Time: 20 mins
Name: **Std. Number:**

Quiz 7

1. The likelihood function can be written as:

$$L(\alpha) = f(x_1, \ldots, x_n; \alpha) = \left(\prod_{i=1}^{n} x_i \right)^{\beta_0 - 1} e^{-\sum_{i=1}^{n} x_i/\alpha} \left(\alpha \beta_0 \Gamma(\beta_0) \right)^{-n}.$$

This implies that a sufficient statistic for α is:

$$S = \sum_{i=1}^{n} X_i.$$

Moreover, the pdf of X can be written as:

$$f(x; \alpha) = \left(\alpha \beta_0 \Gamma(\beta_0) \right)^{-1} x^{\beta_0 - 1} e^{-x/\alpha},$$

which means $f(x; \alpha)$ is a member of the regular exponential class with $t(x) = x$. Thus, S is a complete and sufficient statistic for α.

Since $E(S) = n\alpha\beta_0$, it follows that:

$$T^* = \frac{\sum_{i=1}^{n} X_i}{\beta_0 n}$$

is the UMVUE of α, using the Lehmann-Scheffe theorem. This is because it is a function of the complete sufficient statistic S and is an unbiased estimator of α.