Stochastic Processes November 8, 2022
Sharif University of Technology CE 40-695
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Time: 20 mins

Name: Std. Number:

Quiz 6 (Sufficient Statistics + Estimation Theory)

Questions

1. (50%) Let X_1,X_2,X_3,...,X_n be iid samples from a distribution with the following probability density:

$$f(x|\theta) = \frac{\theta}{(1+x)^{\theta+1}}$$

Find a sufficient statistic for θ.

Answer:

$$f(x_1, x_2, ..., x_n|\theta) = \prod_{i=1}^{n} \frac{\theta}{(1+x)^{\theta+1}} = \frac{\theta^n}{[\prod_{i=1}^{n}(1+x_i)]^{\theta+1}} = \frac{\theta^n}{u^{\theta+1}}$$

We have:

$$u = \prod_{i=1}^{n}(1+X_i)$$

$$g(u,\theta) = \frac{\theta^n}{u^{\theta+1}}$$

$$h(x_1, x_2, ..., x_n) = 1$$

Thus, by factorization Theorem u is a sufficient statistics for θ.

2. (50%) Let X_1,X_2,X_3,...,X_n be iid with pdf:

$$f(x|\theta) = \frac{1}{\theta} \quad 0 \leq x \leq \theta, \theta > 0$$

Find the maximum likelihood estimator (MLE) of θ based on this random sample.

Answer: The likelihood function is:

$$L(\theta|x) = \prod_{i=1}^{n} \frac{1}{\theta^n} I_{[0,\theta]}(x_{(n)}).I_{[0,\infty)}(x_{(1)})$$

where $x_{(1)}$ and $x_{(n)}$ are the smallest and largest order statistics. For $\theta \geq x_{(n)}$, $L = \frac{1}{\theta^n}$ is a decreasing function. So, for $\theta \geq x_{(n)}$, L is maximised at $\hat{\theta} = x_{(n)}$. $L = 0$ for $\theta < x_{(n)}$. So the overall maximum, the MLE is $\hat{\theta}_{MLE} = X_{(n)}$.