SiMLR Simulation Study: Demonstrate basic assumptions for SiMLR and compare to SGCCA in 2 component recovery

Brian B. Avants et al.

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Decode two known latent signals distributed across 3 matrices with SiMLR and SGCCA. For each run, we:

- split the data matrices into train and test groups
- run SiMLR, RGCCA and SGCCA on the triad of matrices
- for SiMLR, we select a similarity measurement (via the energyType variable, default CCA-like and source separation method mixingMethod)
- predict the first latent signal from the low-dimensional space given the embedding data from each method
- predict the second latent signal from the low-dimensional space given the embedding data from each method
- use the amount of variance explained as the outcome measurement for both hidden signals.

Under the above simulation, a better method will more reliably explain the underlying true latent signals.

SiMLR outperforms SGCCA on this task.

```
set.seed( 999 )
library( ANTsR )
## Loading required package: ANTsRCore
## Attaching package: 'ANTsRCore'
## The following objects are masked from 'package:stats':
##
##
       sd, var
## The following objects are masked from 'package:base':
##
##
       all, any, apply, max, min, prod, range, sum
library( RGCCA )
library( smoother )
## Warning: package 'smoother' was built under R version 4.0.2
## Loading required package: TTR
doA = TRUE
smoothRows <- function( x ) {</pre>
  window = sample(25:150, 1)
 nr = nrow(x)
 nc = ncol(x)
  xout = x * 0
 for ( k in 1:nr ) {
```

```
vex = x[k,]
   xout[ k, ] = smoother::smth.gaussian( vex, window=window, tails=TRUE )
 antsrimpute( xout )
}
doCorruption = TRUE
if ( ! exists( "energyType" ) ) energyType = 'regression'
if ( ! exists( "mixingMethod" ) ) mixingMethod = 'ica'
nComp = 4 # n components
if ( ! exists( "nsims" ) ) nsims = 120
nits = 100 # n-iterations
            # simulated signal parameter - not sensitive to this choice
nzs = 1.5
            # simulated signal parameter - not sensitive to this choice
nzs2 = 1
           # simulated signal parameter - not sensitive to this choice
nna = rep( NA, nsims )
# this data frame will hold the results and allow us to answer questions about how noise impacts the ou
simdatafrm = data.frame(
   symRSQ1 = nna,
   sgccaRSQ1 = nna,
   symRSQ2 = nna,
   sgccaRSQ2 = nna,
   corrupt1 = nna,
   corrupt2 = nna,
   corrupt3 = nna,
   nTrueEmbeddings = nna,
   nForSim = nna
   )
sim = 1
for ( sim in c(sim:nsims) ) {
  smaller = rnorm( 1, 0.8, 0.2 )
  nsub = round( 400 * smaller ) # number of subjects
  npix = c( nsub*4, nsub*2, nsub * 8 ) # size of matrices are wildly different
  ntrain = 0.8 * round( nsub )
  train = sample(c(rep(T, ntrain), rep(F, nsub - ntrain))) # train and test split
  test = !train
  nEmbeddings = nk = sample( 5:25, 1 )# for latent signal
    # the outcome's first column is the latent signal that we are seeking
   mixmats = diag( nk )
   outcome = scale( matrix(runif( nsub * nk, nz, nzs2 ),ncol=nk) )
   # the 3 matrices below represent modality specific distributions
   view1tx = scale(matrix( rnorm( npix[1] * nk, nz, nzs ), nrow=nk ))
   view2tx = scale(matrix( rnorm( npix[2] * nk, nz, nzs*1.5 ), nrow=nk ))
   view3tx = scale(matrix( rnorm( npix[3] * nk, nz, nzs*0.8 ), nrow=nk ))
   # below we mix the independent basis matrices with the true signal
   outcomex = outcome
    # throw some difference in here - so really just the first column is the latent signal
   reo=3:nk
   outcomex[,reo]=sample(outcomex[,reo])
    # here, we resample the diagonal matrix to provide some variability about
    # where the signals appear across different modalities
    # we also smooth to provide some modality specific covariation
```

```
smoosig = abs( rnorm(3, 6, 1.5)) # draw from a distribution of smoothing parameters
if ( doA ) mixmat = as.matrix( smoothImage( as.antsImage( mixmats[(1:nrow(mixmats)),] ** view1tx)
# mix in the real signal --- repeat the same procedures for all 3 views of data
mat1 = (outcomex %*% mixmat )
outcomex=outcome
outcomex[,reo] = sample(outcomex[,reo])
if ( doA ) mixmat = as.matrix( smoothImage( as.antsImage( mixmats[(1:nrow(mixmats)),] %*% view2tx),
mat2 = (outcomex %*% mixmat )
outcomex = outcome
outcomex[,reo] = sample(outcomex[,reo])
if ( doA ) mixmat = as.matrix( smoothImage( as.antsImage( mixmats[(1:nrow(mixmats)),] %*% view3tx),
mat3 = (outcomex %*% mixmat )
# small additive noise for each matrix
mat1 = mat1 + matrix( rnorm( prod(dim(mat1)), 0, 0.25 ), nrow=nsub)
mat2 = mat2 + matrix( rnorm( prod(dim(mat2)), 0, 0.25 ), nrow=nsub)
mat3 = mat3 + matrix( rnorm( prod(dim(mat3)), 0, 0.25 ), nrow=nsub)
if ( doCorruption ) {
  # corrupt a portion of matrices - with random amounts of corruption each simulation
  ruinRate = runif(3,0.1,0.9)
  corrSDs = abs(rnorm(3, 10, 10))
  corrMNs = rnorm(3, 0, 10)
  corrInds = round(npix[3] * ruinRate[3] ):npix[3]
  mat3[ ,corrInds] = matrix( rnorm( prod(dim(mat3[ , corrInds])), corrMNs[3], corrSDs[3] ), nrow=ns
  corrInds = round(npix[2] * ruinRate[2] ):npix[2]
  mat2[ ,corrInds] = matrix( rnorm( prod(dim(mat2[ , corrInds])), corrMNs[2], corrSDs[2] ), nrow=ns
  corrInds = round(npix[1] * ruinRate[1] ):npix[1]
  mat1[ ,corrInds] = matrix( rnorm( prod(dim(mat1[ , corrInds])), corrMNs[1], corrSDs[1] ), nrow=ns
# automate the regularization selection using up to 50 neighbors for each matrix
inmats = list( vox = mat1[train,], vox2 = mat2[train,], vox3 = mat3[train,] )
regs = regularizeSimlr( list( mat1[train,], mat2[train,], mat3[train,] ),
                       rep(50, 3), sigma = rep(3.0, 3))
result = simlr(
  inmats,
  smoothingMatrices = regs,
  energyType = energyType,
  initialUMatrix = nComp,
  verbose = FALSE,
 iterations = nits,
   constraint = 'Grassmann', jointInitialization=FALSE,
  constraint = 'Stiefel', jointInitialization=TRUE,
  mixAlg = mixingMethod ) # allows different methods to be compared
p1 = mat1 %*% abs(result$v[[1]]); colnames(p1) = paste0("PC",1:ncol(p1))
p2 = mat2 %*% abs(result$v[[2]]); colnames(p2) = paste0("PC",1:ncol(p1))
p3 = mat3 %*% abs(result$v[[3]]); colnames(p3) = paste0("PC",1:ncol(p1))
nnn = 1:nComp
temp=data.frame(
  outc = outcome[,1],
```

```
sym1=p1[,nnn], sym2=p2[,nnn], sym3=p3[,nnn])
    mdlsym=lm( outc~.,data=temp[train,])
   dfPred = data.frame( true_test_outcome = temp$outc[test], predicted_outcome = predict(mdlsym,newdat
    mdlsymPred = lm( true_test_outcome ~ predicted_outcome, data=dfPred )
   rsqsym = cor( temp$outc[test], predict(mdlsym,newdata=temp[test,]) )^2
   temp=data.frame(
      outc = outcome[,2],
      sym1=p1[,nnn], sym2=p2[,nnn], sym3=p3[,nnn] )
   mdlsym=lm( outc~.,data=temp[train,])
   dfPred = data.frame( true_test_outcome = temp$outc[test], predicted_outcome = predict(mdlsym,newdat
   mdlsymPred = lm( true_test_outcome ~ predicted_outcome, data=dfPred )
   rsqsym2 = cor( temp$outc[test], predict(mdlsym,newdata=temp[test,]) )^2
    # compare to SGCCA - follow suggested glioma example in documentation
   myrgcca = sgcca( # this initializes with SVD
          A = inmats,
          scheme = "centroid",
         ncomp = rep( nComp, 3 ),
         scale = TRUE,
          c1 = rep(0.5, 3), # use something like simlr
         verbose = FALSE )
   prgcca = cbind(
      mat1 %*% myrgcca$a[[1]][,nnn],
     mat2 %*% myrgcca$a[[2]][,nnn],
     mat3 %*% myrgcca$a[[3]][,nnn] )
    temp=data.frame( outc = outcome[,1], prgcca[,1:(max(nnn)*3)] )
    mdlrgcca=lm( outc~.,data=temp[train,])
   rsqsgcca = cor( temp$outc[test], predict(mdlrgcca,newdata=temp[test,]) )^2
   temp=data.frame( outc = outcome[,2], prgcca[,1:(max(nnn)*3)] )
   mdlrgcca=lm( outc~.,data=temp[train,])
   rsqsgcca2 = cor( temp$outc[test], predict(mdlrgcca,newdata=temp[test,]) )^2
  simdatafrm[sim,] = c(
   rsqsym,
   rsqsgcca,
   rsqsym2,
   rsqsgcca2,
   ruinRate, nk, nsub )
  print( paste("Simulation:",sim,"rsqsym",rsqsym, "rsqsgcca", rsqsgcca")
  print( simdatafrm[sim,] )
  cat("<<<<******>>>\n")
 }
## [1] "Simulation: 1 rsqsym 0.423508764480075 rsqsgcca 0.435484617163221"
       symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 1 0.4235088 0.4354846 0.4233509 0.1902529 0.1528079 0.4730103 0.3470637
##
   nTrueEmbeddings nForSim
## 1
                 19
                         297
## <<<<******
## [1] "Simulation: 2 rsqsym 0.602896818444094 rsqsgcca 0.392994152628325"
       symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
```

```
## 2 0.6028968 0.3929942 0.330915 0.2049194 0.4869572 0.8479605 0.2439738
## nTrueEmbeddings nForSim
## 2
                11
## <<<<*******>>>
## [1] "Simulation: 3 rsqsym 0.468059511904875 rsqsgcca 0.407108638025823"
      symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 3 0.4680595 0.4071086 0.2863559 0.2934708 0.3309367 0.6530688 0.565707
    nTrueEmbeddings nForSim
## 3
                20
## <<<<*******>>>
## [1] "Simulation: 4 rsqsym 0.160566500798315 rsqsgcca 0.181787267317681"
      symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 4 0.1605665 0.1817873 0.3552943 0.4022963 0.2857894 0.3844166 0.5029228
## nTrueEmbeddings nForSim
## 4
                11
                       218
## <<<<******>>>
## [1] "Simulation: 5 rsqsym 0.407247342761032 rsqsgcca 0.384645966347922"
      symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 5 0.4072473 0.384646 0.2209504 0.09723628 0.2085293 0.8914678 0.1920348
## nTrueEmbeddings nForSim
## 5
                19
                       399
## <<<<******
## [1] "Simulation: 6 rsqsym 0.561125114061976 rsqsgcca 0.567337910678449"
      symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 6 0.5611251 0.5673379 0.2405937 0.2074819 0.8636401 0.8061877 0.354167
## nTrueEmbeddings nForSim
## 6
                       417
## <<<<******>>>
## [1] "Simulation: 7 rsqsym 0.42744261620474 rsqsgcca 0.464482431383991"
      symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 7 0.4274426 0.4644824 0.4125061 0.4362907 0.6414853 0.1984416 0.6308267
   nTrueEmbeddings nForSim
## 7
                23
                       268
## <<<<******
## [1] "Simulation: 8 rsqsym 0.332924088015538 rsqsgcca 0.260237463616256"
      symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 8 0.3329241 0.2602375 0.07470364 0.06861664 0.3343895 0.598908 0.6975472
## nTrueEmbeddings nForSim
## 8
                24
                       243
## <<<<******
## [1] "Simulation: 9 rsqsym 0.536016775210126 rsqsgcca 0.559196445306127"
      symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 9 0.5360168 0.5591964 0.2067606 0.1653589 0.4846665 0.4283398 0.6475849
## nTrueEmbeddings nForSim
                       250
## <<<<******>>>
## [1] "Simulation: 10 rsqsym 0.40673455201131 rsqsgcca 0.423744999191642"
       symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
nTrueEmbeddings nForSim
## 10
                 16
                        367
## <<<<******
## [1] "Simulation: 11 rsqsym 0.393922864563505 rsqsgcca 0.446441965351105"
       symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
```

```
## 11 0.3939229 0.446442 0.1799417 0.06144816 0.8869186 0.6413863 0.2593609
   nTrueEmbeddings nForSim
## 11
                   9
## <<<<*******>>>
## [1] "Simulation: 12 rsqsym 0.549180282083445 rsqsgcca 0.465646938867039"
       symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 12 0.5491803 0.4656469 0.4365524 0.3206162 0.2112665 0.4690948 0.4236031
     nTrueEmbeddings nForSim
## 12
                  13
## <<<<******>>>
## [1] "Simulation: 13 rsqsym 0.425278648443758 rsqsgcca 0.539068642106174"
       symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 13 0.4252786 0.5390686 0.3065442 0.1747128 0.758147 0.4701019 0.2800488
##
     nTrueEmbeddings nForSim
## 13
                  21
                         310
## <<<<******>>>
## [1] "Simulation: 14 rsqsym 0.196224686313602 rsqsgcca 0.00108499595128204"
       symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 14 0.1962247 0.001084996 0.3591156 0.06918068 0.1378957 0.4512808 0.2691725
     nTrueEmbeddings nForSim
## 14
                  23
                        191
## <<<<******>>>
## [1] "Simulation: 15 rsqsym 0.412843100766646 rsqsgcca 0.425292591055341"
       symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 15 0.4128431 0.4252926 0.2605954 0.2608751 0.2746883 0.7960618 0.4102065
   nTrueEmbeddings nForSim
## 15
                         239
                  11
## <<<<******>>>
## [1] "Simulation: 16 rsqsym 0.667247183857775 rsqsgcca 0.671616002149964"
       symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 16 0.6672472 0.671616 0.2258464 0.1776277 0.5950145 0.7540498 0.5977314
     nTrueEmbeddings nForSim
## 16
                   6
                         374
## <<<<******
## [1] "Simulation: 17 rsqsym 0.453867415519179 rsqsgcca 0.369735756372679"
       symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 17 0.4538674 0.3697358 0.325499 0.1907631 0.834107 0.3217941 0.4684049
##
     nTrueEmbeddings nForSim
## 17
                  20
                         302
## <<<<******>>>
## [1] "Simulation: 18 rsqsym 0.151511318718421 rsqsgcca 0.203717552955127"
       symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 18 0.1515113 0.2037176 0.003682725 0.1080787 0.1664618 0.1158756 0.3925958
##
     nTrueEmbeddings nForSim
                  16
                         198
## <<<<******>>>
## [1] "Simulation: 19 rsqsym 0.320883873377213 rsqsgcca 0.363935301023848"
       symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 19 0.3208839 0.3639353 0.3861397 0.3438188 0.2127765 0.5465544 0.4562401
     nTrueEmbeddings nForSim
## 19
                   8
                         469
## <<<<******
## Warning in sgccak(R, C, c1 = c1, scheme = scheme, init = init, bias = bias, :
```

```
## The SGCCA algorithm did not converge after 1000 iterations.
## [1] "Simulation: 20 rsqsym 0.444259565595044 rsqsgcca 0.452208384031017"
       symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 20 0.4442596 0.4522084 0.4726916 0.3844132 0.6861961 0.4684361 0.4483833
##
     nTrueEmbeddings nForSim
## 20
                 22
                        447
## <<<<******
## [1] "Simulation: 21 rsqsym 0.612998546164954 rsqsgcca 0.45476303781094"
       symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
nTrueEmbeddings nForSim
## 21
                 13
                        162
## <<<<******>>>
## [1] "Simulation: 22 rsqsym 0.537131743862852 rsqsgcca 0.296115955223362"
       symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 22 0.5371317 0.296116 0.2922205 0.1242977 0.2510803 0.6638064 0.157919
##
     nTrueEmbeddings nForSim
                 23
## <<<<******>>>
## [1] "Simulation: 23 rsqsym 0.491477459044192 rsqsgcca 0.442476647163536"
       symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 23 0.4914775 0.4424766 0.4428007 0.3725482 0.8820172 0.257116 0.8844816
     nTrueEmbeddings nForSim
## 23
                  8
                       386
## <<<<******>>>
## [1] "Simulation: 24 rsqsym 0.558655046551794 rsqsgcca 0.437188218040455"
      symRSQ1 sgccaRSQ1
                       symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 24 0.558655 0.4371882 0.4746795 0.1332137 0.2275583 0.1067845 0.3267447
     nTrueEmbeddings nForSim
## 24
                 19
                        382
## <<<<******>>>
## [1] "Simulation: 25 rsqsym 0.433760587301254 rsqsgcca 0.467912333912212"
       symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 25 0.4337606 0.4679123 0.2779834 0.1274827 0.7449202 0.8346043 0.1607545
##
     nTrueEmbeddings nForSim
## 25
                 14
                        406
## <<<<******>>>
## [1] "Simulation: 26 rsqsym 0.483959508001027 rsqsgcca 0.476565931341537"
       symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 26 0.4839595 0.4765659 0.2021536 0.2434862 0.2988959 0.7332857 0.5903049
##
     nTrueEmbeddings nForSim
## 26
                 16
                        232
## <<<<******>>>
## [1] "Simulation: 27 rsqsym 0.218726789413722 rsqsgcca 0.281060953442568"
       symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
##
     nTrueEmbeddings nForSim
                 18
## <<<<******>>>
## Warning in sgccak(R, C, c1 = c1, scheme = scheme, init = init, bias = bias, :
## The SGCCA algorithm did not converge after 1000 iterations.
## [1] "Simulation: 28 rsqsym 0.288988440089472 rsqsgcca 0.0415435576908198"
```

```
symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 28 0.2889884 0.04154356 0.3351967 0.008740797 0.5877902 0.2059184 0.1188846
     nTrueEmbeddings nForSim
## 28
                  13
## <<<<******>>>
## [1] "Simulation: 29 rsqsym 0.46564725732052 rsqsgcca 0.446869439254422"
       symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 29 0.4656473 0.4468694 0.4828166 0.4236607 0.5426349 0.1619521 0.5641378
     nTrueEmbeddings nForSim
## <<<<******>>>
## [1] "Simulation: 30 rsqsym 0.539798740985163 rsqsgcca 0.535991684734143"
       symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 30 0.5397987 0.5359917 0.2051578 0.2252613 0.5122877 0.2884313 0.2333043
     nTrueEmbeddings nForSim
## 30
                  18
                         346
## <<<<******>>>
## [1] "Simulation: 31 rsqsym 0.483124274044693 rsqsgcca 0.472137786138055"
       symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 31 0.4831243 0.4721378 0.3087922 0.1954034 0.4979643 0.5330915 0.430925
##
     nTrueEmbeddings nForSim
                  14
## <<<<******>>>
## [1] "Simulation: 32 rsqsym 0.618171493808309 rsqsgcca 0.60794827743663"
       symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 32 0.6181715 0.6079483 0.2768289 0.1555765 0.2079414 0.8760748 0.3906488
     nTrueEmbeddings nForSim
## 32
                   6
                         342
## <<<<******>>>
## [1] "Simulation: 33 rsqsym 0.637655523936728 rsqsgcca 0.630170643693716"
       symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 33 0.6376555 0.6301706 0.3273998 0.3090232 0.5541301 0.7810494 0.4066829
     nTrueEmbeddings nForSim
## 33
                   5
                         412
## <<<<******>>>
## [1] "Simulation: 34 rsqsym 0.362442236986508 rsqsgcca 0.440371562789379"
       symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 34 0.3624422 0.4403716 0.2176157 0.218577 0.6570363 0.8091384 0.4742028
     nTrueEmbeddings nForSim
                  10
                         381
## 34
## <<<<******>>>
## [1] "Simulation: 35 rsqsym 0.45058141174496 rsqsgcca 0.41208376407335"
       symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 35 0.4505814 0.4120838 0.285278 0.1673379 0.5685066 0.5193919 0.3912626
     nTrueEmbeddings nForSim
                   8
                         363
## 35
## <<<<******>>>
## [1] "Simulation: 36 rsqsym 0.413528184208726 rsqsgcca 0.38807694875069"
       symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 36 0.4135282 0.3880769 0.1221272 0.219212 0.7377817 0.5349642 0.5001206
##
     nTrueEmbeddings nForSim
## 36
                  12
## <<<<******>>>
## [1] "Simulation: 37 rsqsym 0.413824322532293 rsqsgcca 0.471550820389872"
```

```
symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 37 0.4138243 0.4715508 0.3715787 0.3835567 0.6250241 0.5786765 0.4790985
     nTrueEmbeddings nForSim
## 37
                  18
                        463
## <<<<******>>>
## [1] "Simulation: 38 rsqsym 0.464378979194589 rsqsgcca 0.513773266921626"
      symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 38 0.464379 0.5137733 0.3136395 0.3583196 0.1619121 0.5016935 0.4688382
     nTrueEmbeddings nForSim
## 38
                  19
## <<<<******>>>
## [1] "Simulation: 39 rsqsym 0.43618315707526 rsqsgcca 0.446678961074918"
       symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
nTrueEmbeddings nForSim
## 39
                  19
                        326
## <<<<******>>>
## [1] "Simulation: 40 rsqsym 0.644658554013448 rsqsgcca 0.562654577502725"
       symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 40 0.6446586 0.5626546 0.3922975 0.3753444 0.8727865 0.5542072 0.7723171
##
     nTrueEmbeddings nForSim
## 40
                  19
## <<<<******
## [1] "Simulation: 41 rsqsym 0.316133987997781 rsqsgcca 0.358732195159431"
      symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 41 0.316134 0.3587322 0.195772 0.2125657 0.7228659 0.1712609 0.4665875
     nTrueEmbeddings nForSim
## 41
                  16
## <<<<******>>>
## [1] "Simulation: 42 rsqsym 0.446743071445536 rsqsgcca 0.246632821072626"
       symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 42 0.4467431 0.2466328 0.2975752 0.1990916 0.1838669 0.4354277 0.5549797
     nTrueEmbeddings nForSim
## 42
                  14
                        410
## <<<<******>>>
## [1] "Simulation: 43 rsqsym 0.594975131527109 rsqsgcca 0.538203843295017"
       symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 43 0.5949751 0.5382038 0.2863066 0.2454453 0.7421124 0.7611756 0.6793826
     nTrueEmbeddings nForSim
## 43
                   5
                        349
## <<<<******>>>
## [1] "Simulation: 44 rsqsym 0.404227012773244 rsqsgcca 0.458589140052937"
      symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 44 0.404227 0.4585891 0.4328985 0.3882897 0.2844905 0.7824221 0.7410174
     nTrueEmbeddings nForSim
## 44
                  18
                        397
## <<<<******>>>
## [1] "Simulation: 45 rsqsym 0.446025855558032 rsqsgcca 0.588894311544007"
       symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 45 0.4460259 0.5888943 0.1279134 0.1963391 0.2064804 0.583958 0.6615154
##
     nTrueEmbeddings nForSim
## 45
                  22
## <<<<******>>>
## [1] "Simulation: 46 rsqsym 0.51069934866645 rsqsgcca 0.42057915474007"
```

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symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 46 0.5106993 0.4205792 0.3492178 0.3174525 0.7958481 0.2901325 0.7011819
     nTrueEmbeddings nForSim
## 46
                  25
                         310
## <<<<******>>>
## [1] "Simulation: 47 rsqsym 0.476738050498996 rsqsgcca 0.428715143422349"
       symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 47 0.4767381 0.4287151 0.2503024 0.2993654 0.517597 0.3640791 0.5288238
     nTrueEmbeddings nForSim
                  22
                         373
## <<<<******>>>
## [1] "Simulation: 48 rsqsym 0.409558747678858 rsqsgcca 0.335793624327952"
       symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 48 0.4095587 0.3357936 0.1658419 0.1806561 0.4683663 0.2596516 0.4279413
     {\tt nTrueEmbeddings}\ {\tt nForSim}
## 48
                  20
                         487
## <<<<******
## [1] "Simulation: 49 rsqsym 0.400839678619019 rsqsgcca 0.144232221663116"
       symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 49 0.4008397 0.1442322 0.299217 0.1479208 0.3132661 0.5130508 0.1222794
##
     nTrueEmbeddings nForSim
## 49
                  23
## <<<<******>>>
## Warning in sgccak(R, C, c1 = c1, scheme = scheme, init = init, bias = bias, :
## The SGCCA algorithm did not converge after 1000 iterations.
## [1] "Simulation: 50 rsqsym 0.548614812776767 rsqsgcca 0.55812010901291"
       symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 50 0.5486148 0.5581201 0.2543359 0.1649547 0.4244511 0.8887626 0.6784549
     nTrueEmbeddings nForSim
## 50
                   5
## <<<<******
## [1] "Simulation: 51 rsqsym 0.388673508112558 rsqsgcca 0.372065265605415"
       symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 51 0.3886735 0.3720653 0.2627962 0.04766187 0.6794811 0.7984887 0.2327572
     nTrueEmbeddings nForSim
## 51
                  21
                         120
## <<<<******>>>
## [1] "Simulation: 52 rsqsym 0.591902109779209 rsqsgcca 0.561831942733463"
       symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 52 0.5919021 0.5618319 0.3644149 0.3751828 0.5734654 0.4420546 0.460786
     nTrueEmbeddings nForSim
##
## 52
                  21
## <<<<*******>>>
## [1] "Simulation: 53 rsqsym 0.49421838752385 rsqsgcca 0.489848031838596"
       symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
##
## 53 0.4942184 0.489848 0.2510261 0.02737021 0.3299497 0.6254233 0.839792
     nTrueEmbeddings nForSim
## 53
                  23
                         136
## <<<<******>>>
## [1] "Simulation: 54 rsqsym 0.510168614963973 rsqsgcca 0.468803362703102"
       symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 54 0.5101686 0.4688034 0.3262951 0.360053 0.3714408 0.2927066 0.3813086
     nTrueEmbeddings nForSim
```

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247
## <<<<******
## [1] "Simulation: 55 rsqsym 0.219164292584147 rsqsgcca 0.2222120666972"
       symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 55 0.2191643 0.2222121 0.2452796 0.1974284 0.5108066 0.1644917 0.4873366
     nTrueEmbeddings nForSim
## 55
                  25
## <<<<******>>>
## [1] "Simulation: 56 rsqsym 0.333666883107422 rsqsgcca 0.314117447944115"
       symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 56 0.3336669 0.3141174 0.2549724 0.1758256 0.2689334 0.6017487 0.8247073
     nTrueEmbeddings nForSim
##
## 56
                  15
## <<<<*******>>>
## [1] "Simulation: 57 rsqsym 0.512766505590528 rsqsgcca 0.478176501315565"
       symRSQ1 sgccaRSQ1
                           symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 57 0.5127665 0.4781765 0.04905919 0.1011179 0.4837212 0.1470487 0.1336431
     nTrueEmbeddings nForSim
## 57
                  16
                         237
## <<<<******>>>
## [1] "Simulation: 58 rsqsym 0.672661443536249 rsqsgcca 0.657561350710122"
       symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 58 0.6726614 0.6575614 0.1246424 0.01415097 0.1016929 0.1889634 0.3395229
     nTrueEmbeddings nForSim
##
                         229
## 58
                  16
## <<<<******>>>
## [1] "Simulation: 59 rsqsym 0.475504955844789 rsqsgcca 0.378353776679428"
      symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 59 0.475505 0.3783538 0.2346235 0.07190521 0.4197073 0.8143427 0.3800812
     nTrueEmbeddings nForSim
## 59
                  19
                         307
## <<<<******>>>
## [1] "Simulation: 60 rsqsym 0.513461714962024 rsqsgcca 0.428871759637976"
       symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 60 0.5134617 0.4288718 0.1397901 0.09503094 0.8085137 0.3007819 0.7733959
     nTrueEmbeddings nForSim
## 60
                  15
                         308
## <<<<******>>>
## [1] "Simulation: 61 rsqsym 0.487704100602317 rsqsgcca 0.381244697963929"
       symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 61 0.4877041 0.3812447 0.2107391 0.1845455 0.6643969 0.8379465 0.6321312
     nTrueEmbeddings nForSim
##
## 61
                  19
## <<<<******>>>
## [1] "Simulation: 62 rsqsym 0.466391817203409 rsqsgcca 0.39971804003301"
       symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
##
## 62 0.4663918 0.399718 0.3204575 0.3703567 0.3956174 0.4745393 0.817485
     nTrueEmbeddings nForSim
## 62
                  10
                         284
## <<<<******>>>
## [1] "Simulation: 63 rsqsym 0.355790743726059 rsqsgcca 0.245606688313936"
       symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 63 0.3557907 0.2456067 0.3214198 0.02472152 0.1464791 0.4201793 0.1416975
     nTrueEmbeddings nForSim
```

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417
## <<<<******
## [1] "Simulation: 64 rsqsym 0.562105071023776 rsqsgcca 0.579963474773422"
       symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 64 0.5621051 0.5799635 0.1352763 0.1953119 0.4948438 0.6401077 0.7359811
     nTrueEmbeddings nForSim
## 64
                   5
                         335
## <<<<******>>>
## [1] "Simulation: 65 rsqsym 0.624454068831319 rsqsgcca 0.615447787849176"
       symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 65 0.6244541 0.6154478 0.3483742 0.3390995 0.5708133 0.2593225 0.6372742
     nTrueEmbeddings nForSim
##
## 65
                   6
## <<<<******>>>
## [1] "Simulation: 66 rsqsym 0.354838833867149 rsqsgcca 0.367184833609197"
       symRSQ1 sgccaRSQ1
                           symRSQ2
                                      sgccaRSQ2 corrupt1 corrupt2 corrupt3
##
## 66 0.3548388 0.3671848 0.06024169 0.008596782 0.5219328 0.77096 0.2131016
     nTrueEmbeddings nForSim
## 66
                   9
                         217
## <<<<******>>>
## [1] "Simulation: 67 rsqsym 0.477464004632046 rsqsgcca 0.351467511826976"
      symRSQ1 sgccaRSQ1
                         symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 67 0.477464 0.3514675 0.2194754 0.1202089 0.1918682 0.6976846 0.459312
##
     nTrueEmbeddings nForSim
## 67
                  13
                         437
## <<<<******>>>
## [1] "Simulation: 68 rsqsym 0.407638059005117 rsqsgcca 0.412511949213816"
       symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 68 0.4076381 0.4125119 0.3392755 0.3150783 0.3526001 0.2593282 0.4147314
     nTrueEmbeddings nForSim
## 68
                  19
                         429
## <<<<******>>>
## [1] "Simulation: 69 rsqsym 0.250566038252212 rsqsgcca 0.279145408358372"
      symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 69 0.250566 0.2791454 0.3005286 0.4082336 0.2825821 0.5111464 0.8163508
     nTrueEmbeddings nForSim
##
## 69
                  21
                         233
## <<<<******>>>
## [1] "Simulation: 70 rsqsym 0.27221839817641 rsqsgcca 0.213711423312469"
       symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 70 0.2722184 0.2137114 0.1267984 0.03966809 0.8529719 0.7037634 0.2893305
     nTrueEmbeddings nForSim
##
## 70
                  11
## <<<<*******>>>
## [1] "Simulation: 71 rsqsym 0.251728925601898 rsqsgcca 0.328688740845694"
       symRSQ1 sgccaRSQ1
                           symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
##
## 71 0.2517289 0.3286887 0.04558217 0.07317162 0.6458563 0.5544131 0.5678915
     nTrueEmbeddings nForSim
## 71
                  18
                         164
## <<<<******>>>
## [1] "Simulation: 72 rsqsym 0.497963960764412 rsqsgcca 0.491378881265256"
      symRSQ1 sgccaRSQ1
                        symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 72 0.497964 0.4913789 0.1421302 0.204866 0.7854737 0.7760154 0.1042435
     nTrueEmbeddings nForSim
```

```
## 72
                         350
## <<<<******
## [1] "Simulation: 73 rsqsym 0.228734074479321 rsqsgcca 0.606866936926101"
       symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 73 0.2287341 0.6068669 0.4182471 0.4237917 0.8560702 0.1607534 0.6813074
     nTrueEmbeddings nForSim
##
## 73
                         171
## <<<<******>>>
## [1] "Simulation: 74 rsqsym 0.503246862286375 rsqsgcca 0.348381158764383"
       symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 74 0.5032469 0.3483812 0.1774558 0.1437948 0.6123991 0.2624095 0.8066568
     nTrueEmbeddings nForSim
##
## 74
                   8
## <<<<******>>>
## [1] "Simulation: 75 rsqsym 0.460016395637136 rsqsgcca 0.412388715808705"
       symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
##
## 75 0.4600164 0.4123887 0.3866481 0.4463403 0.5548297 0.131352 0.7470957
     nTrueEmbeddings nForSim
## 75
                         329
## <<<<******>>>
## [1] "Simulation: 76 rsqsym 0.337197316859733 rsqsgcca 0.25486580726165"
       symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 76 0.3371973 0.2548658 0.101456 0.1272535 0.4358554 0.2644651 0.8486031
##
     nTrueEmbeddings nForSim
## 76
                  22
                         223
## <<<<******>>>
## [1] "Simulation: 77 rsqsym 0.494072280164883 rsqsgcca 0.381129081575396"
       symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 77 0.4940723 0.3811291 0.2726069 0.1295647 0.3081183 0.5878796 0.2262874
##
     nTrueEmbeddings nForSim
## 77
                  24
                         295
## <<<<******>>>
## [1] "Simulation: 78 rsqsym 0.654065173699139 rsqsgcca 0.599162819435131"
       symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 78 0.6540652 0.5991628 0.4625908 0.4192834 0.4846375 0.7301931 0.6823927
     nTrueEmbeddings nForSim
##
## 78
                   6
                         322
## <<<<******>>>
## [1] "Simulation: 79 rsqsym 0.551702616409293 rsqsgcca 0.761797803488695"
       symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 79 0.5517026 0.7617978 0.401564 0.1477253 0.2006784 0.4993553 0.2210576
     nTrueEmbeddings nForSim
##
## 79
                   5
## <<<<******>>>
## [1] "Simulation: 80 rsqsym 0.476221393320365 rsqsgcca 0.126967769597915"
       symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
##
## 80 0.4762214 0.1269678 0.446546 0.3830049 0.8756047 0.6818467 0.2863321
     nTrueEmbeddings nForSim
## 80
                  16
                         288
## <<<<******>>>
## [1] "Simulation: 81 rsqsym 0.47584134492208 rsqsgcca 0.409022087991925"
       symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 81 0.4758413 0.4090221 0.2867083 0.2583069 0.2001104 0.4423019 0.3564712
     nTrueEmbeddings nForSim
```

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311
## <<<<******
## [1] "Simulation: 82 rsqsym 0.650569013815316 rsqsgcca 0.627424965980112"
      symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 82 0.650569 0.627425 0.3771305 0.3836585 0.6722291 0.8963098 0.7994494
     nTrueEmbeddings nForSim
## 82
## <<<<******>>>
## [1] "Simulation: 83 rsqsym 0.3294776551551 rsqsgcca 0.33165549659564"
       symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 83 0.3294777 0.3316555 0.2370113 0.1990512 0.1285838 0.735307 0.2908897
     nTrueEmbeddings nForSim
##
## 83
                  18
## <<<<******>>>
## [1] "Simulation: 84 rsqsym 0.471308058789125 rsqsgcca 0.370662817302746"
       symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
##
## 84 0.4713081 0.3706628 0.3276988 0.2036059 0.289626 0.4666353 0.1141572
     nTrueEmbeddings nForSim
                  12
## 84
                         303
## <<<<******>>>
## [1] "Simulation: 85 rsqsym 0.420102081439084 rsqsgcca 0.392003263957976"
       symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 85 0.4201021 0.3920033 0.2870685 0.09740123 0.1934827 0.7494017 0.1804987
##
     nTrueEmbeddings nForSim
## 85
                  24
                         224
## <<<<******>>>
## [1] "Simulation: 86 rsqsym 0.488684496503759 rsqsgcca 0.51334901300592"
       symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 86 0.4886845 0.513349 0.1473393 0.2989333 0.4748731 0.506586 0.7978548
     nTrueEmbeddings nForSim
## 86
                  11
                         211
## <<<<******>>>
## [1] "Simulation: 87 rsqsym 0.281102676311363 rsqsgcca 0.250281418301648"
       symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 87 0.2811027 0.2502814 0.1816158 0.1513996 0.1212744 0.5678796 0.8278991
     nTrueEmbeddings nForSim
##
## 87
                  24
                         448
## <<<<******>>>
## [1] "Simulation: 88 rsqsym 0.334651989956658 rsqsgcca 0.425516141775913"
      symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 88 0.334652 0.4255161 0.3189763 0.2657895 0.8205069 0.6799475 0.3415895
     nTrueEmbeddings nForSim
##
## 88
                  25
## <<<<******>>>
## [1] "Simulation: 89 rsqsym 0.579438981505153 rsqsgcca 0.641585409937794"
                         symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
##
      symRSQ1 sgccaRSQ1
## 89 0.579439 0.6415854 0.3204669 0.1786059 0.2644734 0.3922191 0.197737
     nTrueEmbeddings nForSim
## 89
                  10
                         241
## <<<<******>>>
## [1] "Simulation: 90 rsqsym 0.542481624939354 rsqsgcca 0.540422302728259"
       symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 90 0.5424816 0.5404223 0.3132501 0.2244465 0.1047087 0.465873 0.668139
     nTrueEmbeddings nForSim
```

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## 90
                         460
## <<<<*******>>>
## Warning in sgccak(R, C, c1 = c1, scheme = scheme, init = init, bias = bias, :
## The SGCCA algorithm did not converge after 1000 iterations.
## [1] "Simulation: 91 rsqsym 0.646770997468049 rsqsgcca 0.475267084600012"
      symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 91 0.646771 0.4752671 0.408815 0.340636 0.5932293 0.2578457 0.4376522
     nTrueEmbeddings nForSim
## 91
## <<<<******>>>
## [1] "Simulation: 92 rsqsym 0.439373232951051 rsqsgcca 0.476402998084094"
       symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 92 0.4393732 0.476403 0.11564 0.1308203 0.8253608 0.8424116 0.3257419
     {\tt nTrueEmbeddings}\ {\tt nForSim}
## 92
                  18
                         367
## <<<<******
## [1] "Simulation: 93 rsqsym 0.490759939097487 rsqsgcca 0.454767660904333"
       symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 93 0.4907599 0.4547677 0.4360947 0.4916065 0.5043511 0.597646 0.634375
##
     nTrueEmbeddings nForSim
## 93
## <<<<******>>>
## [1] "Simulation: 94 rsqsym 0.165068139466299 rsqsgcca 0.0910135884692659"
       symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 94 0.1650681 0.09101359 0.1774087 0.2109157 0.1401681 0.3079609 0.5048407
     nTrueEmbeddings nForSim
                   8
## 94
                         193
## <<<<******>>>
## [1] "Simulation: 95 rsqsym 0.601396930095418 rsqsgcca 0.569578913284281"
       symRSQ1 sgccaRSQ1
                          symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 95 0.6013969 0.5695789 0.5099276 0.5235029 0.8154165 0.3897922 0.8007865
     nTrueEmbeddings nForSim
                  23
## 95
                         240
## <<<<******>>>
## [1] "Simulation: 96 rsqsym 0.530418230865245 rsqsgcca 0.418879898647659"
       symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 96 0.5304182 0.4188799 0.3034562 0.2590539 0.7398697 0.2556735 0.7227137
     nTrueEmbeddings nForSim
                         242
## 96
                   5
## <<<<*****
## [1] "Simulation: 97 rsqsym 0.513190434882077 rsqsgcca 0.517721849560541"
       symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 97 0.5131904 0.5177218 0.2983547     0.39941 0.4698408 0.2892295 0.8089506
     nTrueEmbeddings nForSim
                  12
## 97
                         344
## <<<<******>>>
## [1] "Simulation: 98 rsqsym 0.295402006686886 rsqsgcca 0.356360685803985"
      symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 98 0.295402 0.3563607 0.3161387 0.1627723 0.110053 0.736788 0.6602833
##
     nTrueEmbeddings nForSim
## 98
                  15
## <<<<******>>>
## [1] "Simulation: 99 rsqsym 0.578787872788604 rsqsgcca 0.564969825374481"
```

```
symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 99 0.5787879 0.5649698 0.3320243 0.3938128 0.8649713 0.678966 0.2438317
     nTrueEmbeddings nForSim
## 99
                   5
                         286
## <<<<******>>>
## [1] "Simulation: 100 rsqsym 0.672546662635403 rsqsgcca 0.705982889113691"
        symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 100 0.6725467 0.7059829 0.1605994 0.1536762 0.3760107 0.1639654 0.6361637
##
      nTrueEmbeddings nForSim
## 100
## <<<<******>>>
## [1] "Simulation: 101 rsqsym 0.471410628189108 rsqsgcca 0.2583883811326"
        symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 101 0.4714106 0.2583884 0.207847 0.206863 0.441508 0.6127449 0.3145526
      nTrueEmbeddings nForSim
## 101
                   13
                          445
## <<<<******
## [1] "Simulation: 102 rsqsym 0.559544835094354 rsqsgcca 0.388289259016514"
        symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 102 0.5595448 0.3882893 0.2885642
                                     0.2811 0.6494493 0.7471799 0.5918866
##
      nTrueEmbeddings nForSim
## 102
                   18
## <<<<******>>>
## [1] "Simulation: 103 rsqsym 0.67593427652584 rsqsgcca 0.672368612760641"
        symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 103 0.6759343 0.6723686 0.3117751 0.3413562 0.1255618 0.7788559 0.8471632
      nTrueEmbeddings nForSim
## 103
                          515
## <<<<******>>>
## [1] "Simulation: 104 rsqsym 0.480328246955971 rsqsgcca 0.483937653413406"
        symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 104 0.4803282 0.4839377 0.3870641 0.422326 0.3736461 0.8978278 0.868112
##
      nTrueEmbeddings nForSim
## 104
                   16
                          361
## <<<<******>>>
## [1] "Simulation: 105 rsqsym 0.428172234952916 rsqsgcca 0.44211027980344"
        symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 105 0.4281722 0.4421103 0.1791217 0.1096631 0.594447 0.7863907 0.5695057
      nTrueEmbeddings nForSim
                   25
## 105
                          403
## <<<<******>>>
## [1] "Simulation: 106 rsqsym 0.278385063999525 rsqsgcca 0.306017901430545"
        symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 106 0.2783851 0.3060179 0.1133166 0.1809248 0.7388901 0.3063571 0.4309404
      nTrueEmbeddings nForSim
                          259
## 106
                   20
## <<<<******>>>
## [1] "Simulation: 107 rsqsym 0.617020411120902 rsqsgcca 0.510640581647732"
        symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 107 0.6170204 0.5106406 0.2197862   0.23306 0.2034699 0.7454509 0.1469591
##
      nTrueEmbeddings nForSim
## 107
                   19
## <<<<******>>>
## [1] "Simulation: 108 rsqsym 0.471270959463565 rsqsgcca 0.467352680249141"
```

```
symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 108 0.471271 0.4673527 0.3877556 0.3858443 0.857093 0.6986732 0.7077201
      nTrueEmbeddings nForSim
## 108
                   10
## <<<<******>>>
## [1] "Simulation: 109 rsqsym 0.484724269919906 rsqsgcca 0.407900280199314"
        symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 109 0.4847243 0.4079003 0.2500876 0.1218499 0.2865298 0.8040656 0.2956018
##
      nTrueEmbeddings nForSim
## 109
## <<<<******>>>
## [1] "Simulation: 110 rsqsym 0.196580145409646 rsqsgcca 0.134752912464734"
        symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 110 0.1965801 0.1347529 0.1635228 0.146297 0.3332688 0.4480866 0.7756696
      nTrueEmbeddings nForSim
## 110
                   25
                          178
## <<<<******
## [1] "Simulation: 111 rsqsym 0.394867614491467 rsqsgcca 0.406502444333483"
        symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 111 0.3948676 0.4065024 0.3451389 0.4052655 0.6475272 0.4720507 0.5124654
##
      nTrueEmbeddings nForSim
                   19
## <<<<******>>>
## [1] "Simulation: 112 rsqsym 0.454013383994139 rsqsgcca 0.236200119335614"
        symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 112 0.4540134 0.2362001 0.2696289 0.1137563 0.6880995 0.8346916 0.2432443
      nTrueEmbeddings nForSim
## 112
                          113
## <<<<******>>>
## [1] "Simulation: 113 rsqsym 0.44149284949681 rsqsgcca 0.249411110183143"
        symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 113 0.4414928 0.2494111 0.4174107 0.2745403 0.5979778 0.2912934 0.3509965
      nTrueEmbeddings nForSim
## 113
                          356
## <<<<******>>>
## [1] "Simulation: 114 rsqsym 0.310232161811422 rsqsgcca 0.357697246768259"
        symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 114 0.3102322 0.3576972 0.3262102 0.4028846 0.3026995 0.7481733 0.4977857
      nTrueEmbeddings nForSim
## 114
                   19
                          237
## <<<<******>>>
## [1] "Simulation: 115 rsqsym 0.253451308518997 rsqsgcca 0.142609836821939"
        symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 115 0.2534513 0.1426098 0.2944198 0.153556 0.4219188 0.1915054 0.1881554
      nTrueEmbeddings nForSim
                   22
## 115
                          348
## <<<<******>>>
## [1] "Simulation: 116 rsqsym 0.425566810846943 rsqsgcca 0.434218114002224"
        symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 116 0.4255668 0.4342181 0.1996378 0.2521373 0.8993718 0.8467455 0.7006626
##
      nTrueEmbeddings nForSim
## 116
                   10
## <<<<******>>>
## [1] "Simulation: 117 rsqsym 0.526846812253803 rsqsgcca 0.485860816993957"
```

```
symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 117 0.5268468 0.4858608 0.3009444 0.157504 0.6876109 0.2071291 0.2320612
      nTrueEmbeddings nForSim
## 117
                    8
                          259
## <<<<******>>>
  [1] "Simulation: 118 rsqsym 0.334252499618229 rsqsgcca 0.227420415521914"
        symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 118 0.3342525 0.2274204 0.226816 0.2473636 0.1780876 0.6906137 0.2790877
##
      nTrueEmbeddings nForSim
## 118
                   18
## <<<<*****
  [1] "Simulation: 119 rsqsym 0.35703860702543 rsqsgcca 0.490206350216824"
        symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 119 0.3570386 0.4902064 0.2111053 0.2070203 0.8266162 0.3726313 0.5619724
##
      nTrueEmbeddings nForSim
## 119
                   17
                          338
## <<<<******>>>
  [1] "Simulation: 120 rsqsym 0.434523381459949 rsqsgcca 0.35404902262921"
        symRSQ1 sgccaRSQ1 symRSQ2 sgccaRSQ2 corrupt1 corrupt2 corrupt3
## 120 0.4345234 0.354049 0.3759627 0.2702019 0.1340498 0.6486141 0.86644
##
      nTrueEmbeddings nForSim
## 120
                   15
## <<<<******>>>
```

Look over results, statistically

The r-squared value is most informative as it tells us how well the omnibus model predicts the known latent signal.

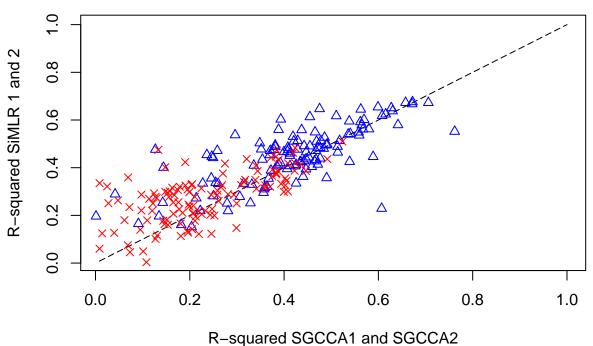
Compare SiMLR \mathbb{R}^2 vs. sgcca \mathbb{R}^2 with paired t-test for the first hidden signal and for the second hidden signal – we concatenate the outcomes to test both at once.

```
# first signal
simvec = c( simdatafrm[,"symRSQ1"], simdatafrm[,"symRSQ2"] )
sgcvec = c( simdatafrm[,"sgccaRSQ1"], simdatafrm[,"sgccaRSQ2"] )
print(t.test(simvec,sgcvec,paired=T))
##
##
   Paired t-test
##
## data: simvec and sgcvec
## t = 6.4133, df = 239, p-value = 7.551e-10
\mbox{\tt \#\#} alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
## 0.02745947 0.05180759
## sample estimates:
## mean of the differences
                0.03963353
mean performance
print( colMeans( simdatafrm , na.rm = TRUE ) )
##
           symRSQ1
                          sgccaRSQ1
                                             symRSQ2
                                                                             corrupt1
                                                           sgccaRSQ2
                                                           0.2324597
##
         0.4498444
                         0.4169872
                                          0.2788696
                                                                            0.4806618
##
          corrupt2
                          corrupt3 nTrueEmbeddings
                                                             nForSim
##
         0.5347564
                         0.4946096
                                         14.6500000
                                                         317.8750000
```

Visualize overall results

```
simdatafrm = na.omit( simdatafrm )
myline = 1:nrow( simdatafrm )
plot( myline/max(myline), myline/max(myline), type='1', lty=5, main='Signal recovery comparison (r-squa
points( simdatafrm[,"sgccaRSQ1"], simdatafrm[,"symRSQ1"], col='blue', ylab='Rsq - SiMLR', xlab='Rsq - R
points( simdatafrm[, "sgccaRSQ2"], simdatafrm[, "symRSQ2"], col='red', ylab='Rsq - SiMLR', xlab='Rsq - SG
```

Signal recovery comparison (r-squared): SiMLR vs SGCCA1 (blue triangle) and SGCCA2 (red x)



Look at results via histogram

```
library( rtemis )
##
     .:rtemis 0.8.0: Welcome, stnava
     [x86_64-apple-darwin17.0 (64-bit): Defaulting to 16/16 available cores]
##
     Documentation & vignettes: https://rtemis.lambdamd.org
##
myMeth = paste0("SiMLR-mix-", mixingMethod, '-E-', energyType)
# Build dataset with different distributions
mdata <- data.frame(</pre>
  method = rep(
    c( paste0(myMeth,1), paste0(myMeth,2),
    "SGCCA1", "SGCCA2" ), each = nsims ),
  value = c( simdatafrm[,1], simdatafrm[,3], simdatafrm[,2], simdatafrm[,4] )
mplot3.x( split( mdata$value, mdata$method ) )
```

