Stochastic Processes November 14, 2022
Sharif University of Technology CE 40-695
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Time: 20 mins

Name: Std. Number:

Quiz 4 (Poisson Processes) Solutions

Questions

1. Consider a poisson process with the rate of 4 events per minute:
   (a) T1 is the time of the first event. What is E[T1]?
   (b) If we know that we only have 1 event in the first 3 minutes, what is the probability of it happening in the first one minute?
   (c) What is the probability of having 4 events in the first 4 minutes?

sol 1. (a) inter-arrival times are from an exponential RV with $\lambda = 4$, the expected time of the first arrival is 15 seconds.
   (b) since we know only one event has happened, then the conditional probability is from Uniform distribution. The answer is $\frac{1}{3}$.
   (c)
$$P(N(4) = 4) = \frac{e^{16}16^4}{4!}$$

2. We have two independent poisson processes P1 and P2 with the rates of λ_1 and λ_2, what is the probability of the following events?
   (a) The first event of P1 is before the first event of P2.
   (b) The third event of P1 is exactly before the fourth event of P2.

sol 2. P1 and P2 create a new process named P3 with the rate of $\lambda_1 + \lambda_2$. the probability of one event being associated with P1 is $\frac{\lambda_1}{\lambda_1+\lambda_2}$. For P2 we have $\frac{\lambda_2}{\lambda_1+\lambda_2}$.
   (a) The first event should be from P1 so: $\frac{\lambda_1}{\lambda_1+\lambda_2}$
   (b) with this criteria we know that the 6th event is from P1 and the seventh one is from P2. For the first 5 events, we must have three events from P2 and two events from P1.
$$\binom{5}{2}(\frac{\lambda_1}{\lambda_1+\lambda_2})^3(\frac{\lambda_2}{\lambda_1+\lambda_2})^4$$