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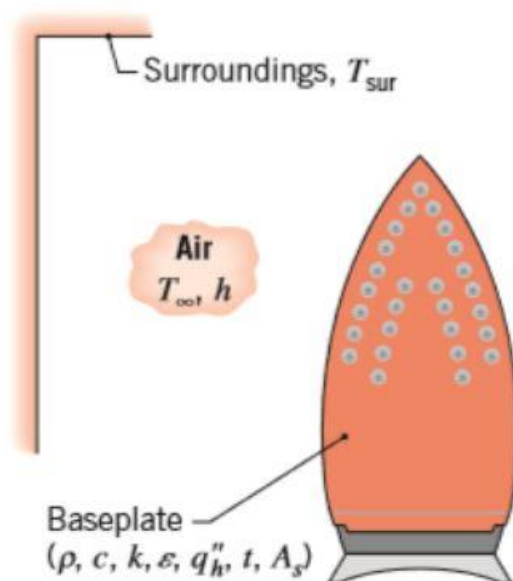
Project 2

BME 333

Given: $L = 7\text{mm}$, $\rho = 2800\text{ kg/m}^3$, $c = 900\text{J/kg}\cdot\text{K}$, $k = 180\text{ W/m}\cdot\text{K}$, $\epsilon = 0.80$, $T_{\infty} = T_{\text{sur}} = 25^\circ\text{C}$, $A_s = 0.040\text{ m}^2$, $q_h'' = 1.25 \times 10^4\text{ W/m}^2\cdot\text{K}$, $h = 10\text{W/m}^2\cdot\text{K}$.

Find: estimate the time required for the plate to reach a temperature of 135°C .

Sketch:

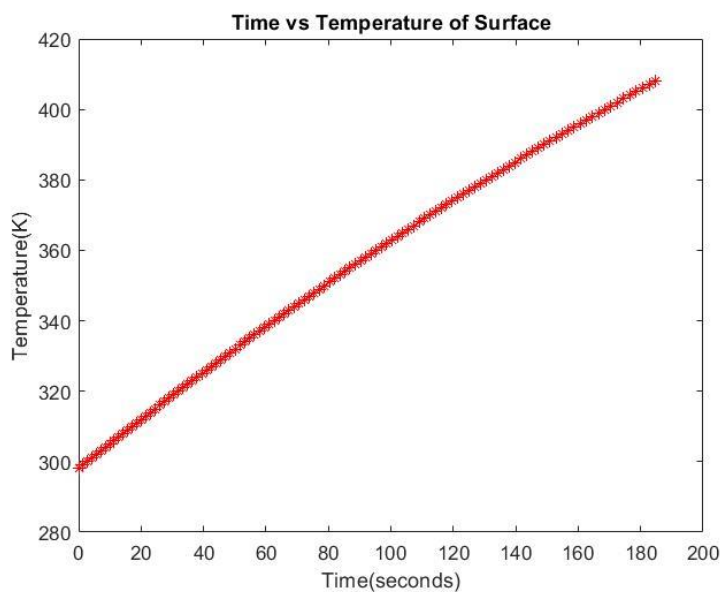


Analytical Solution:

$$\begin{aligned}
 \bullet \quad Bi &= \frac{hL}{k} = \frac{(10 \frac{W}{m^2 \cdot K})(0.007m)}{130 W/m^2 \cdot K} = 3.339 \times 10^{-4} < 1 \\
 \bullet \quad Bi_r &= \frac{hrL}{k} \rightarrow hr = \epsilon \sigma (T_s + T_{\infty})(T_s^2 + T_{\infty}^2) \\
 &= (0.3)(5.67 \times 10^{-8} \frac{W}{m^2}) (403K + 293K)(403K^2 + 293K^2) \\
 &= 3.17 W/m^2 \cdot K \\
 \bullet \quad Bi_r &= \frac{(3.17 \frac{W}{m^2 \cdot K})(0.007m)}{130 W/m^2 \cdot K} = 3.177 \times 10^{-4} < 1 \\
 \bullet \quad \int_0^t A_s + \vec{E}_g - [h(T - T_{\infty}) + \epsilon \sigma (T^4 - T_{sur}^4)] A_g dt &= \int_0^t \rho V_c \frac{dT}{dt} dt \\
 A_s \int_0^t [q'' - h(T - T_{\infty}) - \epsilon \sigma (T^4 - T_{sur}^4)] dt &= \rho V_c (T - T_{\infty}) \\
 A_s [q'' - h(T - T_{\infty}) - \epsilon \sigma (T^4 - T_{sur}^4)] t &= \rho V_c (T - T_{\infty}) \\
 \bullet \quad t &= \frac{\rho V_c (T - T_{\infty})}{A_s [q'' - h(T - T_{\infty}) - \epsilon \sigma (T^4 - T_{sur}^4)]} \\
 t &= \frac{(2300 \frac{kg}{m^3})(0.007m \cdot 0.007m)(135 - 25)(900 \frac{J}{kg \cdot K})}{(0.007m) [(1.29 \times 10^4 \frac{W}{m^2 \cdot K}) - 10 \frac{W}{m^2 \cdot K}(135 - 25) - (0.3)(5.67 \times 10^{-8} \frac{W}{m^2})(403K^4 - 293K^4)]} \\
 &= 184 \text{ seconds}
 \end{aligned}$$

As seen in the calculations, the time required to heat the surface to 135°C is about 184 seconds.

For the code, the exact time that it was able to calculate is 184.79 seconds, in the graph right below we see the comparison of the time vs the temperature of the surface:



The graph looks almost linear with a little bit of a bend out in the middle which means that right before it hit the temperature required, it was approaching the max temperature it would have been able to reach based off the parameters given.