Itro to category theory A category C comsists of: 1) a class of dojects Ob(e) "AEC" ¥ A/B € 06(e) a set Home (A, B) of morphisms "A => B" 3) a composition law of morphisms: Y A,B,CEC Home (B, C) × Home (A, B) -> Home (A, C) (c, D) \$ (A,B) A,B,C,D∈ € such that: Home (S,D) disjoint Home (A,B), VAECJida: A-> A st  $B \xrightarrow{f} A \xrightarrow{id_A} A \xrightarrow{g} C$ f = idA of and g = C) associatity of the composition. A FB in l'is an isomorphism B => A in e st f.g = idB, gof = idA

Examples Sets, Top, Rings, CRings, Grp, Ab Y ring R: R-Mod Mod-R division ring D: Vect D, D-Vect field k: Vection - Recall - R-Mod = Vector - Z-Mod = Ab · Paint of category theory: don't think only about diects, but also about "amous" C, D categories Fil - Disa a (covaniant) functor law assigning : V c ∈ C an object F(C) ∈ D C + C' a morphism in D  $F(C) \xrightarrow{F(P)} F(C')$ satisfying: @ F (ide) = idf(e) B F ( f.g) = F(f) . F(g) contravariant functor: - F(C) & F(F)  $F(f \circ g) = F(g) \circ F(f)$  $e^{\circ p}$ :  $ob(e^{\circ p}) = ob(e)$ Hampop (A,B) = Home (B,A) A contravaniant functor & -> D is a covariant functor e op -> D

Example

Consider the following category. C

Ob (E) = {\*\*}

Home (\*, \*) = G a group.

Then a representation of G over a field to
is a function

F: C -> Vector

homosphism G -> Aut (V)

i.e. we fix a k[G]-module structure on V

( Rmks: - every morphism of C is an isomorphism)

( Y f = g st fog = idx = g o f)

- any functor sends iso to iso

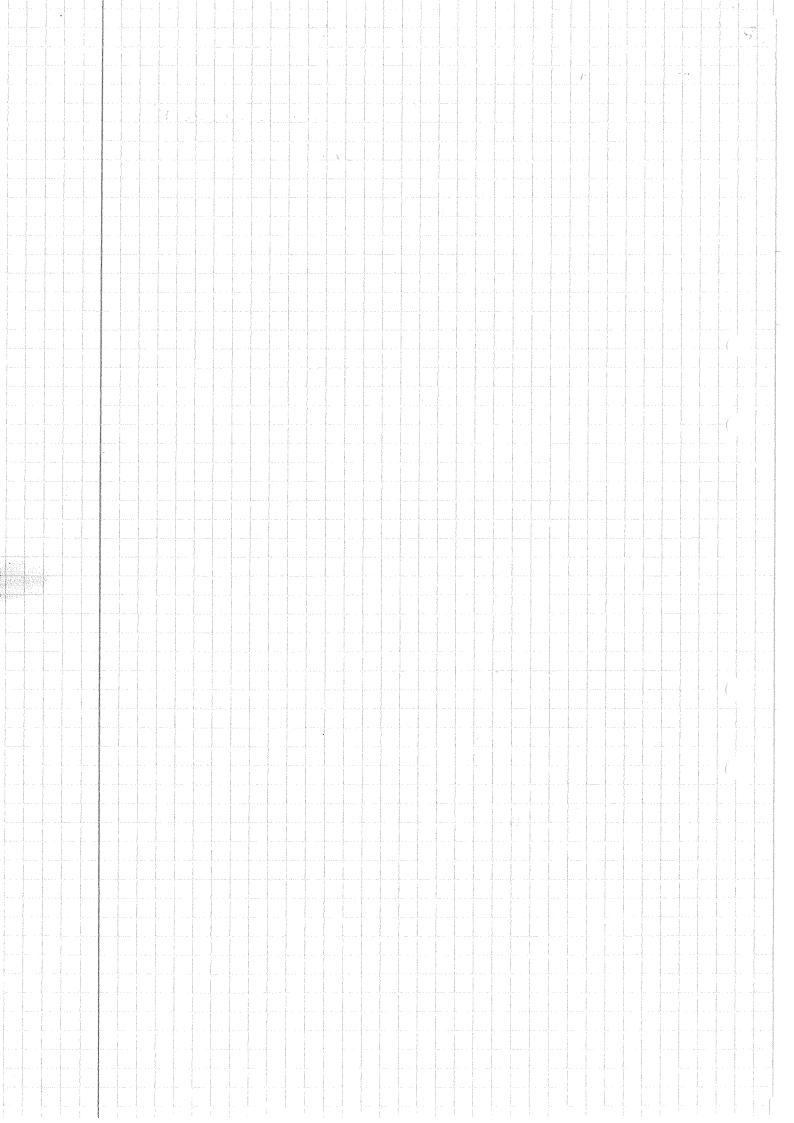
Idea: nather than studying directly G,
we study its representations.

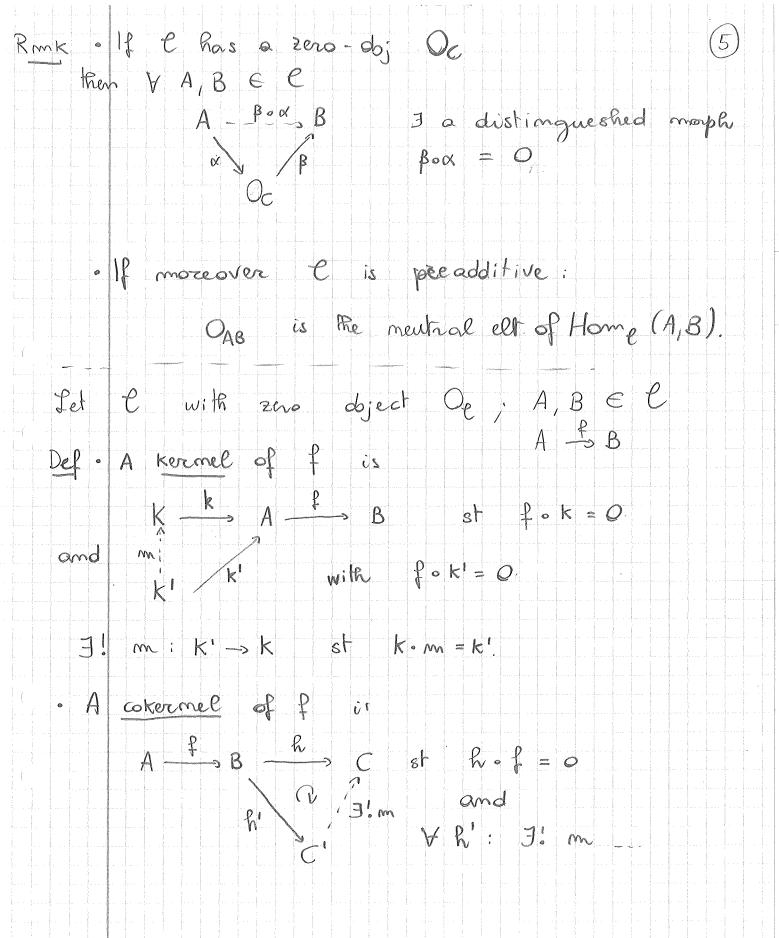
Turens out that we can get a lot of
impo from those: - mounal subgroubs,
- #elements

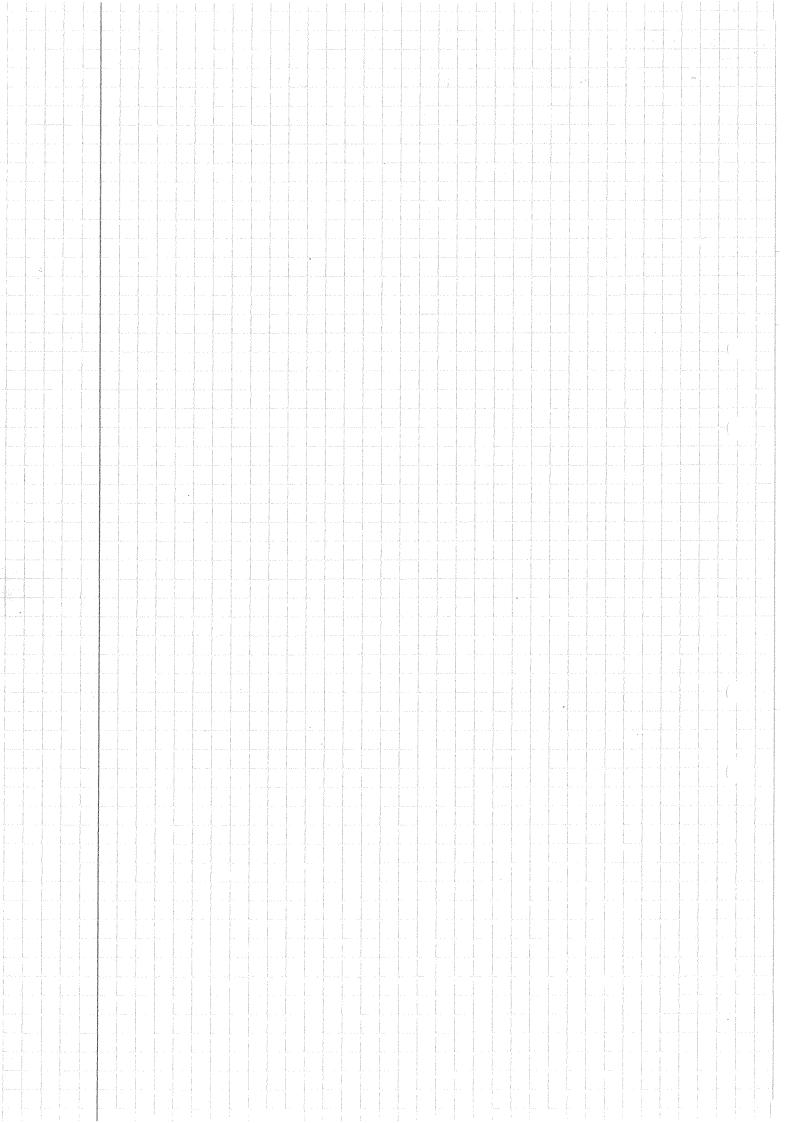
- # elts in the conjugacy classes

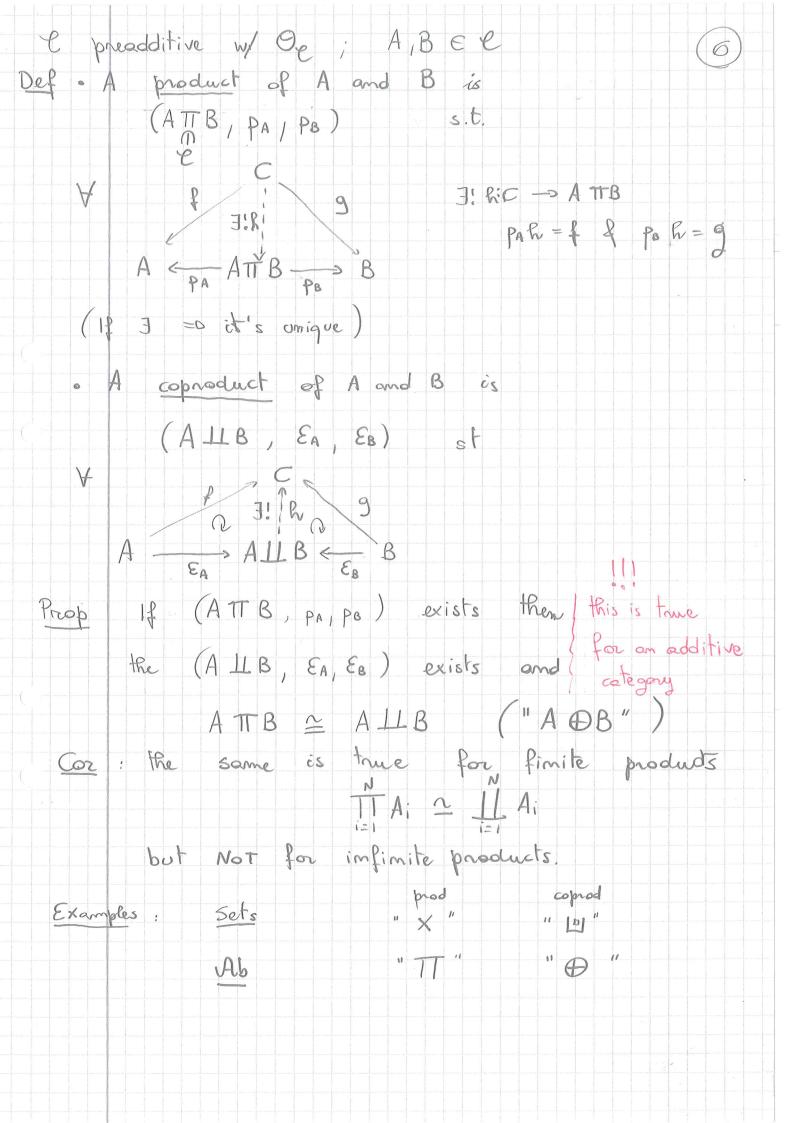
TALK ABOUT NATURAL TRANSFORMATIONS & the caregory Funct (C,D)

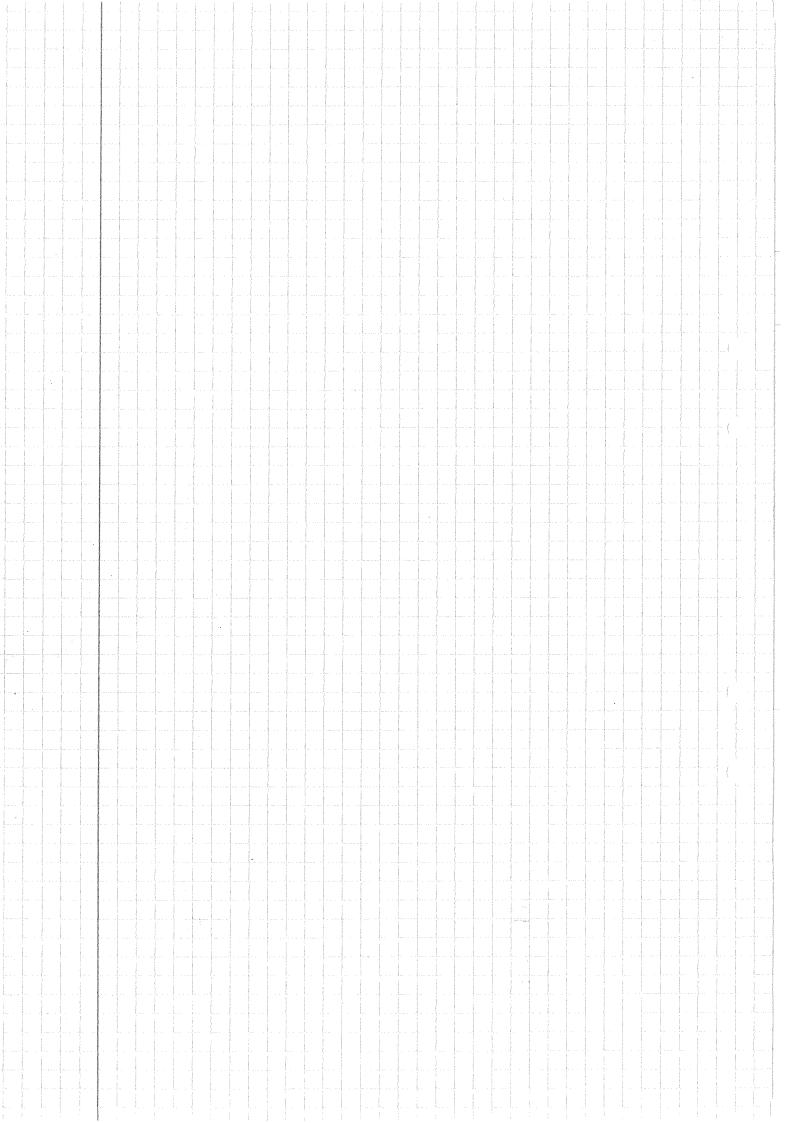
commu - (R-linear) A category e is called preadditive if Def Home (A,B) is an abelian group and the composition is bilimean: (R-module)
(R-bilimean) (f+f') o (g+g') = (f+g) o (f+g') o (f+g') o (f'og) - R-Mod, Mod-R, Ab, Vectro and pre-additive Ring, Sets, Top one mot preadditive  $\Rightarrow ((f+g)(1) = f(1) + g(1) \neq 1)$ Def C category  $-Z \in C$  is animitial object if Home(Z, C) consist of only one morphism  $\forall C \in C$ a terminal object if the same holds for Home (G2) - they one unique up to iso - Oe both initial & terminal is a zero objects Ex - Sets:
\$\phi\$ initial, \lambda \pm is terrminal, no zero-doj. Ab, R. Mod, Mod-R,. Pouve a zero object - Ring: Z is initial, no terminal object.

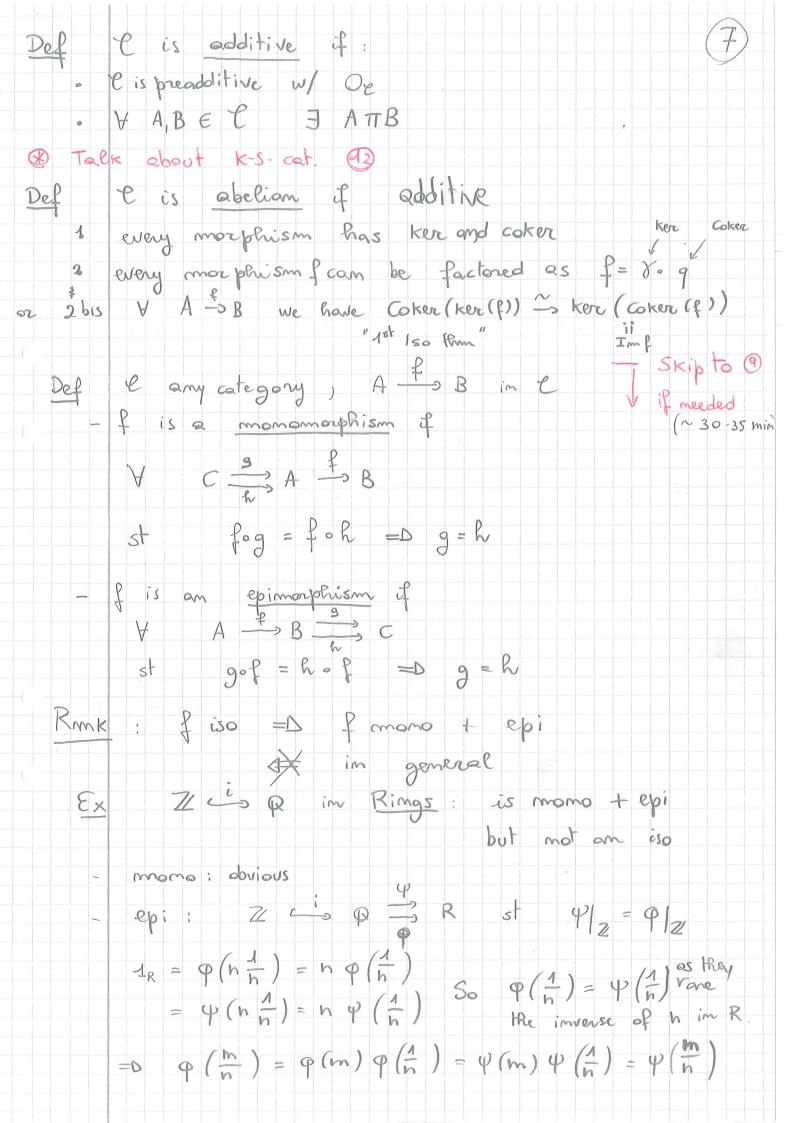


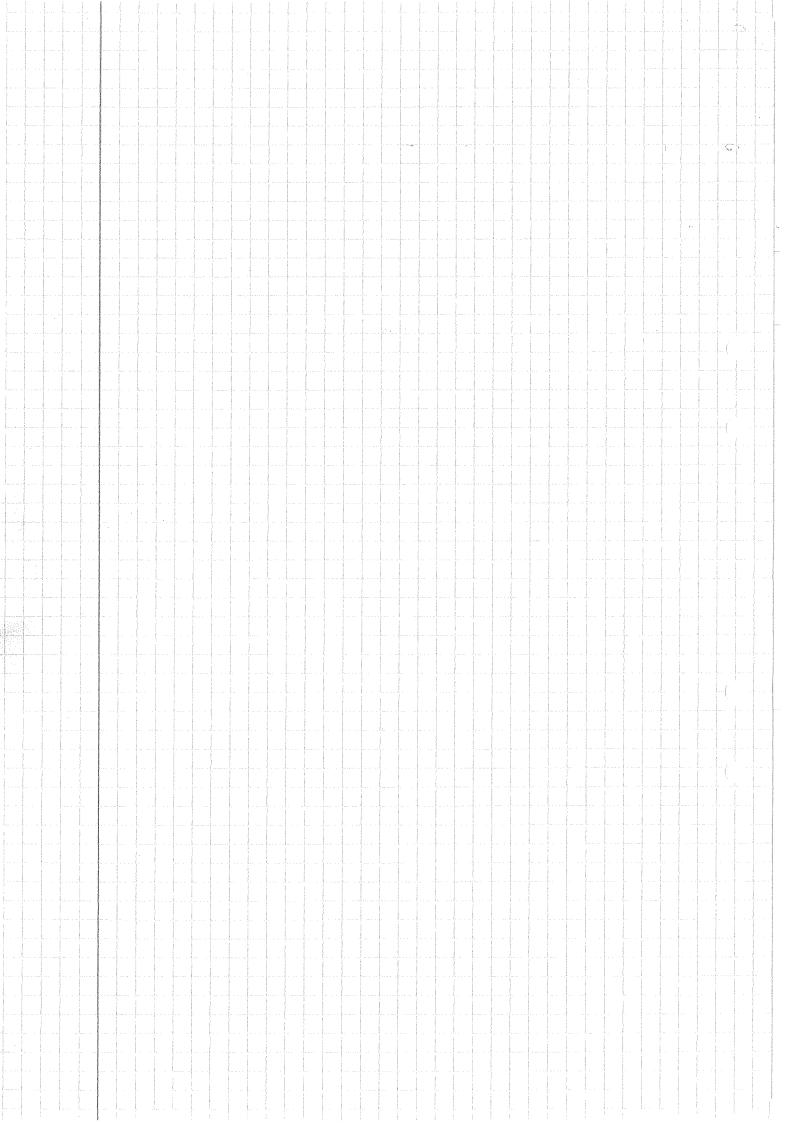


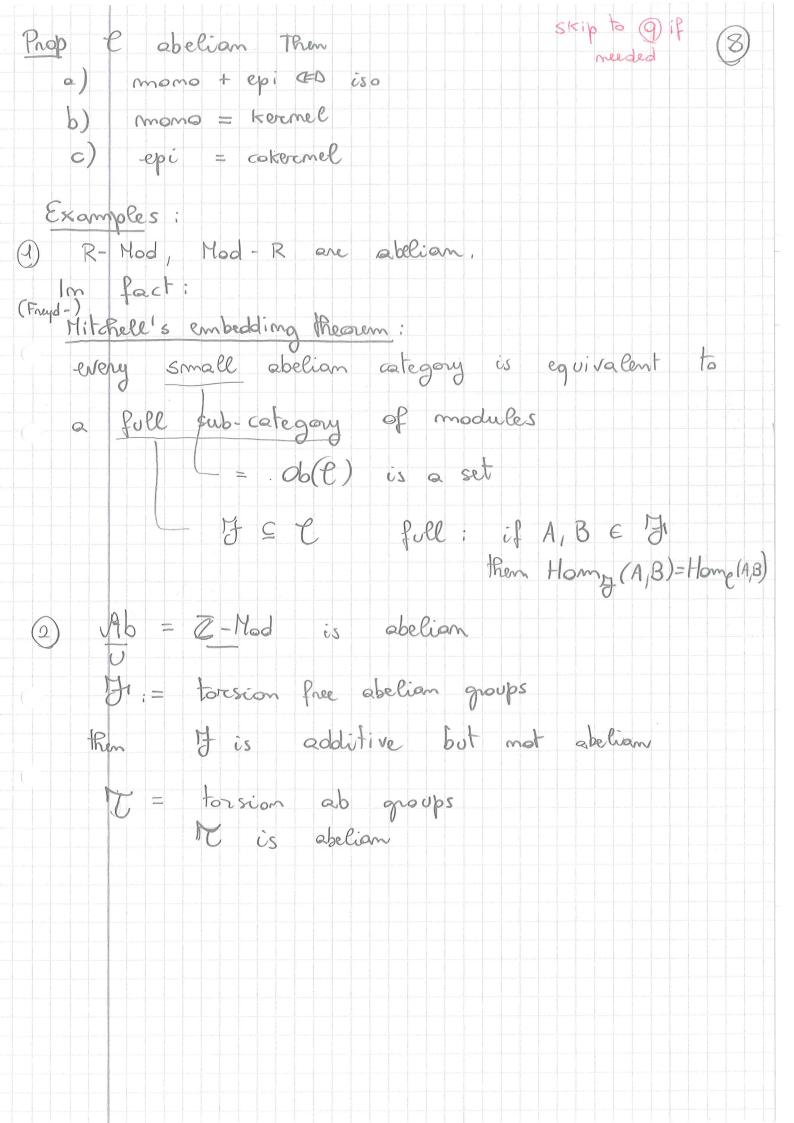


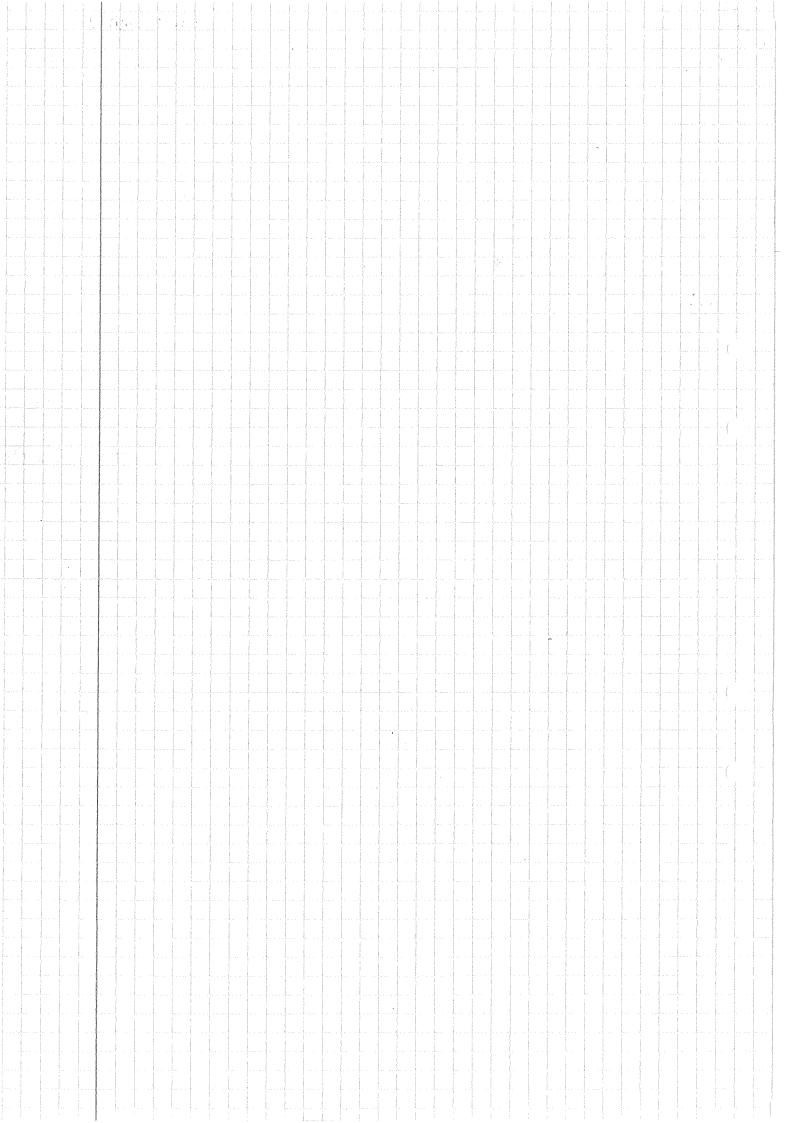












Def.  $\longrightarrow$  Ci,  $f_{i-1}$  Ci  $f_{i-1}$  Sequence

is exact if  $\forall i$   $Imf_{i-1} = ker f_{i}$ •  $\ell$ , D ab. cat,  $F_{i}$   $\ell$   $\longrightarrow$  D a functor

1 F is exact if it sends exact seq's to exact seq's

2 F is left exact if  $\forall v o \longrightarrow A \longrightarrow B \longrightarrow C \longrightarrow o$  S.  $\ell$ . S

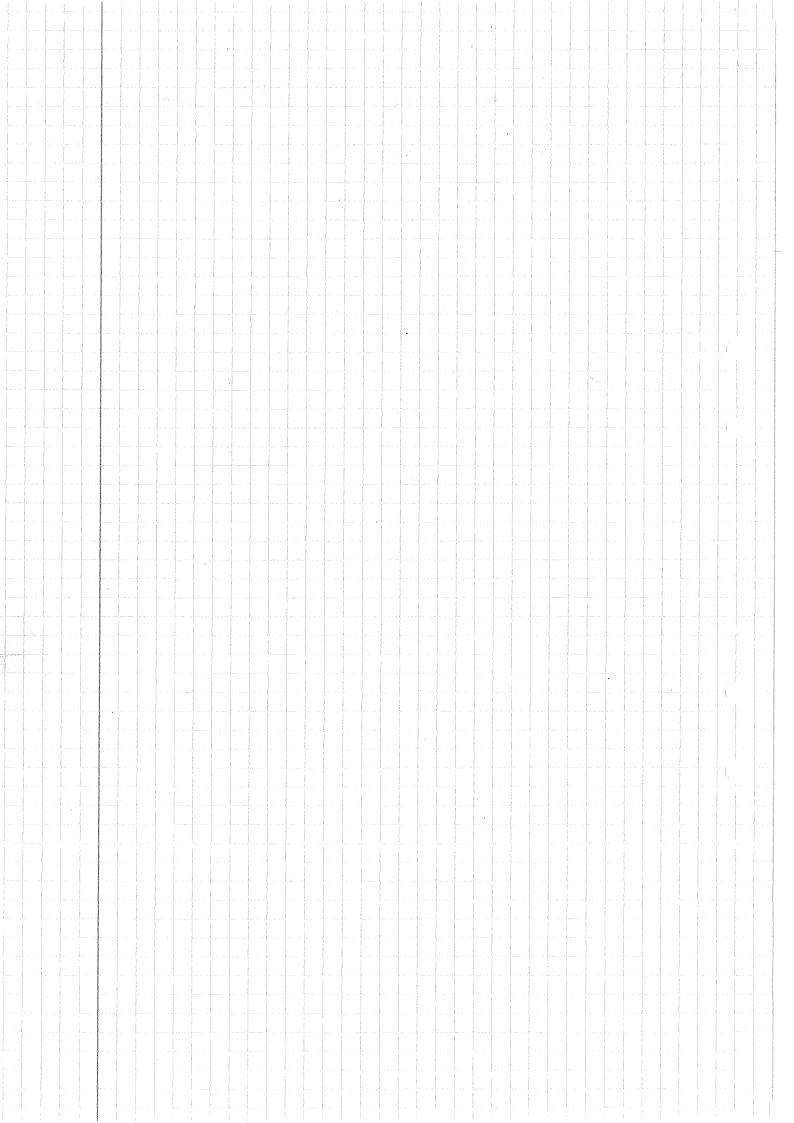
=0  $O \longrightarrow F(A) \longrightarrow F(B) \longrightarrow F(C)$  is exact

3 F is night exact  $F(A) \longrightarrow F(B) \longrightarrow F(C) \longrightarrow C$ Lemma F exact  $C \longrightarrow C$  both left and night exact

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Important functor  $e^{\circ f} \times e \ni (c, D), c, D \in e$  $\begin{cases} (f,g) & \iff g:C' \to C \\ g:D \to D' \end{cases}$ (d', D') Copx C > Sets (C,D) +> Home (C,D) Home (-,-):  $(C',D') \mapsto (J',D')$   $(C',D') \mapsto Home(C',D')$ Rmk Home (-,-) is covariant in both entires Prop Cabelian; Home (-,-) is left exact in both entries. Def  $\ell$  ab ;  $P \in \ell$  is projective if  $Hom_{\ell}(P, -)$  is exact (i.e. also night exact) · I ∈ C is imjective of Homp (-, I) is exact e has enough projectives if ∀ C ∈ e 3 PEl projective and P = 0 enough imjective if VCEC JI ∈ e imjective

0 → C → I



Examples

C = R-Mod Ros enough projectives and injectives

To tassion ab groups:

-  $G_i \in Ob(T) = 0$  II  $G_i \in Ob(T)$ but  $TTG_i$  is not tassion TT = 0 in tassion

Ros enough injectives

17 has not enough proj: 3 P is projective (=0 P=0

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