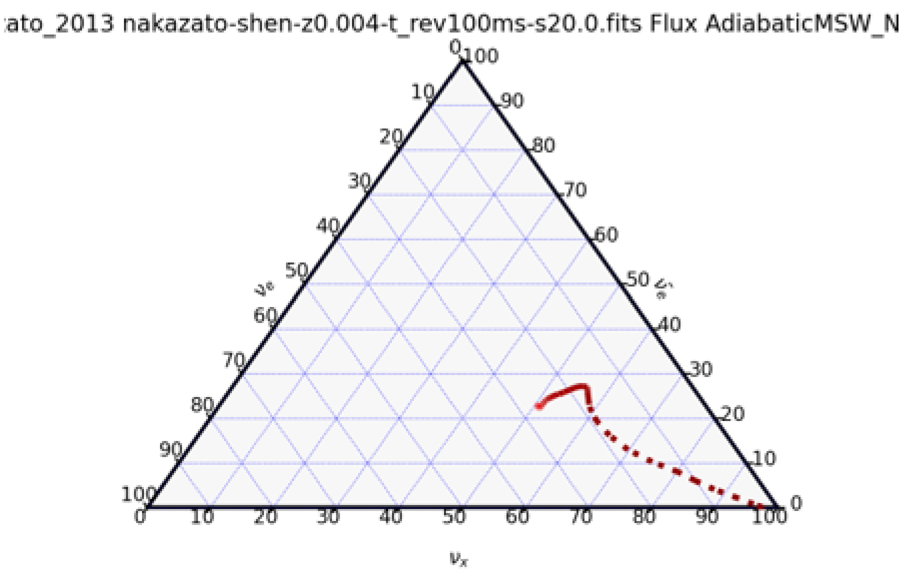




Nakazato 2013

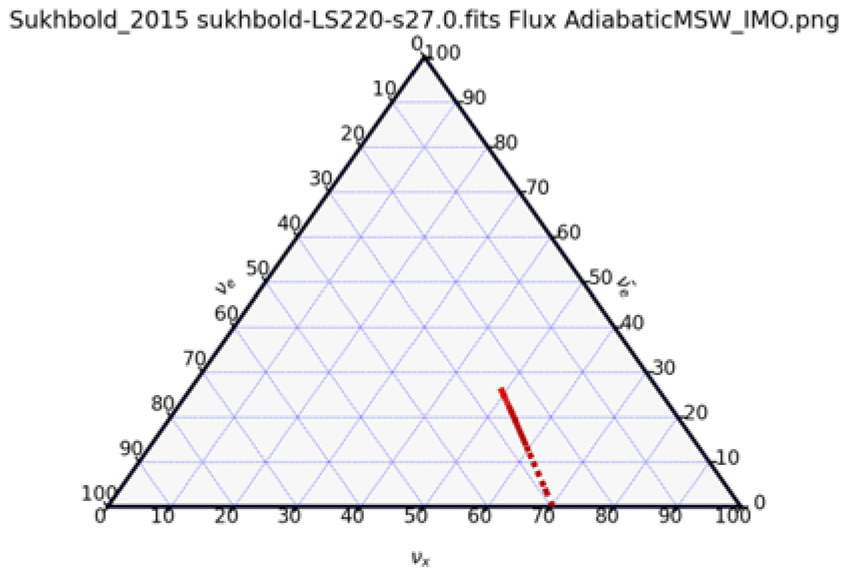


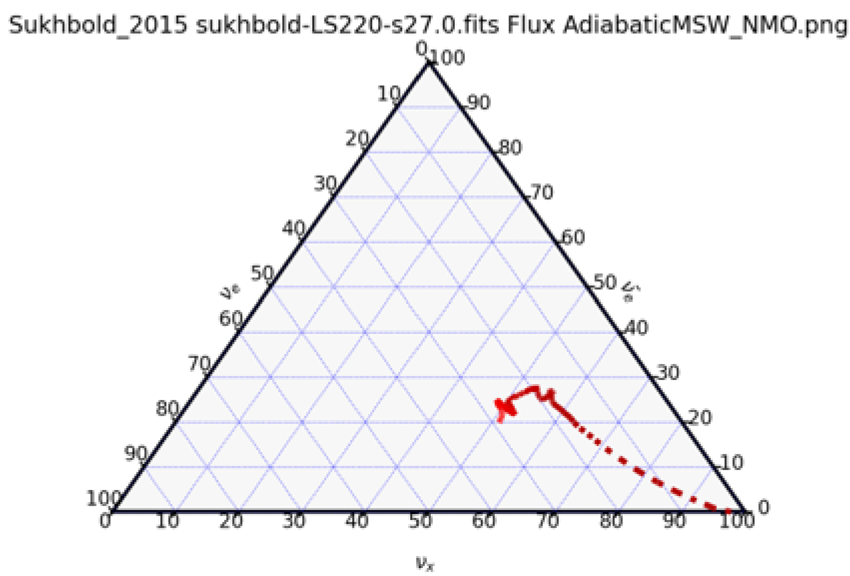
(Shown Above) Nakazato 2013...Flux AdiabaticMSW IMO



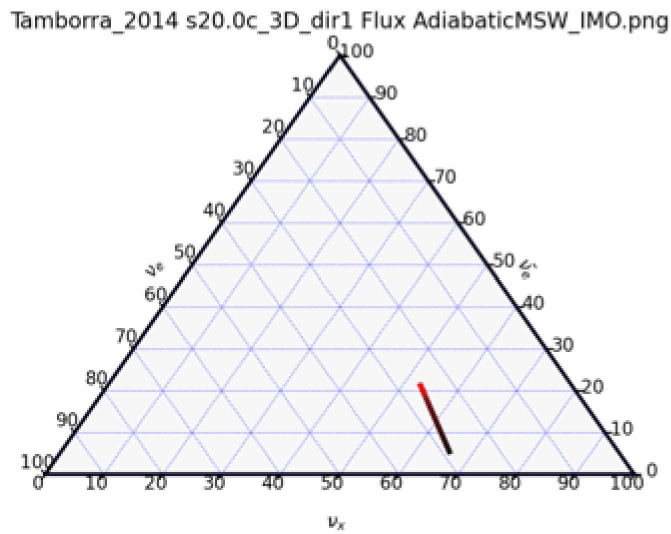
(Shown Above) Nakazato 2013...Flux AdiabaticMSW NMO

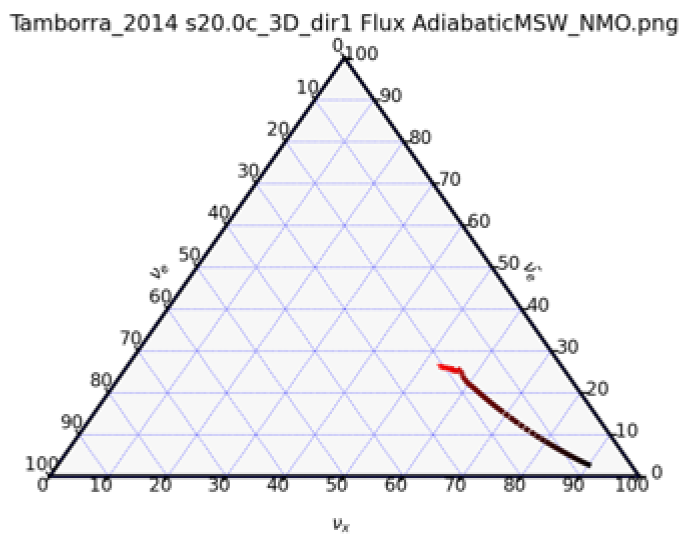
Sukhbold 2015



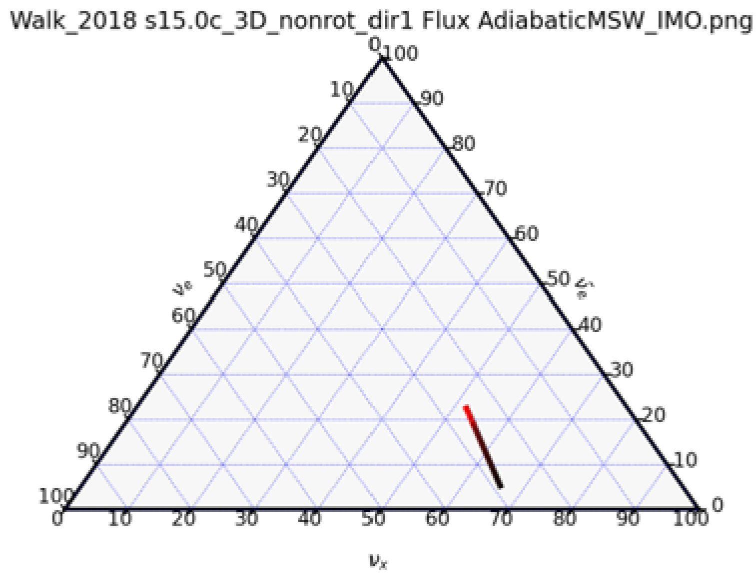


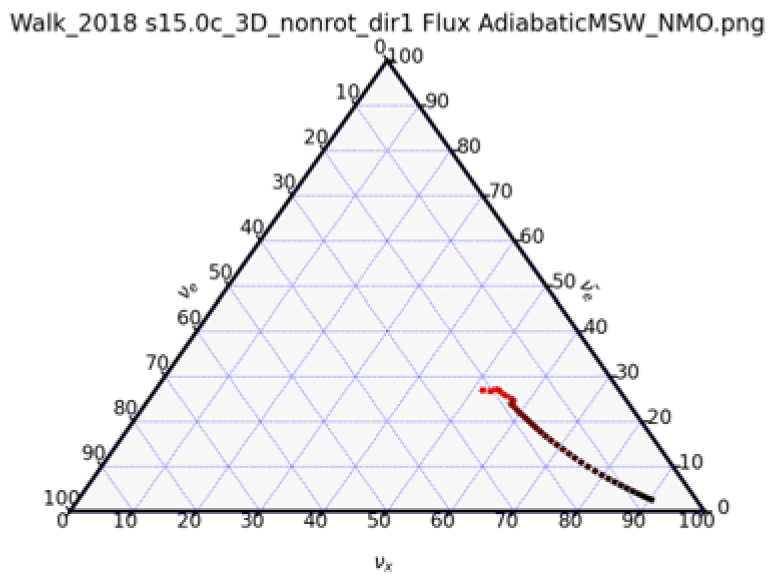
Tamborra 2014


Tamborra_2014 s20.0c_3D_dir1 Flux AdiabaticMSW_IMO.png
ν_e
$\bar{\nu}_e$
ν_x
0 10 20 30 40 50 60 70 80 90 100

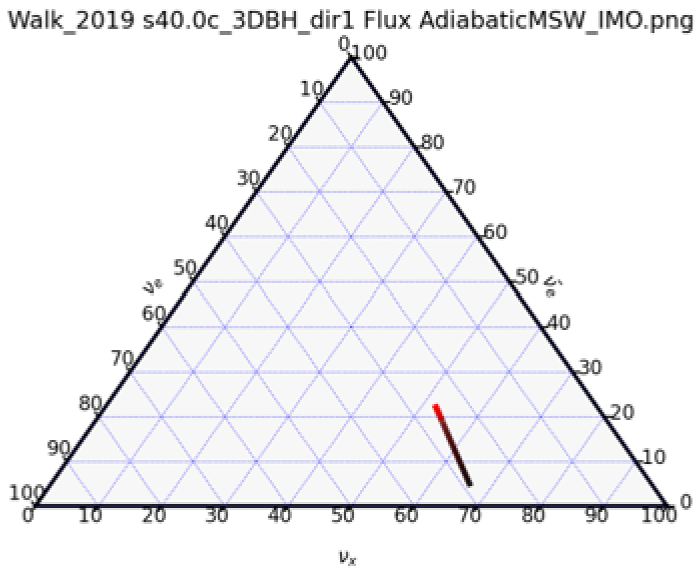

Tamborra_2014 s20.0c_3D_dir1 Flux AdiabaticMSW_NMO.png
ν_e
$\bar{\nu}_e$
ν_x
0 10 20 30 40 50 60 70 80 90 100

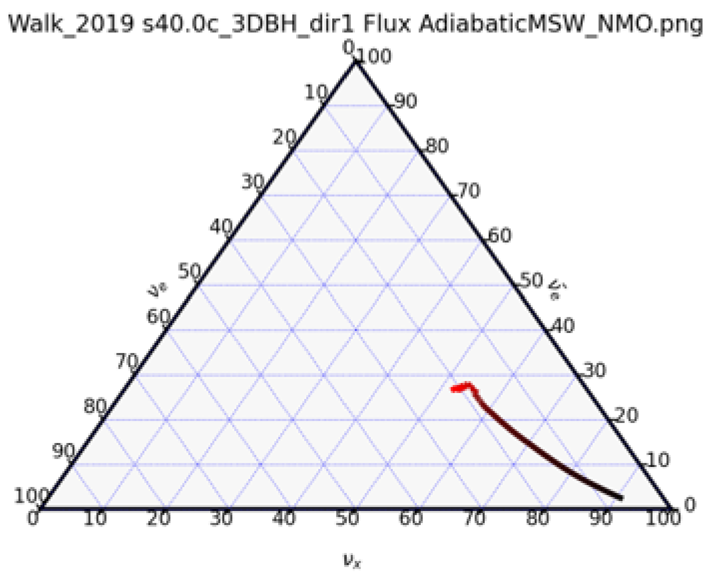
Walk 2018


Walk_2018 s15.0c_3D_nonrot_dir1 Flux AdiabaticMSW_IMO.png
ν_e
$\bar{\nu}_e$
ν_x


Walk_2018 s15.0c_3D_nonrot_dir1 Flux AdiabaticMSW_NMO.png
ν_e
$\bar{\nu}_e$
ν_x

Walk 2019


Walk_2019 s40.0c_3DBH_dir1 Flux AdiabaticMSW_IMO.png
ν_e
$\bar{\nu}_e$
ν_x


Walk_2019 s40.0c_3DBH_dir1 Flux AdiabaticMSW_NMO.png
ν_e
$\bar{\nu}_e$
ν_x

5 Next Steps and Unfolding

From the promising findings of the IMO Tell-Tale, we believe that, with sufficient "unfolding" of the event rates, we will be able to reverse-engineer the original truth flux. If we were able to do so, we would likely find the IMO tracks would differ significantly from the NMO tracks. Unfolding these events is somewhat difficult, however, so an approximation will be necessary. Below we describe our current thinking and methodologies for unfolding these events. Specifically, we describe a simple method for unfolding; While we are currently working with a simplified algorithm, we also discuss potential nuances of a future algorithm.

5.1 Concepts in a Simple Unfolding

To find an approximation of the unfolded flux event count (for each time bin):

$$\phi_{est} = \frac{N_{det}}{N_t \langle \sigma \rangle} \tag{1}$$

Where N_{det} is the calculated total events for the time slice, $\langle \sigma \rangle$ is the flux-averaged cross-section, and the "total number of targets in the detector" is:

$$N_t = \frac{M N_A}{A} \tag{2}$$

Where N_A is Avogadro's Number, A is the atomic mass of the target, and M is the mass of the tank.

5.2 N_t for Best Channel Configuration

For each target we assume different values for each detector:

Scint 20kt

Proxy	Target	N_t
$\overline{\nu_e}$	-	-
ν_e	-	-
ν_x	C_{12}	$\frac{(20kt)*(100*10^9)*N_A}{12}$

Argon 40kt

Proxy	Target	N_t
$\overline{\nu_e}$	-	-
ν_e	$Ar_{(l)}$	$\frac{(40kt)*(100*10^9)*N_A}{39.9}$
ν_x	-	-

It should be noted that the actual target is nueAr40 + e

Water Cherenkov 100kt

Proxy	Target	N_t
$\overline{\nu_e}$	p	$\frac{(100kt)*(100*10^9)*N_A*2}{18}$
ν_e	-	-
ν_x	-	-

5.3 Finding $\langle\sigma\rangle$ for a Time Bin

To find $\langle\sigma\rangle$, which is the average cross-section in a given time bin from SNoWGLoBES, we follow:

$$\langle\sigma\rangle = \frac{\int \sigma(E_\nu) \cdot \phi(E_\nu) dE_\nu}{\phi_{tot}} \tag{3}$$

Where:

$$\phi_{tot,\Delta t} = \int \int \phi(E_\nu) dE_\nu dt \tag{4}$$

And $\phi(E_\nu)$ is a flux "kernel," that is not known. It is "not known" in the sense that we, on Earth, can only see what our detectors give us. Hence, we will need to choose a kernel that approximates the physics surrounding a given progenitor. Finally, Δt is the change in time for a given time bin and $\sigma(E_V)$ is the cross-section as a function of energy.

5.4 Programmatic Challenges for a Complete Unfolding

Programatically, there are some nuances that need addressing. First, we realize that there will be a ϕ_{est} for each flavor proxy. In other words, we actually have three: ϕ_{est,ν_x}, ϕ_{est,ν_e}, and $\phi_{est,\bar{\nu_x}}$. For each flavor estimation, there are additional nuances with calculating $\langle\sigma\rangle$, since each σ term is not composed of one single detector channel. Let us take the example of the Best-Channel Proxy Configuration, and find ϕ_{est,ν_x}. First, we must find $\sigma(E_\nu)$. In this example, it is constructed using only the *nc* channel (Neutral Current) channel from the scint20kt detector; however, another configuration may call for an element-wise summation between all three available detectors. This is the case when using the "Aggregate Detectors" configuration, which must do an element-wise summation from the *nc* channels from all three detectors. We then have the energy-dependent flavor-specific cross section across all available detectors. We then use choose a kernel to use in equation 3. As a simplification, we can use the ϕ_{ν_x} flux. To calculate the integral found in the numerator of equation 3, we first multiply element-wise between the elements of $\sigma(E_\nu)$ and $\phi_t(E_\nu)$, and sum the resulting matrix product.

5.5 Formal Description of Programmatic Approach

Let $\boldsymbol{\sigma_f}(E_\nu)$ be the matrix that describes the energy-dependent cross-section for a given flavor. $\boldsymbol{\sigma_f}(E_\nu)$ is to be interpreted as a function of E_ν in that the i-th element in $\boldsymbol{\sigma_f}(E_\nu)$ is associated with the ith energy bin in the associated energy spectrum: every detector output from SNoWGLoBES will give an energy spectrum as a matrix that has the same dimensions as each channel's energy-dependent cross-section. $\boldsymbol{\sigma_f}(E_\nu)$ is found by summing the channels in each detector (as prescribed by the proxy configuration):

$$\boldsymbol{\sigma_f}(E_\nu) = \sum_{\forall i} \sum_{\forall j} \boldsymbol{\sigma_{f,i,j}}(E_\nu) \tag{5}$$

Where f is the flavor, i is the detector, and j is the channel within that detector. $\boldsymbol{\sigma_{f,i,j}}(E_\nu)$ is a matrix with $\dim(\boldsymbol{\sigma_{f,i,j}}(E_\nu)) = \dim(\boldsymbol{\sigma_f}(E_\nu))$. To calculate $\int \sigma(E_\nu) \cdot \phi_t(E_\nu) dE_\nu$:

$$\sum_{\forall l} (\boldsymbol{\sigma_f}(E_\nu) \cdot \boldsymbol{\phi}(E_\nu)^T) \tag{6}$$

Where l is the index of the resultant matrix. Finally, we select a ϕ kernel to use as part of the ϕ_{tot} calculation and compute it as an element-wise sum.

6 Conclusion

As mentioned previously, we have yet to find a robust model for discriminating between the mass orderings. However, this project has thus far been able to clearly visualize some key relational models and has hinted at potential ways to identify a true robust model. Specifically, our next phase of the project is to "unfold" the events from the BstChnl proxy configuration. With enough unfolding, we should be able to reverse-engineer the truth flux plot, which we've already noted has a seemingly-robust way of discriminating between the orderings.

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