Homework 5-Buying a Pair of Jeans: Model Simulation I

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Due April 10th, Tuesday.

Buying a Pair of Jeans

a. Write a file with all inputs for three(3) experts, *agents 1,2,3*.
Let us consider six(6) pair of jeans, i.e., x=1,…,6.
The inputs for each expert are: $Budget(i), Price(x), Trust(x), Fit(x)$, and initial state $\hat{S}_{Fash(i,x,t=0)}$. From the initial state we build the initial probability

$$P_i\left(Fash(i,x,t=0)\middle|\overrightarrow{Adv}(x,t=0)\right)=\begin{cases}1 & Fash(i,x,t=0)=\widehat{Fash}(i,x,t=0)\\ 0 & Fash(i,x,t=0)\neq\widehat{Fash}(i,x,t=0)\end{cases}$$

where $\widehat{Fash}(i,x,t=0)=0.25*\hat{S}_{Fash(i,x,t=0)}$.
Also we need an advertising campaign for 10 periods, i.e., up to *t=10*, i.e., a set of values for

$$\overrightarrow{Adv}(x,t=10)=\{(Adv(j,x,t'));j=1,\dots,N_i\ \text{and}\ t'=1,\dots,t=10\}$$

Finally we will need values for the parameters of the model(s):
$\rho_i, \mathrm{P_i}, \lambda_i, \Lambda_\mathrm{i}, \phi_i, \Phi_\mathrm{i}, \tau_i, \mathrm{T_i}, \gamma_i\ \Gamma_\mathrm{i}, \Delta_\mathrm{i,x}, \alpha_{ij}, \mathrm{A_{ij}}$.

Put all these data in the excel spread sheet provided with this homework, where your program can read from. We want to build a data base consistent with all students so that we can exchange spread sheets/data.

BuyingJeans.Hw5.xlsx

b. Write a subroutine to simulate the purchase probability or expected purchase value and plot its values over time, for t=1,…,10.

The expected purchase value is given by

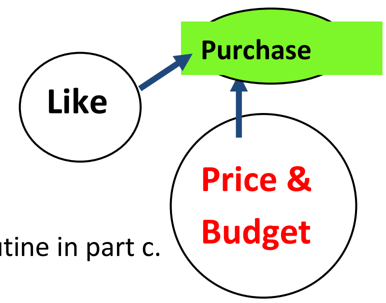
$$\langle Purchase(i,x,t)\rangle = \frac{1}{1+e^{M_{i,price-Like}(Purchase(i,x,t)=1)}} \qquad (11)$$

where

$$M_{i,price-Like}\big(Purchase(i,x,t)\big) = M_{i,price}\big(Purchase(i,x,t)\big) + M_{i,Like}\big(Purchase(i,x,t)\big)$$

$$= Purchase(i,x,t)\left[\rho_i\left(\frac{Price(x)}{Budget(i)-Price(x)}\right)^{\mathrm{P_i}} + \lambda_i\left|1-0.25*S_{Like(i,x,t)}\right|^{\Lambda_i} - 2\right] \qquad (9)$$

Now, the parameters ρ_i and λ_i can be interpreted as weights for combining the two methods. The input is $Price(x), Like(i,x,t)$ and output is the expected value $\langle Purchase(i,x,t)\rangle$. If this expected value is larger than a threshold *T*, say *T*=0.5, than a purchase is made.



The input $Like(i,x,t)$ will need to be estimated from the subroutine in part c.

c. Write a subroutine to simulate the probability

$$P_{i.x}\big(Like(i,x,t)\,|\,Fash(i,x,t), Trust(x), Fit(x)\big) = \frac{1}{Z_{Like}}e^{-M_{i,FTFa}(Like(i,x,t))} \qquad (19)$$

with the normalization constant

$$Z_{Like} = \sum_{S_{Like(i,x,t)}=0}^{4} e^{-M_{i,FTFa}(Like(i,x,t)=0.25*S_{Like(i,x,t)})} \qquad (20)$$

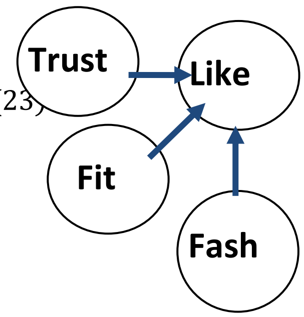
and

$$\begin{aligned} M_{i,FTFa}\big(Like(i,x,t)\big) &= M_{i,Fit}\big(Like(i,x,t)\big) + M_{i,Trust}\big(Like(i,x)\big) + M_{i,Fasht}\big(Like(i,x)\big) \qquad (21)\\ &= \phi_i\left|0.25*S_{Like(i,x,t)} - Fit\right|^{\Phi_i} + \tau_i\left|0.25*S_{Like(i,x,t)} - Trust\right|^{\mathrm{T_i}} \\ &+ \gamma_i\, 0.25^{\Gamma_i}\left|S_{Like(i,x,t)} - S_{Fash(i,x,t)}\right|^{\Gamma_i} \end{aligned}$$

Thus, the coefficients $(\phi_i, \tau_i, \gamma_i)$ can be thought as the weights to combine the experts and one can think of using Boosting to estimate their values. We may then obtain the expected value

$$\langle Like(i,x,t)\rangle = \sum_{S_{Like(i,x,t)}=0}^{4} Like(i,x,t) \times \frac{1}{Z_{Like}} e^{-M_{i,FTFa}(Like(i,x,t))} \quad (22)$$

$$= \sum_{S_{Like(i,x,t)}=0}^{4} 0.25 \times S_{Like(i,x,t)} \times \frac{1}{Z_{Like}} e^{-M_{i,FTFa}(S_{Like(i,x,t)})}$$



Alternatively we estimate the optimal value

$$\widehat{Like}(i,x,t) = 0.25 \times \hat{S}_{Like(i,x,t)} = argmin_{S_{Like(i,x,t)}} M_{i,FTFa}(S_{Like(i,x,t)}) \quad (23)$$

and in this case one may not need to estimate Z_{Like}. You can choose which one to use. The inputs are $Fash(i,x,t), Trust(x), Fit(x)$ and one must have coefficients $(\phi_i, \Phi_i, \tau_i, T_i, \gamma_i\ \Gamma_i)$.

The input $Fash(i,x,t)$ is obtained from subroutine developed in part d.

d. Write a subroutine to simulate the probability

$$P_i\left(Fash(i,x,t)\middle|\overrightarrow{Adv}(x,t)\right)$$

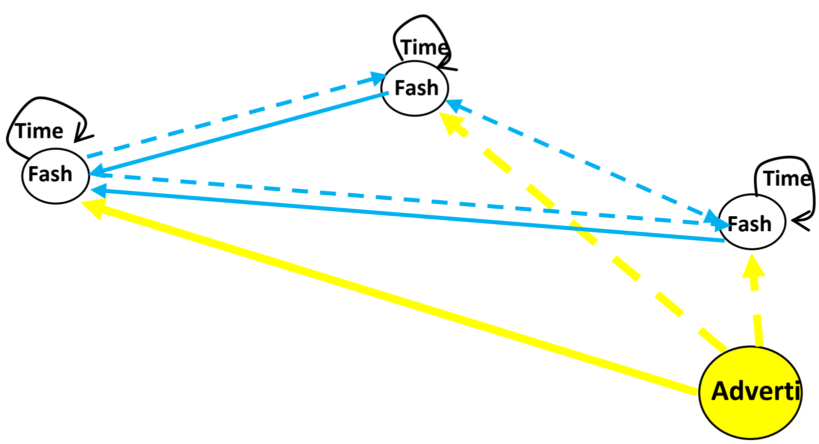
Here $\overrightarrow{Adv}(x,t)$ is a vector

$$\overrightarrow{Adv}(x,t) = \{(Adv(j,x,t')); j = 1, \dots, N_i \text{ and } t' = 1, \dots, t\} \quad (24)$$

where $Adv(j,x,t')$ is a parameter controlling the amount of advertising being absorbed by *agent j* about *pair of jeans x* at a moment in *time t*.

We assume a state in fashion to be given at *t=0*, $\widehat{Fash}(i,x) = 0.25 * \hat{S}_{Fash(i,x,)}$, i.e.,

$$P_i\left(Fash(i,x,t=0)\middle|\overrightarrow{Adv}(x,t=0)\right) = \begin{cases} 1 & Fash(i,x,t=0) = \widehat{Fash}(i,x) \\ 0 & Fash(i,x,t=0) \neq \widehat{Fash}(i,x) \end{cases}$$



Use the formula

$$P_i\left(Fash(i,x,t)\middle|\overrightarrow{Adv}(x,t)\right) = \tag{37}$$

$$= \frac{1}{Z_{Fash}}\left\{\sum_{S_{Fash(i,x,t-1)}=0}^{4}\left[e^{-\left[Adv(i,x,t)\,\left|S_{Fash(i,x,t)}-min(S_{Fash(i,x,t-1)}+\mathbf{1},\ \mathbf{4}))\right|^{\Delta_{\mathrm{i,x}}}+\alpha_{ii}\left|S_{Fash(i,x,t)}-S_{Fash(i,x,t-1)}\right|^{\mathrm{A_{ii}}}\right]}\right.\right.$$

$$\left.\left.\times P_i\left(Fash(i,x,t-1)\middle|\overrightarrow{Adv}(x,t-1)\right)\right]\right\}$$

$$\times\left\{\prod_{\substack{j=1\\ j\neq i}}^{N_i}\sum_{S_{Fash(j,x,t-1)}=0}^{4}\left[e^{-\alpha_{ij}\left|S_{Fash(i,x,t)}-S_{Fash(j,x,t-1)}\right|^{\mathrm{A_{ij}}}}\times P_i\left(Fash(j,x,t-1)\middle|\overrightarrow{Adv}(x,t-1)\right)\right]\right\}$$

From this subroutine, you can iterate in time to estimate a sequence of probabilities $P_i\left(Fash(i,x,t)\middle|\overrightarrow{Adv}(x,t)\right)$ over time and for each *agent i* and each *pair of jeans x*.

We can then estimate the expected value

$$\langle Fash(i,x,t)\rangle = \sum_{S_{Fash(i,x,t)}=0}^{4} Fash(i,x,t)\times P_i\left(Fash(i,x,t)\middle|\overrightarrow{Adv}(x,t)\right) \tag{38}$$

$$= \sum_{S_{Fash(i,x,t)}=0}^{4} 0.25\times S_{Fash(i,x,t)}\times P_i\left(S_{Fash(i,x,t)}\middle|\overrightarrow{Adv}(x,t)\right)$$

as output. Alternatively, select the state $\hat{S}_{Fash(i,x,t)}\in\{0,1,2,3,4\}$ that maximizes the probability

$$\hat{S}_{Fash(i,x,t)} = argmax_{S_{Fash(i,x,t)}}P_i\left(Fash(i,x,t) = 0.25 * S_{Fash(i,x,t)}\middle|\overrightarrow{Adv}(x,t)\right)$$

Then, use $\widehat{Fash}(i,x,t) = 0.25 * \hat{S}_{Fash(i,x,t)}$ as input to the subroutine of part c.