

Fig. A.1. Two rooms

Fig. A.2. A hybrid automaton

#### A.1 Networked thermostat controllers

We consider networked thermostat controllers to control the temperatures of two rooms, adapted from [32]. Figure A.1 shows a hybrid automaton. The temperature  $x_i$  of room<sub>i</sub> is controlled by the heater and the temperatures of both rooms, where  $A_i$  is the set of the adjacent rooms, and  $K_i$ ,  $h_i$ ,  $d_i$  depend on the size of the room, the heater's power, and the size of the door. If the temperature is low, the heater turns on, and if the room temperature is high, the heater turns off.

We consider two thermostats for linear and polynomial dynamics, and one thermostat for ODE dynamics. We consider STL formulas in Table 2.

1. Linear dynamics

$$\dot{x}_1 = \begin{cases} -0.4 & \text{(On)} \\ 0.7 & \text{(Off)} \end{cases} \qquad \dot{x}_2 = \begin{cases} -0.6 & \text{(On)} \\ 1 & \text{(Off)} \end{cases}$$

2. Polynomial dynamics

$$x_1(t) = \begin{cases} 0.05 * t^2 + 0.5 * t + x_1(0) & (\text{On}) \\ -0.0175 * t^2 - 0.35 * t + x_1(0) & (\text{Off}) \end{cases}$$

$$x_2(t) = \begin{cases} 0.06 * t^2 + 0.6 * t + x_2(0) & \text{(On)} \\ -0.275 * t^2 - 0.55 * t + x_2(0) & \text{(Off)} \end{cases}$$

3. ODEs

$$\dot{x} = \begin{cases} 0.035 * x & \text{(On)} \\ -0.02 * x & \text{(Off)} \end{cases}$$

## A.2 Networked watertank controllers

There are two watertanks and the water falls from the first watertank to the second watertank. Networked watertank controller controls the pump of each watertank, adapted from [26]. Figure A.2 shows a hybrid automaton. The water level  $x_i$  of each tank is controlled by a pump and the adjacent watertank, where a and  $q_i$  depend on the the pump's power and the width of the pipe, and g is

Dynamics	STL Formulas	
Linear	$f_1^{\top}: & \Diamond_{[2,10]}(x_2 > 20) \\ f_2^{\top}: \Box_{[0,5]}((x_1 > 19) \to \Diamond_{[2,5)}(x_1 < 24)) \\ f_1^{\perp}: & \Box_{[0,10]}((\text{on}_1\text{on}_2) \to (x_1 > 21)) \\ f_2^{\perp}: & (\Diamond_{[0,2]}\text{on}_2) \mathbf{R}_{[0,8]}(x_1 > 21)$	
Poly	$f_{1}^{\top}: & \Diamond_{[2,10]}(x_{2} > 20) \\ f_{2}^{\top}: \Box_{[0,5]}((x_{1} > 19) \to \Box_{[2,5)}(x_{1} < 24)) \\ f_{1}^{\perp}: & \Box_{[0,5]}(\operatorname{on}_{1} \to (x_{1} > 22.5)) \\ f_{2}^{\perp}: & (\Diamond_{(0,2]}\operatorname{on}_{2}) \mathbf{R}_{[0,8]}(x_{1} > 23)$	
ODE	$f_{1}^{\top}: \qquad \Diamond_{[1,5]}(x < 22)$ $f_{2}^{\top}: \qquad \Diamond_{[0,10]}((x > 23) \lor \Box_{[5,8]} \text{off})$ $f_{1}^{\perp}: \qquad on \mathbf{R}_{[10,20]}(x < 19.5)$ $f_{2}^{\perp}: \qquad (\Diamond_{(2,5)} \text{off}) \mathbf{U}_{[0,10]}(x < 19)$	

**Table 2.** STL properties of thermostat model

 $x_i = (q_i + a\sqrt{2y}\sqrt{x_{i-1}}) - a\sqrt{2y}\sqrt{x_i}$ 

the standard gravity constant. If the water level is low, the pump turns on, and if the water level is high, the pump turns off.

Fig. A.3. Hybrid automata of the watertank

We consider two watertanks linear and polynomial dynamics, and one watertank for ODE dynamics. We consider STL formulas in Table 3.

## 1. Linear dynamics

$$\dot{x}_1 = \begin{cases} -0.8 & \text{(On)} \\ 0.9 & \text{(Off)} \end{cases} \qquad \dot{x}_2 = \begin{cases} -0.6 & \text{(On)} \\ 1 & \text{(Off)} \end{cases}$$

## 2. Polynomial dynamics

$$x_1(t) = \begin{cases} 0.02 * t^2 + 0.4 * t + x_1(0) & \text{(On)} \\ 0.005 * t^2 - 0.4 * t + x_1(0) & \text{(Off)} \end{cases}$$

$$x_2(t) = \begin{cases} 0.02 * t^2 + 0.5 * t + x_2(0) & \text{(On)} \\ 0.005 * t^2 - 0.6 * t + x_2(0) & \text{(Off)} \end{cases}$$

## 3. ODEs

$$\dot{x} = \begin{cases} 0.8 - 0.01 * \sqrt{2 * g} * \sqrt{x} & \text{(On)} \\ -0.01 * \sqrt{2 * g} * \sqrt{x} & \text{(Off)} \end{cases}$$

Table 3. STL properties of watertank model

Dynamics		STL Formulas
Linear	$f_1^{ op}$ : $f_2^{ op}$ :	$\Diamond_{[3,8]}(x_1 \le 7)$ $\Diamond_{[1,3]}((x_2 > 2) \mathbf{R}_{[1,2]} \text{ on}_2)$
	$f_1^{\perp}$ :	$\Diamond_{[5,9]}(x_2 < 5)$
	$f_2^{\perp}$ :	$(\lozenge_{[1,3)}x_2 > 5.8) \mathbf{R}_{[4,7]} \text{ off}_2$
Poly	$f_1^{ op}$ : $f_2^{ op}$ :	$\Diamond_{[3,8]}(x_1 \le 6)$ $(\Diamond_{[0,3]} \text{off}_2) \mathbf{R}_{[0,4]}(y_1 > 4)$
	$f_1^{\perp}$ :	$\Diamond_{[5,9]}(x_2 < 0.5)$
	$f_2^{\perp}$ :	$(\lozenge_{[1,3)}x_2 > 7) \mathbf{R}_{[4,7]} \text{ off}_2$
ODE	$f_1^ op$ : $f_2^ op$ :	$\Diamond_{[2,4]}(x > 2)$ $\Box_{[0,10]}((x < 2) \to \Diamond_{[0,10]} \text{on})$
	$f_1^{\perp}$ :	off $\mathbf{U}_{[2,10]}$ $(x < 2)$
	$f_2^{\perp}$ :	$(\Box_{[0,10]}(\Diamond_{[0,7]}(x<3))$

## A.3 Autonomous cars

There are two cars that moves in the  $\mathbb{R}^2$  plane. One of the car (leader) drives according to its own scenario and the other (follower) follows the leader, adapted from 3. If the follower is to the left of the leader, the follower rotates counterclockwise. If the follower is to the right of the leader, the follower rotates clockwise. Otherwise, the follower drives in a straight way. For polynomial and ODE cases, we assume that we can obtain relative positions between the follower and the leader and the follower changes it's velocity based on the relative distance. We consider STL formulas in Table 4.

#### 1. Linear dynamics

$$\begin{cases} \dot{x}_1 = 3, \ \dot{y}_1 = 0 & (1) \ \mathrm{straight} \\ \dot{x}_1 = 1.5, \ \dot{y}_1 = 3 & (2) \ \mathrm{counter\text{-}clockwise} \\ \dot{x}_1 = 1.5, \ \dot{y}_1 = -3 & (3) \ \mathrm{clockwise} \end{cases}$$

## 2. Polynomial dynamics

$$\begin{cases} \dot{r}_x = v_x, \ \dot{r}_y = v_y, \ \dot{v}_x = 1.2, \ \dot{v}_y = 1.4 & (1) \ (\text{x-close}, \, \text{y-close}) \\ \dot{r}_x = v_x, \ \dot{r}_y = v_y, \ \dot{v}_x = 1.2, \ \dot{v}_y = -1.4 & (2) \ (\text{x-close}, \, \text{y-far}) \\ \dot{r}_x = v_x, \ \dot{r}_y = v_y, \ \dot{v}_x = -1.2, \ \dot{v}_y = 1.4 & (3) \ (\text{x-far}, \, \text{y-close}) \\ \dot{r}_x = v_x, \ \dot{r}_y = v_y, \ \dot{v}_x = -1.2, \ \dot{v}_y = -1.4 & (4) \ (\text{x-far}, \, \text{y-far}) \end{cases}$$

## 3. ODEs

$$\begin{cases} \dot{r}_x = 1, \ \dot{r}_y = 2 * sin(\theta), \ \dot{\theta} = 0.05 & (1) \text{ (x-close, y-close)} \\ \dot{r}_x = 1, \ \dot{r}_y = -2 * sin(\theta), \ \dot{\theta} = 0.05 & (2) \text{ (x-close, y-far)} \\ \dot{r}_x = -1, \ \dot{r}_y = 2 * sin(\theta), \ \dot{\theta} = 0.05 & (3) \text{ (x-far, y-close)} \\ \dot{r}_x = -1, \ \dot{r}_y = -2 * sin(\theta), \ \dot{\theta} = 0.05 & (4) \text{ (x-far, y-far)} \end{cases}$$

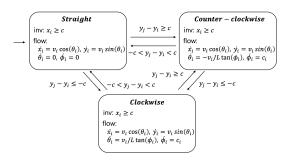


Fig. A.4. Hybrid automata of the autonomous car

## A.4 Battery

There are two fully charged batteries, and a control system switches load between these batteries to achieve longer lifetime out of the batteries, adapted from 42. Figure A.3 shows a hybrid automaton.

As long as the total charge  $g_i$  is is greater than  $c \in \mathbb{R}$ , the battery can be turned on an turned off.

We consider two batteries for linear and polynomial dynamics, and one battery for ODE dynamics. We consider STL formulas in Table 5.

## 1. Linear dynamics

$$\begin{cases} \dot{d}_i = 1, \ \dot{g}_i = -0.3 & \text{if } b_i = \text{on}, \ i \in \{1, 2\} \\ \dot{d}_i = 0.8, \ \dot{d}_j = -0.166, \ \dot{g}_i = -0.5, \ \dot{g}_j = 0 & \text{if } b_i = \text{on}, \ b_j = \text{off} \\ \dot{d}_i = 0.7, \ \dot{d}_j = 0, \ \dot{g}_i = -7, \ \dot{g}_j = 0 & \text{if } b_i = \text{on}, \ b_j = \text{dead} \\ \dot{d}_i = 0, \ \dot{g}_i = 0 & \text{if } b_i = \text{dead} \end{cases}$$

Table 4. STL properties of autonomous car model

Dynamic	STL Formulas		
Linear	$\begin{array}{ll} f_1^\top \colon & \Diamond_{[3,5]}(y_2 - y_1 \le 5) \\ f_2^\top \colon & \Box_{[1,3]}((x_2 - x_1) < -1 \to \Diamond_{[5,7]} \text{ rotate}) \\ f_1^\bot \colon & (y_2 - y_1 \ge 8) \ \mathbf{U}_{[5,10]} \left( (x_2 - x_1 \ge 2) \right. \\ f_2^\bot \colon \Diamond_{[1,5]}(\sim \text{clockwise} \to \Box_{[0,10]}(y_2 - y_1 > 3)) \end{array}$		
Poly	$\begin{array}{ll} f_1^\top \colon & \Diamond_{[2,14]}(vy_1 < 3) \\ f_2^\top \colon & \Diamond_{[0,3]} \mathrm{dec}_x \ \mathbf{R}_{[0,15]} \ (r_x > 3)) \\ f_1^\bot \colon & (r_y \ge 10) \ \mathbf{U}_{[0,10]} \ (r_x \ge 7) \\ f_2^\bot \colon & \Diamond_{[2,10]} (\mathrm{dec}_y \to \Box_{[0,15]}(r_y > 5)) \end{array}$		
ODE	$\begin{array}{ccc} f_1^{\top} \colon & \Box_{[0,3]}(r_x < 5) \\ f_2^{\top} \colon & (\Diamond_{[0,10]}(\operatorname{inc}_y)  \mathbf{R}_{[0,10]}  r_y < 10 \\ f_1^{\perp} \colon & \Box_{[0,20]}(r_x < 6) \\ f_2^{\perp} \colon & \Box_{[0,12]}(\operatorname{inc}_y \to \Diamond_{[5,7]}(r_y < 3)) \end{array}$		

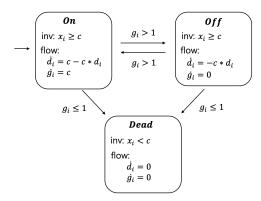


Fig. A.5. Hybrid automata of the battery

## 2. Polynomial dynamics

$$\begin{cases}
d_i(t) = 0.1 * (6 - 2 * t + 2 * t^2) + d_i(0), \ g_i(t) = -0.5 * t + g_i(0) & (1) \text{ On} \\
d_i(t) = -0.1 * (1 - 2 * t + 2 * t^2) + d_i(0), \ g_i(t) = g_i(0) & (2) \text{ Off} \\
d_i(t) = 0, \ g_i(t) = g_i(0) & (3) \text{ Dead}
\end{cases}$$

# 3. ODEs

$$\begin{cases} \dot{d} = 2 + 0.122 * d, \ \dot{g} = -0.5 & (1) \text{ On} \\ \dot{d}_x = -0.05 * d, \ \dot{g}_y = 0 & (2) \text{ Off} \\ \dot{d}_x = 0, \ \dot{g}_y = 0 & (3) \text{ Dead} \end{cases}$$

 $\textbf{Table 5.} \ \mathrm{STL} \ \mathrm{properties} \ \mathrm{of} \ \mathrm{battery} \ \mathrm{model}$ 

Dynamics	STL Formulas
Linear	$f_1^{\top}: \qquad (g_2 < 4) \ \mathbf{R}_{[6,20]}(d_2 \ge 0)$ $f_2^{\top}: \Box_{[4,14]}((g_2 > 4) \to \Diamond_{[0,10]}(d_2 > 1))$ $f_1^{\perp}: \qquad \Box_{[2,23]}((g_1 > 3) \to (d_1 > 1))$ $f_2^{\perp}: \qquad (\Box_{[0,5]}(g_1 > 1)) \ \mathbf{U}_{[0,8]} \ \text{off}_1$
Poly	$f_1^{\top} \colon (g_2 < 4) \ \mathbf{R}_{[6,20]}(d_2 \ge 0)$ $f_2^{\top} \colon \Box_{[4,14]}((g_2 > 4) \to \Diamond_{[0,10]}(d_2 > 1))$ $f_1^{\perp} \colon \Box_{[15,25]}((g_2 > 4.5) \to (d_2 > 15.5))$ $f_2^{\perp} \colon (\Box_{[0,5]}(g_1 > 1)) \ \mathbf{U}_{[0,8]} \ \text{off}_1$
ODE	$\begin{array}{ll} f_1^\top \colon & \Diamond_{[5,10]}(\text{on } \vee \text{off}) \\ f_2^\top \colon & (\Diamond_{[0,10]} \text{ off}) \mathbb{R}_{[0,10]}(g_1 > 2)) \\ f_1^\bot \colon & \Box_{[0,25]}(g_1 > 6) \\ f_2^\bot \colon & (\Box_{[0,10]}(\Diamond_{[0,5]}(d_1 > 1.5)) \end{array}$

Table 6. STL properties of railroad model

Dynamics		STL Formulas
Linear	$f_1^ op : f_2^ op : f_1^\perp : f_2^\perp :$	$ \begin{array}{l} \diamondsuit_{[4,12]}((\text{near }or  \text{past})orb > 10) \\ \diamondsuit_{[0,5)}((t < 50)  \mathbf{U}_{[0,10]}  (b > 5)) \\ (t > 20)  \mathbf{U}_{[10,20]}  (b < 75) \\ (\square_{[0,5)}(\lozenge_{[2,10]} (b > 50)) \end{array} $
Poly	$f_1^ op$ : $f_2^ op$ : $f_1^\perp$ : $f_2^\perp$ :	$ \Diamond_{[3,8]}(tx < 60)  \Diamond_{[0,5)}((t < 50) \mathbf{U}_{[0,10]}(b > 5))  (t > 40) \mathbf{U}_{[10,20]}(b < 40)  (\square_{[0,10)}(\Diamond_{[2,10]}(b > 50)) $
ODE	$f_1^{ op}$ : $f_2^{ op}$ : [ $f_1^{\perp}$ : $f_2^{\perp}$ :	$(vb < 13) \mathbf{U}_{[2,10]} (b > 40)$ $\Box_{[0,10]} ((v_b > 0) \to \Diamond_{[0,20]} (b > 20))$ $\Box_{[0,20]} (v_b > 0)$ $(\Box_{[0,10)} (\text{open} \lor \Diamond_{[3,5]} (b \ge 80))$

## A.5 Railroad

There is a crossing bar on the circular railroad track. Figure A.5 shows a hybrid automaton that controls the bar, adapted from 32. The velocity of the crossing bar  $v_b$  is determined by the current height of the bar. If the height is greater than 70, the crossing bar can jump to close mode. If the height is less than 20, the crossing bar can jump to open mode.

For linear and polynomial dynamics, we assume that there is a train approaching to the bar to increase complexity. There are 4 cases, (i) when the train is far away enough from the crossing bar, (ii) when the train approaches

$$\overrightarrow{\text{linv: } v_b \geq 0} \\ \text{flow:} \\ \dot{b} = v_b \\ \dot{v}_b = c$$
 
$$b > 70 \\ \overleftarrow{\text{linv: } v_b \leq 0} \\ \text{flow:} \\ \dot{b} = v_b \\ \overrightarrow{v}_b = c + c * vb^2$$

 ${f Fig.}$   ${f A.6.}$  Hybrid automata of the railroad

to the crossing bar, (iii) when the train *closes* to the crossing bar, (iv) when the train *passes* the crossing bar. We consider STL formulas in Table [6].

1. Linear dynamics

$$\begin{cases} \dot{t} = -30, \ \dot{b} = 0 & (1) \\ \dot{t} = -5, \ \dot{b} = 5 & (2) \\ \dot{t} = -5, \ \dot{b} = 10 & (3) \\ \dot{t} = -5, \ \dot{b} = -5 & (4) \end{cases}$$

2. Polynomial dynamics

$$\begin{cases} \dot{t} = -30, \ \dot{b} = v_b, \ \dot{v}_b = 0 & (1) \\ \dot{t} = -5, \ \dot{b} = v_b, \ \dot{v}_b = 0.3 & (2) \\ \dot{t} = -5, \ \dot{b} = v_b, \ \dot{v}_b = 0.5 & (3) \\ \dot{t} = -5, \ \dot{b} = v_b, \ \dot{v}_b = -1 & (4) \end{cases}$$

3. ODEs

$$\begin{cases} \dot{b} = v_b, \ \dot{v}_b = 1.2 \\ \dot{b} = v_b, \ \dot{v}_b = -1 + 0.2 * v_b^2 \end{cases}$$
 (1) Open (1) Close