Uncertainty Quantification of Low-Dimensional Models

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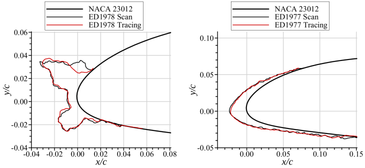
Outline

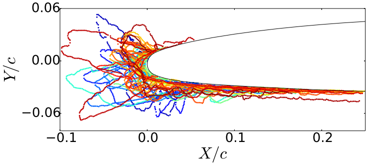
Topic

Motivation

- Many fluid systems have uncertainties associated with them
  - Governing parameters
  - Boundary conditions
- Wing icing [1, 2]



Broeren, 2013



Addy, 2000

[1] Broeren et. al. *Swept-Wing Ice Accretion Characterization and Aerodynamics*. AIAA 2013-2824.

[2] Addy, H. E. *Ice Accretions and Icing Effects for Modern Airfoils*. NASA TP 2000-210031.

Motivation

- Many fluid systems have uncertainties associated with them
  - Governing parameters
  - Boundary conditions
- Airplane cargo hold fires [3]

Colder Source **Hotter Source**

[3] DeGennaro, Lohry et. al. *Uncertainty Quantification for Cargo Hold Fires*. To appear at AIAA Scitech 2016.

Motivation

Goal: apply uncertainty quantification tools to low-dimensional models

- Understand statistical variations in flow with parameters

Potential applications

- Investigate statistical variations in spatial structures and dynamics
  - POD modes, DMD modes
  - DMD eigenvalues
- POD Galerkin models
  - Sensitive to parametric variation/uncertainty

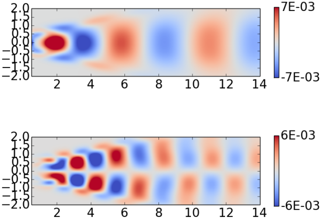
Topic

Background: Low-Dimensional Modeling

- Proper Orthogonal Decomposition (POD)
  - Data compression, dominant spatial features
  - Modes are eigenvectors of the dataset covariance matrix
  - Modes describe dataset better than any other linear basis
- Dynamic Mode Decomposition (DMD)
  - Describe dataset as linear dynamical system
  - Spatial modes + (frequencies, growth/decay rates)

POD/DMD Modes

Cylinder, Re = 100

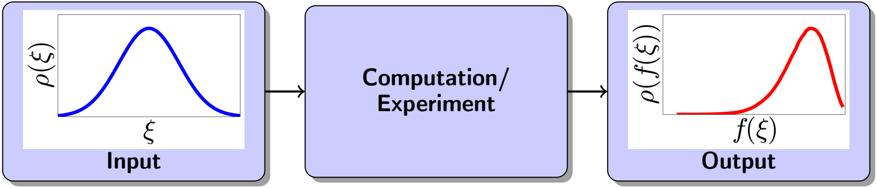


Background: Polynomial Chaos Expansions (PCE)

- Polynomial Chaos Expansions (PCE)
  - Method for quantifying parametric uncertainty efficiently
  - Spectral method in probability space
  - Expand output in terms of basis polynomial functions of random variables

$$f(\xi) \approx \sum_{i}^{N} a_i \phi_i(\xi)$$

$$\langle f, g \rangle = \int_{\Gamma} f(\xi) g(\xi) \rho(\xi) d\xi \quad , \quad \langle \phi_i, \phi_j \rangle = \delta_{ij}$$



Methodology

Quantify Uncertain Input

- Identify source of uncertainty
  - Physical parameters (eg. Reynolds number)
  - Boundary conditions
- Write a probabilistic description of uncertainty (ie. PDF)

Explore Uncertain Parameter Space

- Utilize efficient sampling of probability space
  - Quadrature nodes corresponding to a spectral basis
- Collect simulation data using discrete points in probability space
  - Immersed boundary projection method (IBPM) code

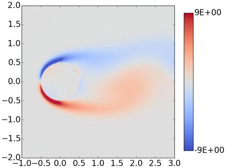
Quantify Uncertain Output

- Quantify uncertainty in outputs
  - POD modes
  - DMD modes
  - DMD eigenvalues

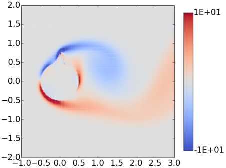
Topic

Setup

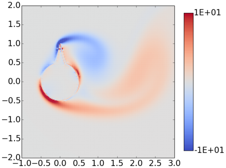
Small Spike

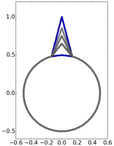


Medium Spike



Large Spike



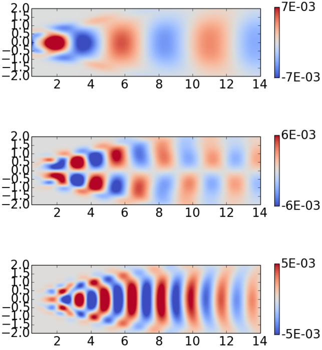


- Assume spike height is uniformly distributed between limits shown
- Re = 100
- Output = wake POD modes, DMD eigenvalues

Range of Flow Behavior

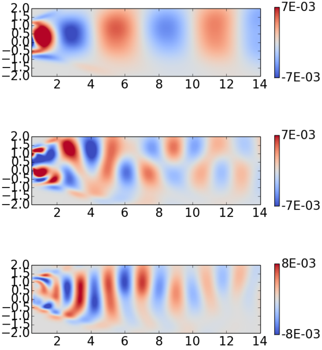
Cylinder, Re = 100

POD Modes

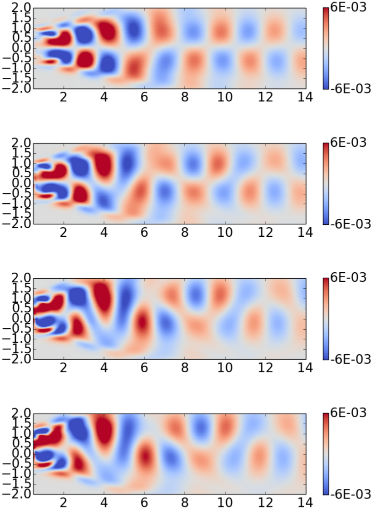


Perturbed Cylinder, Re = 100

POD Modes

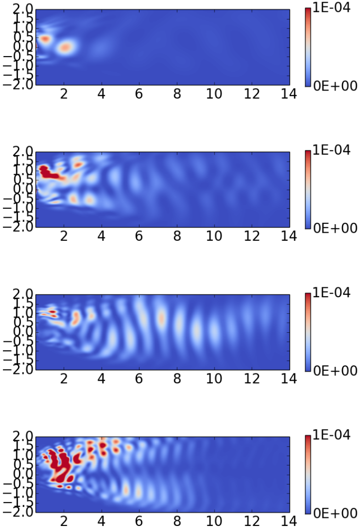


Query the Surrogate



- Can query the surrogate PCE model at different spike heights
  - Height = 0.14
  - Height = 0.36
  - Height = 0.63
  - Height = 0.86
- Behavior consistent with simulations

Statistical Variance in Modes



Projection Error

- Choose the $Q-1$ points halfway between Q quadrature nodes
- Calculate true modes and interpolated modes at $Q-1$ points
- Compare error between true modes and interpolated modes vs. true modes and mean modes

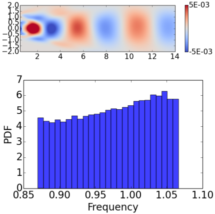
$$N(Y) \equiv max(||Y(\xi_k) - \Phi(\xi_k)||_2) \quad , \quad k = 1 \ldots Q-1$$

MODE	N(y_P)	N($\overline{y}$)	N($\overline{y}$)/$N(y_P)$
1	4e-3	3e-1	75
2	3e-2	9e-1	30
3	2e-1	1.2	6
4	7e-1	1.5	2
5	2e-1	1.8	9

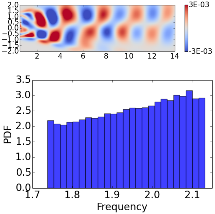
- PCE model captures range of symmetrical to asymmetrical modes

DMD Eigenvalues

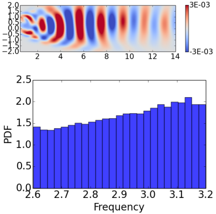
Low Frequency



Medium Frequency



High Frequency



- Output is the imaginary part of DMD eigenvalues
- Histograms are based on 10,000 Monte Carlo samples of the PCE surrogate
- Modest deformation of uniform distribution for all frequencies

Conclusions

- Uncertainty quantification techniques provide a fast, efficient, and accurate methodology for quantifying how low-dimensional models change with parametric uncertainty
  - POD modes
  - DMD modes/eigenvalues
- Further research
  - Multiple parameters
  - Apply to POD Galerkin models