*Stochastic Processes* November 22, 2022
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Time: 25 mins

Name: Std. Number:

# Quiz 5 (Gaussian Processes) Solution

**Questions**

1. Consider Process $\{X(t); t \in R\}$ that $X(t) = At$ and $A \sim N(0, 1)$
   (a) Show that $X(t)$ is a Gaussian Process
   (b) Find the expected value and Auto-Covariance

sol 1. (a) $X(t_i) = t_i A$ for $i = 1, ..., k$ and $A \sim N(0, 1)$. Hence $t_1$, $t_2$, ..., $t_k$ are jointly gaussian. Thus, $\{X(t); t \in R\}$ is a guassian process.

(b)

$$\mu(t) = E[X(t)] = E[tA] = 0$$

$$Cov(X(t_1), (X(t_2)) = E[(X(t_1) - E[X(t_1)])(X(t_2) - E[X(t_2)])]$$

$$E[X(t_1)X(t_2)] = E[t_1 t_2 A^2] = t_1 t_2 = var(A) = t_1 t_2$$

2. Let $\{X(t); t \in R\}$ be a Gaussian process with covariance function $R_X(t)$, let's define the process $Y(t)$ as below:

$$Y(t) = X(t) - 0.4X(t - 2)$$

   (a) Compute covariance function for this process. Is it stationary?
   (b) Is it a Gaussian process?

sol 2. (a) As usual, it is a good idea to start with the definition of the covariance function:

$$\begin{aligned} r_Y(s, t) &= \mathrm{C}[Y(s), Y(t)] = \mathrm{C}[X(s) - 0.4X(s - 2), X(t) - 0.4X(t - 2)] \\ &= r_X(t - s) + 0.16 r_X(t - s) - 0.4 r_X(t - s - 2) - 0.4 r_X(t - s + 2). \end{aligned}$$

Since this does only depend on $t - s$ (convince yourself that $r_Y(s + c, t + c) = r_Y(s, t)$ for any constant $c$ ) and since $m_Y(t)$ is constant, $\{Y(t)\}$ is a weakly stationary process.

(b) linear combination of Gaussian processes is itself a Gaussian processes.