In the name of GOD.

Stochastic Processes

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Quiz 0 (15 minutes) Probability & Statistics + Signal & Systems Date: Oct. 14th

1. What is the difference between "independence" and "mutual exclusivity" (i.e. disjointness) of events? Can two events be mutually exclusive and independent at the same time? (20 points)

   Solution: Mutually exclusive means the events cannot both occur (their intersection has probability zero). Independence means the occurrence of one event gives no information about the occurrence of the other; formally ($P(A \cap B) = P(A)P(B)$). If two events are mutually exclusive and both have positive probability, then they cannot be independent, because if ($P(A) > 0$) and ($P(B) > 0$) then ($P(A \cap B) = 0$) is not equal to ($P(A)P(B) > 0$). The only way mutually exclusive events are independent is if at least one of them has probability zero.

2. Suppose X is a continuous random variable and $P(X = 0) > 0$. Is this possible? Explain.. (20 points)

   Solution: No. Continuous random variables cannot assign positive probability to a single point; $P(X = x) = 0$ for any specific x.

3. Two different experiments give random variables X and Y with the same distribution. Are they necessarily independent? Explain. (20 points)

   Solution: No. Having the same distribution does not imply independence. Independence refers to the relationship between variables, not their individual distributions.

4. Can every system be characterized by its impulse response? If not, which class of systems (linearity, time-invariance, causality, etc.) admit a characterization via impulse response? (20 points)

   Solution: Systems that are linear and time-invariant (LTI) can be fully characterized by their impulse response. For more general systems (nonlinear, time-varying) you cannot define a single impulse response. For time-varying linear systems, you might define a time-varying kernel $h(t,)$ but that's not a traditional impulse response.

5. Does every (reasonable) time-domain function have a Fourier transform? If not always, under what additional conditions or restrictions does a Fourier transform exist (in the classical sense)? (20 points)

   Solution: No, not every function has a classical Fourier transform. The classical integral & only converges under certain conditions (e.g. absolutely integrable, or more weakly, square-integrable plus using tempered distributions). Common sufficient conditions include: (x(t)) is absolutely integrable (($\int |x(t)|, dt < \infty$)) or of finite energy (($\int |x(t)|^2 dt < \infty$)) under some contexts. For more general signals, one uses the theory of distributions (generalized Fourier transforms).