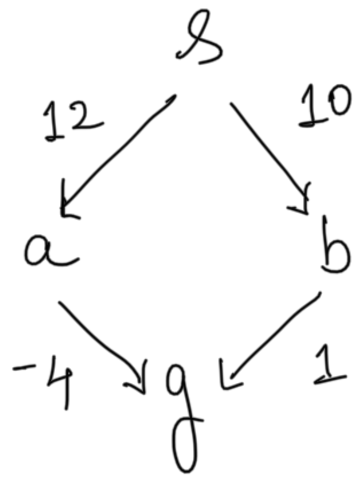
Grad HW 1

Q1

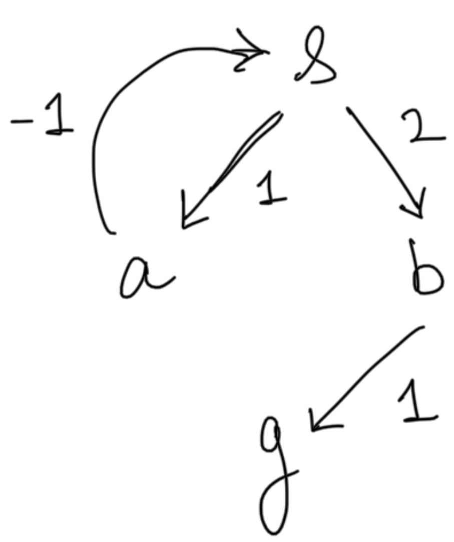
a)



optimal path = {s, sa, sag}, cost = 8

UCS path = {s, sb, sbg}, cost = 11

b)

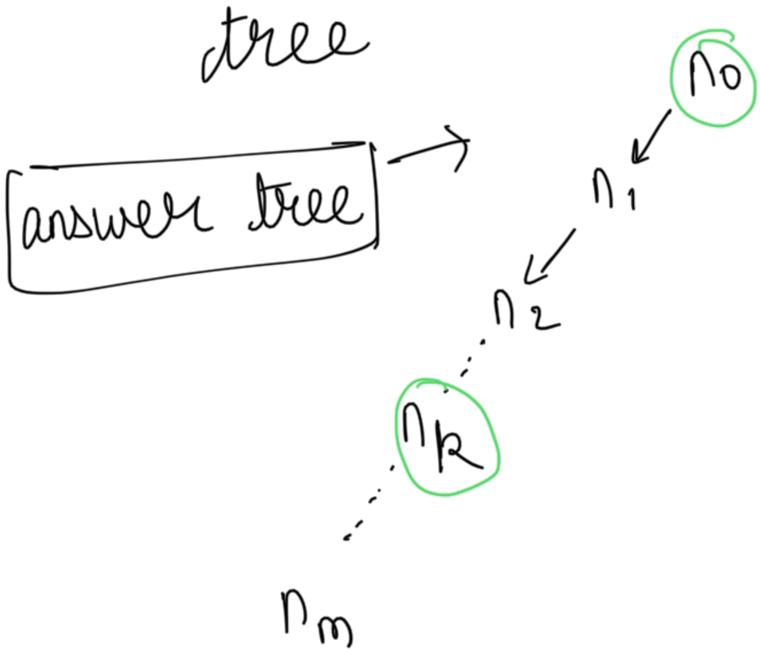


path (dfs, bfs, ids, bidirectional) = {s, sa, sas, sasa, ... }

Q2 a) Intuition :

① Since dfs & bfs take time of similar order, our goal state would be towards the left part of the tree

② Also, to minimize the number of nodes visited in bfs we should lower the branching factor of the tree



n_0 = start state

n_k = goal state

$b = 1$

paths explored by dfs $D = \{n_0, n_0n_1, n_0n_1n_2, \ldots, n_0..n_k\}$

paths explored by bfs $B = \{n_0, n_0n_1, \ldots, n_0..n_k\}$

paths explored by ids $I = \{ n_0,\ n_0, n_0n_1,$

$n_0, n_0n_1, n_0n_1n_2, \ldots$

$n_0, n_0n_1, \ldots\ldots n_0n_1\ldots n_k \}$

$|D| = k \quad |B| = k$

$|I| = 1 + 2 + 3 + \ldots + k$

$= \dfrac{k(k+1)}{2}$

$\therefore$ In our graph dfs & bfs are $O(n)$ while ids is $O(n^2)$

b) in q6 of HW1 we saw that number of nodes visited by ids for a solution at height d

$= b^d + 2b^{d-1} + \ldots + (d+1)$

$$= b^d \underbrace{\left(1 + \frac{2}{b} + \frac{3}{b^2} + \dots + \frac{(d+1)}{b^d}\right)}$$

↳ this series does not converge for $b \geq 1$; hence our time complexity will be a function of d for $b = 1$

substituting $b = 1$ above (for our tree),

$$= 1\left(1 + 2 + 3 + \dots + (d+1)\right) = \frac{(d+1)(d+2)}{2}$$

$\Rightarrow$ for $b = 1$, ids $= O(d^2)$

Q3 Bidirectional search terminates when a common node is explored.

When a common node is explored, we already have a solution ($s \to n$, $g \to n$, where n is the common node explored $\Rightarrow$ solution path = $s \to n \to g$)

$\therefore$ to prove completeness of bidirectional it is sufficient to prove that paths from start & goal meet in a finite time.

We prove this by induction,

invariant : paths from start and goal meet at a common node in finite time for a goal state at height d

initialization : $d = 0 \Rightarrow$ start = goal = common node

$\Rightarrow$ bidirectional terminates

induction : let bidirectional UCS terminate in finite steps for a goal state at height $d = h-1$

then for $d = h$,

let the path from goal state take its first step from $g \to g_{-1}$

we now have a search problem where

start = s, goal = g_{-1}

Since our goal state is at height $h-1$, our bidirectional search will terminate in finite steps from the induction hypothesis

$\Rightarrow$ bidirectional is complete