1. In the production of a particular material, three variables are of interest: *A*, the operator effect (three operators – selected at random): *B*, the catalyst used in the experiment (three catalysts); and *C*, the washing time of the product following the cooling process (15 minutes and 20 minutes). Three runs were made at each combination of factors. The coded yields are given in the following table.

Operator [A]	Washing Time in Mins [C]					
	15			20		
	Catalyst [B]			Catalyst [B]		
	1	2	3	1	2	3
1	10.7	10.3	11.2	10.9	10.5	12.2
	10.8	10.2	11.6	12.1	11.1	11.7
	11.3	10.5	12.0	11.5	10.3	11.0
2	11.4	10.2	10.7	9.8	12.6	10.8
	11.8	10.9	10.5	11.3	7.5	10.2
	11.5	10.5	10.2	10.9	9.9	11.5
3	13.6	12.0	11.1	10.7	10.2	11.9
	14.1	11.6	11.0	11.7	11.5	11.6
	14.5	11.5	11.5	12.7	10.9	12.2

i) Assuming interactions AB and BC are present, write down the linear model and hence derive the expression for EMS of the model terms.

ii) Analyse the data suitably and give your comment.

Answer

Model:

$$y_{ijkl} = \mu + \tau_i + \beta_j + \gamma_k + (\tau\beta)_{ij} + (\beta\gamma)_{jk} + \varepsilon_{(ijk)l} \begin{cases} i = 1,2,3 \\ j = 1,2,3 \\ k = 1,2 \\ l = 1,2,3 \end{cases}$$

where μ = overall mean
τ_i = main effect of factor A
β_j = main effect of factor B
γ_k = main effect of factor C
$(\tau\beta)_{ij}$ = interaction effect of factors A and B
$(\beta\gamma)_{jk}$ = interaction effect of factors B and C
and $\varepsilon_{(ijk)l}$ = Error that is NID$(0, \sigma^2)$.

Expected Mean square:

# of level	3	3	2	3	Expected Mean Square
Fixed/Random	R	F	F	R	
Index	i	j	k	l	
τ_i	1	3	2	3	$\sigma^2 + 18 \times \sigma_\tau^2$
β_j	3	0	2	3	$\sigma^2 + 6 \times \sigma_{\tau\beta}^2 + 18 \times \sum \beta_j^2 /2$
γ_k	3	3	0	3	$\sigma^2 + 27 \sum \gamma_k^2 /1$
$(\tau\beta)_{ij}$	1	0	2	3	$\sigma^2 + 6 \times \sigma_{\tau\beta}^2$
$(\beta\gamma)_{jk}$	3	0	0	3	$\sigma^2 + 9 \times \sum \sum (\beta\gamma)_{jk}^2 /2$
$\varepsilon_{(ijk)l}$	1	1	1	1	σ^2

n	**54**
Grand Total	**606.4**
CF	**6809.647**
RSS	**6872.84**
TSS	**63.19259**

Operator [A]	Washing Time in Mins. [C]					
	15			20		
	Catalyst [B]			Catalyst [B]		
	1	2	3	1	2	3
1	32.8	31	34.8	34.5	31.9	34.9
2	34.7	31.6	31.4	32	30	32.5
3	42.2	35.1	33.6	35.1	32.6	35.7

AB	B1	B2	B3
A1	67.3	62.9	69.7
A2	66.7	61.6	63.9
A3	77.3	67.7	69.3

BC	C1	C2
B1	109.7	101.6
B2	97.7	94.5
B3	99.8	103.1

Factor	Level 1	Level 2	Level 3
A	199.9	192.2	214.3
B	211.3	192.2	202.9
C	307.2	299.2	

SSA	13.9826
SSB	10.1826
SSC	1.18519
SS(AB)	4.77407
SS(BC)	3.6337
SSE	29.4344

			ANOVA			
Source	**SS**	**DF**	**MS**	**F**	**F-Crit**	**Remark**
A	13.983	2	6.991			
B	10.183	2	5.091			
C	1.185	1	1.185			
AB	4.774	4	1.194	1.703	2.59	Insignificant
BC	3.634	2	1.817	2.592	3.22	Insignificant
Error	29.434	42	0.701			
Total	63.193	53				

			Revised ANOVA			
Source	**SS**	**DF**	**MS**	**F**	**F-Crit**	**Remark**
A	13.983	2	6.991	8.868	3.1907	Significant
B	10.183	2	5.091	6.458	3.1907	Significant
C	1.185	1	1.185	1.503	4.0427	Insignificant
Error Pooled	37.842	48	0.788			
Total	63.193	53				