Computing Abelian varieties over finite fields

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Lightning Talk
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Motivation

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- Denote by $\mathcal{C}_h$ the isogeny class of abelian varieties with char. poly. of Frobenius h.

Main result

Theorem (M.)

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- Solution: attack the problem locally (weak equivalence) then use the action of the Pic's.
- Extra 1: in the ordinary case we can compute also polarizations.
- Extra 2: similar results for isogeny classes $\mathcal{C}_{h^r}$ (when $\mathbb{Z}[F,q/F]$ is Bass)

Coming soon on the LMFDB.

Thank you!