



1. (60 Points) Let (X_1, \dots, X_n) be i.i.d. with

$$P(X_i = 1) = \theta, \quad P(X_i = 0) = 1 - \theta, \quad 0 < \theta < 1.$$

A student proposes the following three claims about *sufficient statistics* for θ :

1. **Claim A:** $T_1 = \sum_{i=1}^n X_i$ is sufficient for θ .
2. **Claim B:** $T_2 = \sum_{i=1}^n (2X_i - 1)$ is *not* sufficient for θ , because it can take negative values and “does not look like a count”.
3. **Claim C:** $T_3 = \mathbf{1}\{\text{at least one } X_i = 1\}$ (i.e., 1 if there is at least one success, 0 otherwise) is sufficient for θ , because it still depends on θ .

For each claim (A), (B), and (C):

- State whether it is **correct** or **wrong**, and
- Give a short explanation in words (no detailed calculations needed).

2. (40 Points) Let (X_1, \dots, X_n) be i.i.d. random variables with pdf

$$f(x | \theta) = \begin{cases} \frac{1}{1 - \theta}, & \theta < x < 1, \\ 0, & \text{otherwise,} \end{cases} \quad 0 < \theta < 1.$$

Find a sufficient statistic for θ .