Stochastic Processes January 10, 2023
Sharif University of Technology CE 40-695
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Time: 20 minutes + 5 minutes upload time

Name: Std. Number:

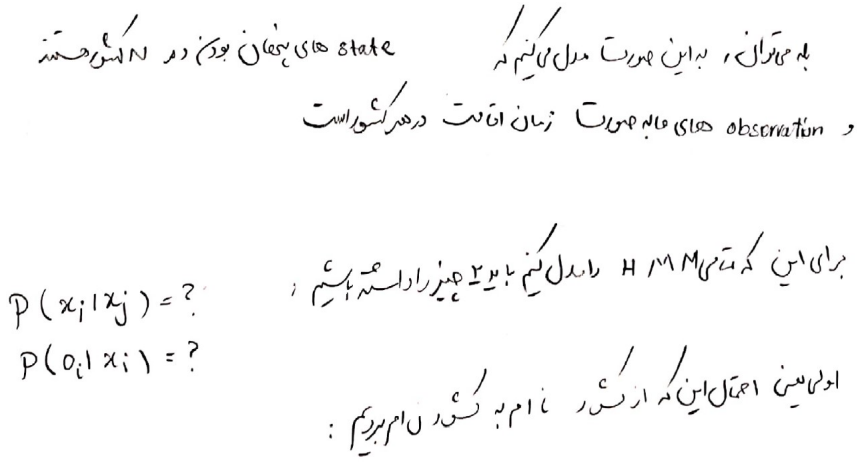
Quiz 9 (Markov Chains)

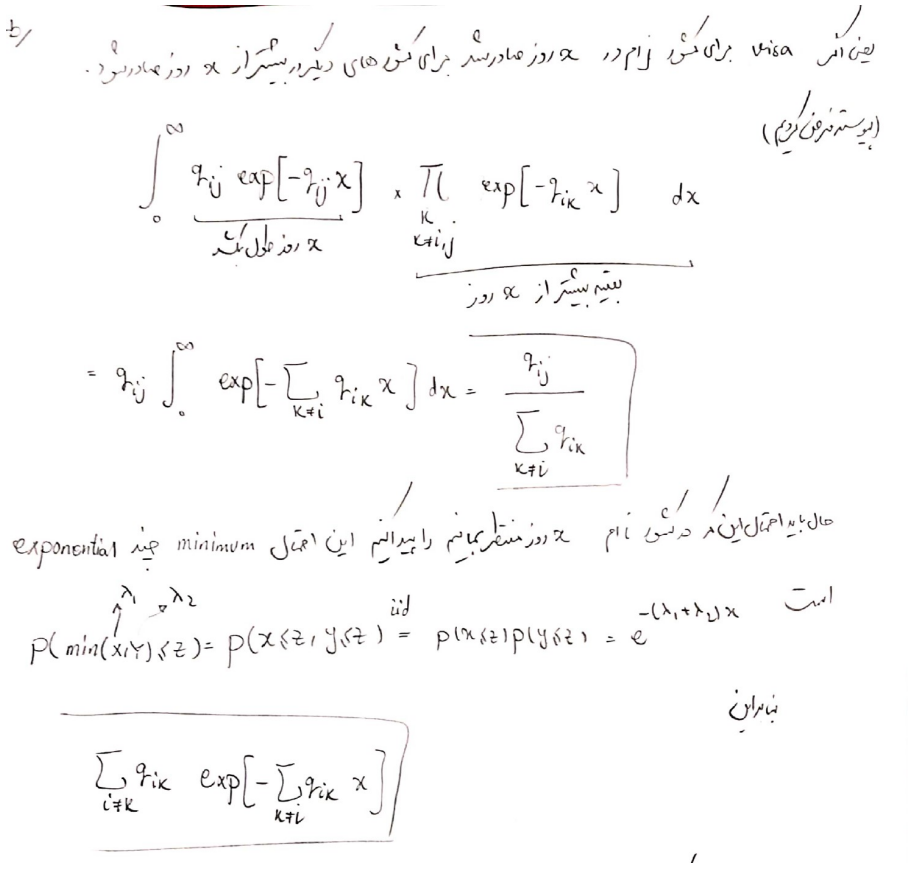
1 Marco Polo (15 points)

You know Marco Polo, always living on the edge! During his travels, he used to apply for the visa of all other countries right at the point of arriving at the next country and leave the country as soon as receiving the first visa! Assume there are N countries in the world, $C_1, ..., C_N$ and the time till receiving the visa of country j being in country i follows an exponential distribution with the mean parameter of q_{ij} . And for the start there is an equal probability of being in each of the countries.

1. Can we model Marco Polo's travels with a HMM? Why? (5 points)
2. Assuming we have modeled it using a HMM (whether it has been the perfect model for the situation or we have just simplified it), find all the required distributions for defining the HMM. (10 points)

Solution :





2 Theran's Weather (10 points)

Suppose we can model Tehran's weather using the following HMM, what is the most probable weather sequence for observation sequence (Outdoor workout, Indoor workout)?

Table 1: State transition probabilities

t-1	t	P
Clean	Clean	0.4
Clean	Polluted	0.6
Polluted	Clean	0.3
Polluted	Polluted	0.7

Table 2: Observation probabilities

Observation	**State**	**P(O|S)**
Indoor workout	Clean	0.2
Indoor workout	Polluted	0.9
Outdoor workout	Clean	0.8
Outdoor workout	Polluted	0.1

Table 3: Initial probabilites

State	$\pi(S)$
Clean	0.3
Polluted	0.7

Solution :

$$P(Out|Polluted) = 0.7 \times 0.1 = 0.07$$
$$P(Out|Clean) = 0.3 \times 0.8 = 0.24$$
$$P((Out, In)|Polluted) = max\{0.7 \times 0.1 \times 0.7, 0.3 \times 0.8 \times 0.6\} \times 0.9 = 0.1296$$
$$P((Out, In)|Clean) = max\{0.7 \times 0.1 \times 0.3, 0.3 \times 0.8 \times 0.4\} \times 0.2 = 0.0192$$

Hence the sequence is (Clean, Polluted).