Modeling Master Planning Problems in Semiconductor Manufacturing with Quantum Annealing

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1 MIP Model

We considered the PhD Thesis of Thomas Ponsignon [1], who formulated the Master Planning in Semiconductor Manufacturing as a Mixed-Integer-Programming (MIP):

Objective (3.1):

$$f(x,u,I,B,s^{sr}) = \sum_{p=1}^{p_{max}} \sum_{t=1}^{t_{max}} (rev_{pt} s_{pt}^{sr} - hc_{pt} I_{pt} - udc_{pt} B_{pt} - \sum_{m=1}^{m_{max}} mc_{pmt} x_{pmt} - \sum_{m=1}^{m_{max}} lc_{pmt} u_{pmt})$$

Constraints:

(3.2)

$$I_{pt} + s_{pt}^{sr} + s_{pt}^{fo} = I_{pt-1} + \textstyle\sum_{m=1}^{m_{max}} x_{pmt} + \sum_{m=1}^{m_{max}} x_{pmt}^{i}$$

$$\forall p \in [p_{max}], t \in [t_{max}]$$

(3.3)

$$s_{pt}^{fo} + B_{pt} = d_{pt}^{fo} + B_{pt-1}$$

$$\forall p \in [p_{max}], t \in [t_{max}]$$

(3.4)

$$s_{pt}^{sr} \leq d_{pt}^{sr}$$

$$\forall p \in [p_{max}], t \in [t_{max}]$$

(3.5)

$$C_{mbt}^{min} \leq \sum_{p=1}^{p_{max}} \sum_{k=0}^{min(k_{max},t_{max}-t)} cc_{bk}^{pm}(x_{p,m,t+k} + x_{p,m,t+k}^{i}) \leq C_{mbt}^{max}$$

$$\forall m \in [m_{max}], p \in [p_{max}], t \in [t_{max}]$$

(3.6)

$$x_{pmt} \leq \delta u_{pmt}$$

$$\forall p \in [p_{max}], m \in [m_{max}], t \in [t_{max}],$$

where δ is large enough.

(3.7)

$$u_{pmt} \in \{0, 1\}$$

$$\forall p \in [p_{max}], m \in [m_{max}], t \in [t_{max}]$$

(3.8) All variables are non-negative.

2 QUBO Model

Let's first tackle the objective function (3.1):

2.1 Objective Function

Each term in the objective function has as left-side a constant value and as right-side a decision variable, e.g. $rev_{pt}s_{pt}^{sr}$, where rev_{pt} is a constant value, while s_{pt}^{sr} is a decision variable. Note that all decision variables are discrete, only the constant ones could be real-valued. For this reason, we will employ the log-trick [2] for the decision variables, representing any value in $[K]$, for a fixed K, with $log_2(K)$ qubits, e.g. we know from (3.4) that $s_{pt}^{sr} \leq d_{pt}^{sr}$, thus we use in this case $K := d_{pt}^{sr}$, removing also this constraint.

For the other decision variables, we must have a context, such that we can extract the maximum possible value. For each variable we will see whether it is possible the choose such a maximum value.

The log-trick means that we write the discrete value as its bounded binary representation, e.g. we write s_{pt}^{sr} as:

$$k = \left\lfloor log_2(d_{pt}^{sr}) \right\rfloor$$
$$s_{pt}^{sr} = \sum_{j=0}^{k-1} \sigma_{pt}^{sr}(j)2^j + (d_{pt}^{sr} + 1 - 2^k)\sigma_{pt}^{sr}(k),$$

where $\sigma_{pt}^{sr}(j)$ are the $k+1$ qubits which will represent s_{pt}^{sr}.

2.2 Constraints

(3.2)

$$I_{pt} + s^{sr}_{pt} + s^{fo}_{pt} = I_{pt-1} + \sum_{m=1}^{m_{max}} x_{pmt} + \sum_{m=1}^{m_{max}} x^{i}_{pmt}$$

$$\forall p \in [p_{max}], t \in [t_{max}]$$

is an equality constraint, thus is can be rewrited as it is into a QUBO formulation, whereby behind each variable we have the encoding from the log-trick. Thus, we have:

$$(I_{pt} + s^{sr}_{pt} + s^{fo}_{pt} - I_{pt-1} - \sum_{m=1}^{m_{max}} x_{pmt} - \sum_{m=1}^{m_{max}} x^{i}_{pmt})^2$$

$$\forall p \in [p_{max}], t \in [t_{max}]$$

which remains quadratic.

(3.3)

$$s^{fo}_{pt} + B_{pt} = d^{fo}_{pt} + B_{pt-1}$$

$$\forall p \in [p_{max}], t \in [t_{max}]$$

This constraint can be handled in the same manner as the previous one.

(3.4)

$$s^{sr}_{pt} \leq d^{sr}_{pt}$$

$$\forall p \in [p_{max}], t \in [t_{max}]$$

Remember that we already covered this one with the log-trick in subsection 2.1.

(3.5)

$$C^{min}_{mbt} \leq \sum_{p=1}^{p_{max}} \sum_{k=0}^{min(k_{max}, t_{max}-t)} cc^{pm}_{bk} (x_{p,m,t+k} + x^{i}_{p,m,t+k}) \leq C^{max}_{mbt}$$

$$\forall m \in [m_{max}], p \in [p_{max}], t \in [t_{max}]$$

We can split it in 2 trivial constraints:

$$C^{min}_{mbt} \leq \sum_{p=1}^{p_{max}} \sum_{k=0}^{min(k_{max}, t_{max}-t)} cc^{pm}_{bk} (x_{p,m,t+k} + x^{i}_{p,m,t+k})$$

$$\sum_{p=1}^{p_{max}} \sum_{k=0}^{min(k_{max}, t_{max}-t)} cc^{pm}_{bk} (x_{p,m,t+k} + x^{i}_{p,m,t+k}) \leq C^{max}_{mbt}$$

$$\forall m \in [m_{max}], p \in [p_{max}], t \in [t_{max}]$$

Such inequalities constraints are handled with slack variables, such that we obtain only equality constraints, thus:

$$C_{mbt}^{min} + l_{mbt}^{min} = \sum_{p=1}^{p_{max}} \sum_{k=0}^{min(k_{max}, t_{max}-t)} cc_{bk}^{pm}(x_{p,m,t+k} + x_{p,m,t+k}^{i})$$

$$\sum_{p=1}^{p_{max}} \sum_{k=0}^{min(k_{max}, t_{max}-t)} cc_{bk}^{pm}(x_{p,m,t+k} + x_{p,m,t+k}^{i}) + r_{mbt}^{max} = C_{mbt}^{max}$$

$$\forall m \in [m_{max}], p \in [p_{max}], t \in [t_{max}],$$

where l_{mbt}^{min} and r_{mbt}^{min} are the aforementioned slack variables. Again, they also have the log-trick behind them, in order to represent non-negative integers. The bound for the log-trick can be, for example, $C_{mbt}^{max} - C_{mbt}^{min}$, for each m, b and t. For l_{mbt} we obtain this upper-bound from $C_{mbt}^{min} + l_{mbt} \leq C_{mbt}^{max}$ and for r_{mbt}, in the same manner, by considering that the double summation is already greater or equal than C_{mbt}^{min}, so r_{mbt} cannot exceed $C_{mbt}^{max} - C_{mbt}^{min}$.

As a quick optimization, we observe that cc_{bk}^{pm} and $x_{p,m,t+k}^{i}$ are constants in the sum, so we can move the extra factor before the sum. The formulation remains quadratic as well.

(3.6)

$$x_{pmt} \leq \delta u_{pmt}$$

$$\forall p \in [p_{max}], m \in [m_{max}], t \in [t_{max}],$$

where δ is large enough.

The author wrote an explanation for this constraint: “It fixes the binary variable to 1 whenever there is a positive production. On the other hand, $u_{pmt} = 0$ leads to $x_{pmt} = 0$” [1].

To represent this inequality we also employ slack variables, τ_{pmt}, s.t. $x_{pmt} + \tau_{pmt} = \delta u_{pmt}$. We get a simple upper-bound for τ_{pmt}, namey exactly δ, as u_{pmt} is a binary variable.

3 Implementation

The implementation can be found here.

References

[1] Thomas Ponsignon. *Modeling and Solving Master Planning Problems in Semiconductor Manufacturing*. PhD thesis, 12 2012.

[2] Andrew Lucas. Ising formulations of many np problems. *Frontiers in Physics*, 2, 2014.