Grad HW 3

Q3 The time complexity of cutset conditioning is $O(d^c (n-c) d^2)$. Consider following cases:

① $c=0$, we already have a tree structure hence we take $O(nd^2)$ time

② Let there be $k (\leq c)$ elements in the cutset that have alldiff constraints on them.

2.1 $k=c$ or $k \approx c$, we still have our algorithm working almost as good as tree-structured CSP since we can assign values to the alldiff constraint variables in the cutset randomly. $O(d^{c-k}(n-c)d^2)$

$\rightarrow O((n-k)d^2)$

2.2 $k=0$ or $k << c$, since the domain size is large, in this case we will be solving a lot of tree CSPs. This scenario won't work well with cutset conditioning

Q2 <u>invariant</u>: if (x_i, x_j) are not in the queue, their current domains D_i, D_j are compatible s.t (x_i, x_j) is arc-consistent

<u>initialization</u>: at the start of AC-3, all the arcs are in the queue. Our initialization would be the first time (X_i, x_j) are popped out of the queue.
When (x_i, x_j) are popped, we perform REMOVE-INCONSISTENT-VALUES(x_i, x_j), this removes the non-compatible values from D_i so that arc (x_i, x_j) is arc-consistent

<u>induction</u>: if after k iterations, (X_i, X_j) are not in the queue and current domains are s.t (X_i, x_j) is arc-consistent, then for $(k+1)^{th}$ iteration,

case I: No values are removed from D_i, D_j.
We have same domains as in the inductive step hence (X_i, X_j) are arc-consistent

Case II: Values are removed from D_i / D_j or both. REMOVE-INCONSITENT-VALUES returns true and neighbours of X_i / X_j are added to the queue, so (X_i, X_j) are in the queue now

In both cases above our invariant is maintained

☒ QED

Q1 Let the A* search with an ε-admissible heuristic find a goal state g'. We need to prove:

$$cost(g') \leq cost(g) + \varepsilon,$$ where g is the optimal goal state

Case 1 : $g' = g$

inequality holds true

Case 2 : $g' \neq g$

$$f(g) = c(g) + h(g) \quad (g \text{ is a goal state } \& \ h^*(\cdot) \text{ is an admissible heuristic})$$

$$\leq c(g) + \cancel{h^*(g)}^{\,0} + \varepsilon$$

$$\Rightarrow f(g) \leq c(g) + \varepsilon \quad — (1)$$

claim: some ancestor n of g is in the queue

If not, then g is expanded an we have case I.

(This ancestor of g can be the root or g trivially)

Since n is the ancestor of g,

$$f(n) \le f(g) \quad \text{— } (2)$$

As n is still in the queue when g' is expanded,

$$f(g') \le f(n) \quad \text{— } (3)$$

Since g' is a goal state with current heuristic, (for A* termination)

$$f(g') = c(g') + \cancel{h(g')}^{\,0}$$
$$= c(g') \quad \text{— } (4)$$

from (1), (2), (3) & (4) we have

$$f(g') = c(g') \le f(n) \le f(g) \le c(g) + \varepsilon$$

$$\Rightarrow \quad c(g') \le c(g) + \varepsilon$$

☒ QED