CIT 596 Homework 3

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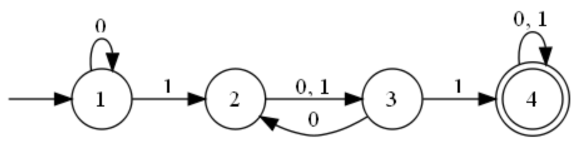
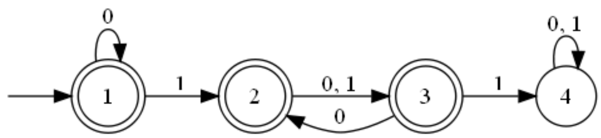
1 Exercise 1.13

Give a DFA that recognizes the language F where F is the language of all strings over $\{0, 1\}$ that do not contain a pair of 1s which are separated by an odd number of symbols.



(a) The NFA A accepts $\overline{F}$



(b) The DFA B accepts F

Figure 1: DFA for Exercise 1.13

2 Exercise 1.16b

Convert the given NFA (omitted) to a DFA.

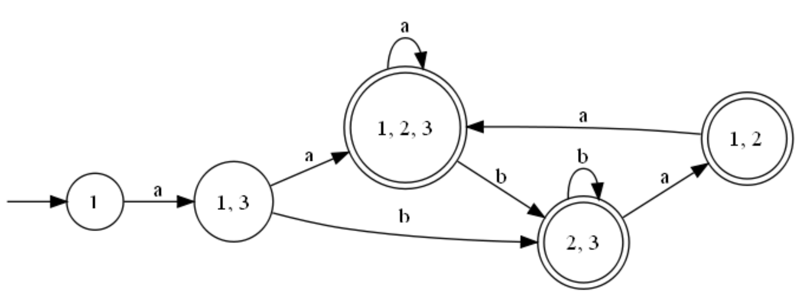


Figure 2: DFA for Exercise 1.16b

3 Exercise 1.17a

Give an NFA recognizing the language $(01 \bigcup 001 \bigcup 010)^{\star}$.

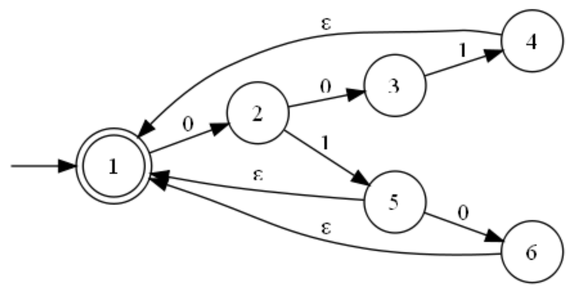


Figure 3: DFA for Exercise 1.17a

4 Exercise 1.17b

Convert the NFA from Exercise 1.17a to an equivalent DFA.

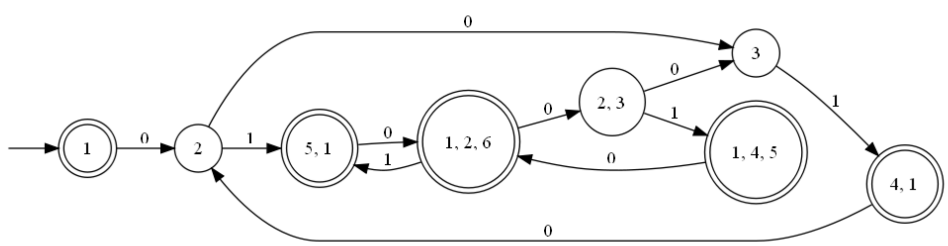


Figure 4: DFA for Exercise 1.17b

5 Exercise 1.19b

Convert the following regular expression to an NFA: $(((00)^{\star}(11)) \bigcup 01)^{\star}$.

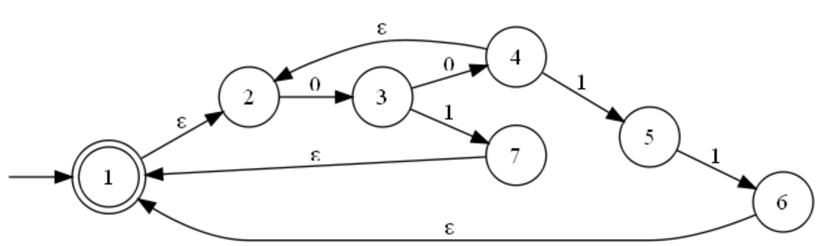
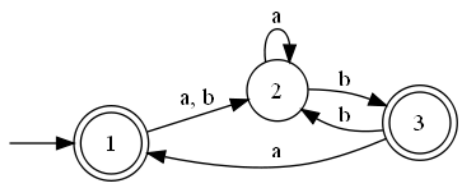


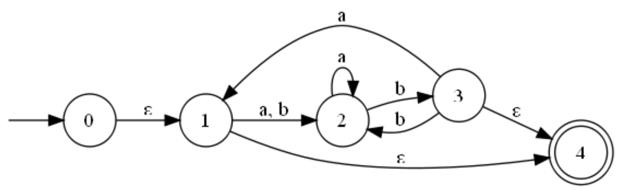
Figure 5: DFA for Exercise 1.19b

6 Exercise 1.21b

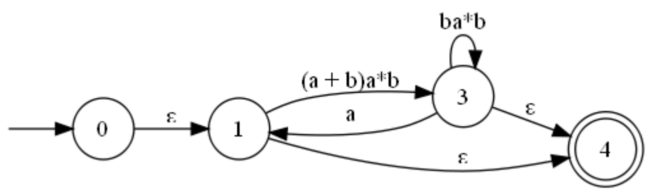
Convert the following NFA to a regular expression.



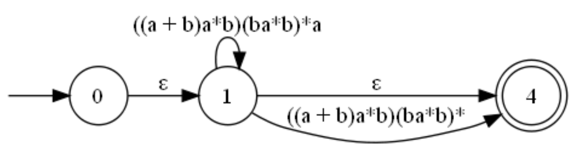
(a) The original NFA.



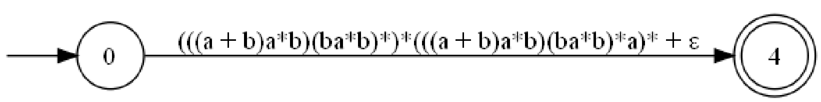
(b) Add a start and end state.



(c) Rip out state 2.



(d) Rip out state 3.



(e) Rip out state 1 to get the final regular expression.

Figure 6: DFA for Exercise 1.21b

7 Exercise 1.28c

Convert the regular expression $(a \bigcup b^+)a^+b^+$ to an NFA, given that $\Sigma = \{a, b\}$.

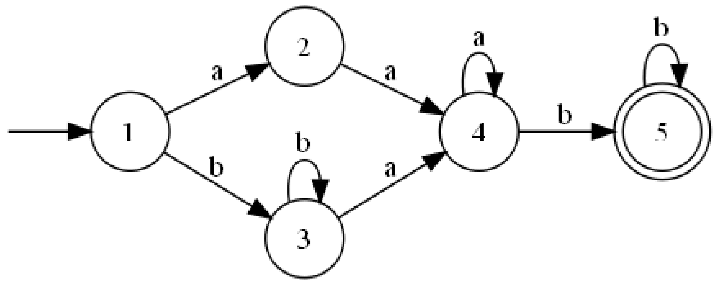


Figure 7: DFA for Exercise 1.28c

8 Exercie 1.29b

Use the pumping lemma to show that the language $A_2 = \{www \mid w \in \{a, b\}^\star\}$ is not regular.

Proof.

1. Assume A_2 is regular.
2. Let the three ws in the language be represented as w_1, w_2, w_3.
3. Then $|w_1| = |w_2| = |w_3|$.
4. Let $xyz = w_1$.
5. Given that $y \neq \epsilon$ and either x or $z \neq \epsilon$, then $y < w_2$ and $y < w_3$.
6. Pumping y up will make $|w_1| \neq |w_2|$, which breaks the language.
7. Therefore, since the langauge A_2 cannot be pumped, is not regular. □

9 Exercie 1.32

Show that the language B (omitted) is regular.

Proof.

1. Let B_R be the reverse language of B.
2. Given that a regular language can be pumped, and that the reverse of a regular language is also a regular language, B_R should be pumpable if B is a regular language.
3. Let b_1 be the top row of B_R, b_2 be the middle row, and b_3 be the bottom, or sum, row.
4. Since b_3 is a summation of b_1, b_2, there must be a carry bit which works its way from left to right through B_R (since B_R is a reverse of the summation B).
5. Let $w = xyz$ be some section of B_R, where y is the section being pumped.

6. Let j be the carry-in bit comming in to y and k be the carry-out bit comming out of y.
7. If $j = k$ then the summation b_3 would still be correct after pumping and y can be pumped.
8. If $j \neq k$, then I'm not sure what to do to make y pumpable, so I'll just say something goes here to prove that B_R can be pumped when $j \neq k$.
9. Since B_R can be pumped in all cases, then B_R is a regular language.
10. Therefore, since B_R is a regular language, than B is a regular language. □

10 Exercise 1.51

If x and y are indistinguishable by L, show that their indistinguishability is an equivalence relation.

I looked up equivalence relations and found that they have to do with set theory. When I took CIT592, we had only the briefest glance into set theory. I'm not sure I know how to prove this, or even where to start with proofs about sets, so I'm skipping it.

11 Exercise 1.53

Let $\Sigma = \{0, 1, +, =\}$ and $ADD = \{x = y + z \mid x, y, z$ are binary integers and x is the sum of y and $z\}$. Show that the language ADD is not regular.

Proof.

1. Assume that ADD is regular.
2. Let p be the pumping length of ADD.
3. Let $w = ADD = x_1 y_1 z_1$, where $0 > y_1 \leq p$.
4. Since the symbols $=$ and $+$ cannot be pumped, the only pumpable segments must be the binary representation of integers in x, y, and z.
5. If we assume that $x_1 = \epsilon$, then $z_1 \neq \epsilon$ and y_1 must either be x or $x = y$. Pumping either one would change the equality of the summation and break the language.
6. If we assume that $z_1 = \epsilon$, then $x_1 \neq \epsilon$ and y_1 must either be $y + z$ or z. Again, pumping either one would change the equality of the summation and break the language.
7. Therefore, since the langauge ADD cannot be pumped, is not regular. □

12 Exercise 1.55c

What is the minimum pumping length for $001 \bigcup 0^\star 1^\star$?

The minimum pumping length is 2 because at that length, either a 0 or a 1 in the second position could be pumped.

13 Exercise 1.55h

What is the minimum pumping length for $10(11^\star 0)^\star 0$?

The minimum pumping length is 5, where the string 10100 can be divided as $x = 10, y = 1, z = 00$.