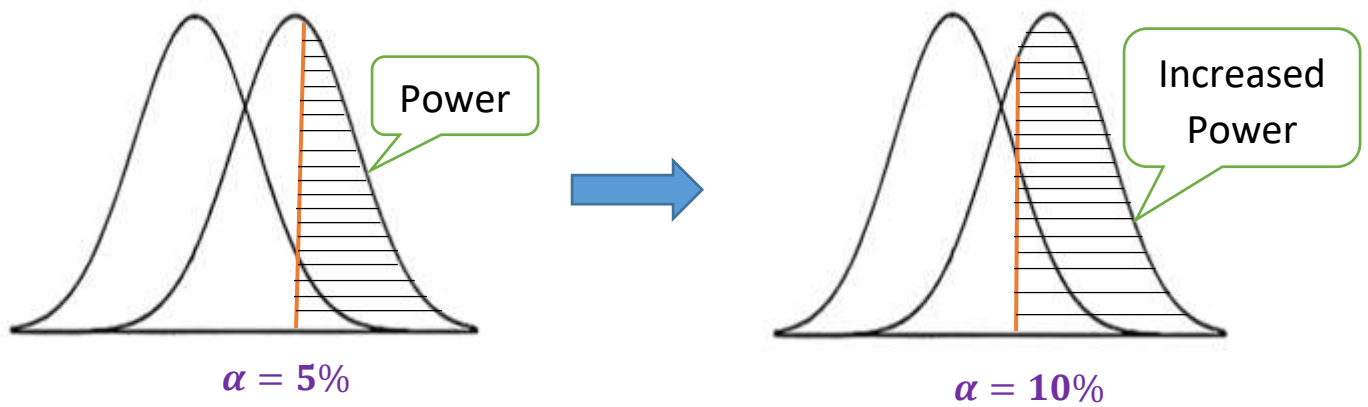


Effect Size Vs Power of Test



Increase in α Vs Power of Test

$$E(\bar{x}) = \mu_0 \text{ under } H_0: \mu = \mu_0.$$

Since $x_i \sim N(\mu, \sigma^2)$,

$$\begin{aligned} E(\bar{x}) &= E\left(\frac{1}{n} \sum x_i\right) = \frac{1}{n} \sum E(x_i) \\ &= \frac{1}{n} \times n\mu = \mu \end{aligned}$$

Page 21 (TOH) :

* M.L.E. of σ_0^2 under H_0

$$\ln L = -\frac{n}{2} \ln(2\pi) - \frac{n}{2} \ln(\sigma_0^2) - \frac{1}{2\sigma_0^2} \sum (x_i - \mu_0)^2$$

$$\left. \frac{\partial \ln L}{\partial \sigma_0^2} \right|_{\sigma_0^2 = \hat{\sigma}_0^2} = -\frac{n}{2\hat{\sigma}_0^2} + \frac{1}{2\hat{\sigma}_0^4} \sum (x_i - \mu_0)^2 = 0$$

$$\Rightarrow \frac{1}{\hat{\sigma}_0^2} \sum (x_i - \mu_0)^2 = n \quad [\because 2\hat{\sigma}_0^2 \neq 0]$$

$$\Rightarrow \underline{\underline{\hat{\sigma}_0^2 = \frac{1}{n} \sum (x_i - \mu_0)^2}}$$

Page 22 (TOH) :

* M.L.E of μ & σ^2 under H_1

$$L' = \ln L = -\frac{n}{2} \ln(2\pi) - \frac{n}{2} \ln(\sigma^2) - \frac{1}{2\sigma^2} \sum (x_i - \mu)^2$$

$$\bullet \left. \frac{\partial L'}{\partial \mu} \right|_{\mu = \hat{\mu}} = 0$$

$$\Rightarrow -\frac{1}{2\sigma^2} \times 2 \sum (x_i - \hat{\mu}) (-1) = 0$$

$$\Rightarrow \sum (x_i - \hat{\mu}) = 0 \quad \Rightarrow \underline{\underline{\hat{\mu} = \frac{1}{n} \sum x_i = \bar{x}}}$$

$$\bullet \left. \frac{\partial L'}{\partial \sigma^2} \right|_{\sigma^2 = \hat{\sigma}^2} = 0$$

$$\Rightarrow -\frac{n}{2\hat{\sigma}^2} + \frac{1}{2\hat{\sigma}^4} \sum (x_i - \hat{\mu})^2 = 0$$

$$\Rightarrow \underline{\underline{\hat{\sigma}^2 = \frac{1}{n} \sum (x_i - \bar{x})^2}}$$

Page 65 (T0H)

2x2 contingency χ^2

$$\text{We have, } \hat{a} = \frac{(a+b)(a+c)}{n}, \hat{b} = \frac{(a+b)(b+d)}{n}$$

$$\hat{c} = \frac{(c+d)(a+c)}{n}, \hat{d} = \frac{(c+d)(b+d)}{n}$$

$$\sum \frac{O_i^2}{E_i} - n \text{ gives}$$

$$\frac{na^2}{(a+b)(a+c)} + \frac{nb^2}{(a+b)(b+d)} + \frac{nc^2}{(c+d)(a+c)} + \frac{nd^2}{(c+d)(b+d)} - n$$

$$= n \left[\frac{a^2(b+d)(c+d) + b^2(a+c)(c+d) + c^2(a+b)(b+d) + d^2(a+b)(a+c) - \frac{(a+b)(a+c)(b+d)(c+d)}{(b+d)(c+d)}}{(a+b)(a+c)(b+d)(c+d)} \right]$$

On simplification, we will get the required expression.