HW 3

Q1

a) by admissibility,

$h(D) \leq cost(D \to Z)$

$\Rightarrow \quad h(D) \leq 4 \quad$ (least cost is $D \to F \to E \to Z$)

(DFEZ)

$\Rightarrow \quad h(D) \in \{1, 2, 3, 4\}$

b) by consistency,

$h(X) - h(D) \leq cost(X \to D)$

$h(D) - h(Y) \leq cost(D \to Y)$

where $X \in \{A, B, C\}$

$Y \in \{E, F, Z\}$

for $x \in X$,

$x = A, \quad h(A) - h(D) \leq 3$

$\Rightarrow \quad h(D) \geq 2$

$x = B, \quad h(B) - h(D) \leq 2$

$\Rightarrow \quad h(D) \geq 1$

$x = C, \quad h(C) - h(D) \leq 1$

$\Rightarrow \quad h(D) \geq 2$

for $y \in Y$,

$x = E$, $h(D) - h(E) \leq 3$

$\Rightarrow h(D) \leq 4$

$x = F$, $h(D) - h(F) \leq 1$

$\Rightarrow h(D) \leq 2$

$x = Z$, $h(D) - h(Z) \leq 4$

$\Rightarrow h(D) \leq 4$

$\therefore$ from above $2 \leq h(D) \leq 2 \Rightarrow h(D) = 2$

Q2 a) Variables : $\{O, N, E, T, W, \underbrace{X_1, X_2}_{\text{carry bits}}\}$

Domain :

→ domain[Z] $\in \{0,1,2,3,4,5,6,7,8,9\}$

where $Z \in \{O, N, E, T, W\}$

→ $X_1, X_2 \in \{0,1\}$

Constraints :

① alldiff (O, N, E, T, W)

② $E + E = O + 10X_1$

③ $N + N + X_1 = W + 10X_2$

④ $0 + 0 + X_2 = T$

b) by min-remaining value heuristic, we start with $X_1 = 0$

① set $X_1 = 0$

reduced domain

X_1	X_2	E	O	$s \in \{N, T, W\}$
0	$\{0, 1\}$	$\{0,1,2,3,4\}$	$\{0,1,\ldots,8\}$	$\{0,1,\ldots,9\}$

② set $X_2 = 0$

reduced domain

X_1	X_2	$u \in \{E, N\}$	$v \in \{W, O\}$	T
0	0	$\{0,1,2,3,4\}$	$\{0,1,\ldots 8\}$	$\{0,1,\ldots,9\}$

③ set $E = 0$
$\Rightarrow O = 0$ violates alldiff

④ set $E = 1$

X_1	X_2	E	O	T	N	W
0	0	1	$\{2\}$	$\{4\}$	$\{0, 3\}$	$\{0, 6\}$

in step ⑤ & ⑥ we set $O = 2$ & $T = 4$
no domain reduction happens

⑦ set $N = 0 \Rightarrow W = 0$ violates all diff

(8) set $N = 3$

X_1	X_2	E	O	T	N	W
0	0	1	2	4	3	$\{6\}$

(9) set $W = 6$ $\Rightarrow$ we found solution

$\therefore$ $X_1 = 0$, $X_2 = 0$

$E = 1$, $O = 2$, $T = 4$, $N = 3$, $W = 6$

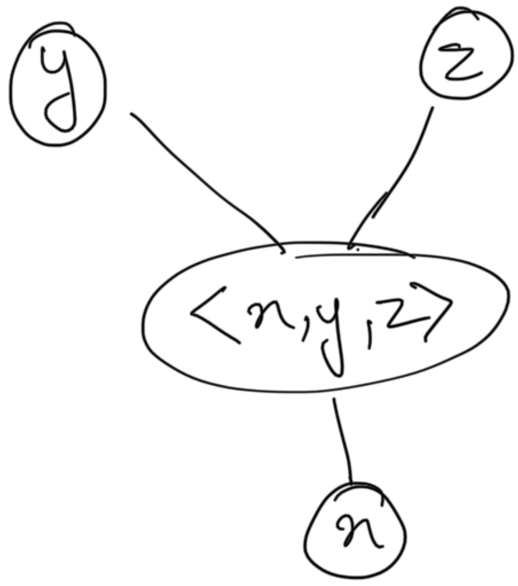
$$\begin{array}{r} O\ N\ E \\ +\ O\ N\ E \\ \hline T\ W\ O \end{array} \equiv \begin{array}{r} 2\ 3\ 1 \\ +\ 2\ 3\ 1 \\ \hline 4\ 6\ 2 \end{array}$$

Q3 Let $Y + Z = K$ and $X \in D_1$, $Y \in D_2$, $Z \in D_3$

$K = \langle x, y, z \rangle$

we generate the domain of K by enumeration

s.t $y + z > x$ and $y \in D_2$, $z \in D_3$, $x \in D_1$



now we are left with binary constraints only:

first element of $K = X$

second " $= Y$

third " $= Z$

Q4 a) Greedy hill climbing : instead of keeping track of k, we track a single variable updating it to the next best possible variable

b) BFS: Since we start with a single variable, and track in non-decreasing order (from $1 \to$ no limit), it's analogous to BFS where we have ↑ number of elements in the fringe at every level

c) Random hill climbing: since $T=0$, $e^{\Delta E/T} = 0$ (as $\Delta E < 0$), we reject the node (don't go down hill) and our next state is random uphill node.

d) Random walk: since $T=\infty$, $e^{\Delta E/T} = 1$ hence we do not reject any node. The next state is chosen completely at random.