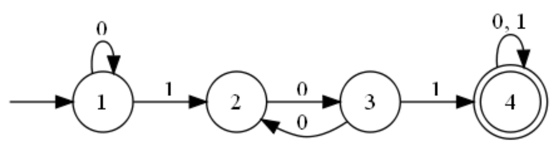
CIT 596 Homework 3

Steven Tomcavage
stomcava@seas.upenn.edu

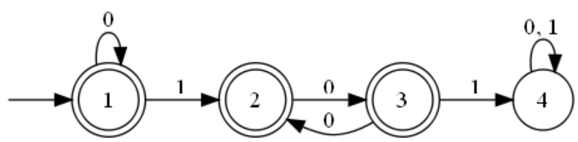
February 17, 2011

1 Exercise 1.13

Give a DFA that recognizes the language F where F is the language of all strings over $\{0, 1\}$ that do not contain a pair of 1s which are separated by an odd number of symbols.



(a) The NFA A accepts $\overline{F}$



(b) The DFA B accepts F

Figure 1: DFA for Exercise 1.13

2 Exercise 1.16b

Convert the given NFA (omitted) to a DFA.

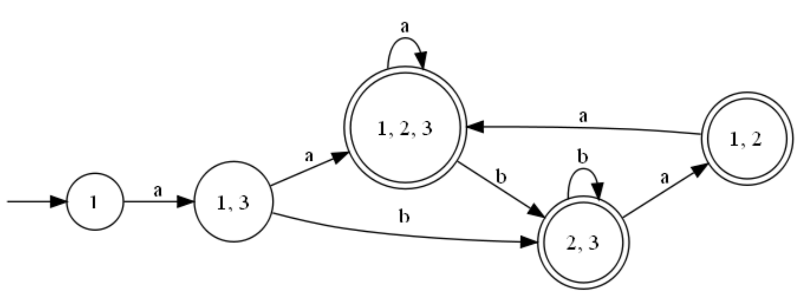


Figure 2: DFA for Exercise 1.16b

3 Exercise 1.17a

Give an NFA recognizing the language $(01 \bigcup 001 \bigcup 010)^{\star}$.

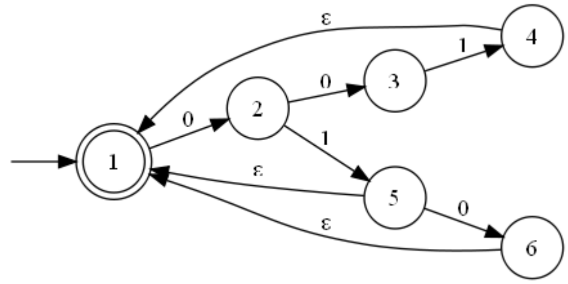


Figure 3: DFA for Exercise 1.17a

4 Exercise 1.17b

Convert the NFA from Exercise 1.17a to an equivalent DFA.

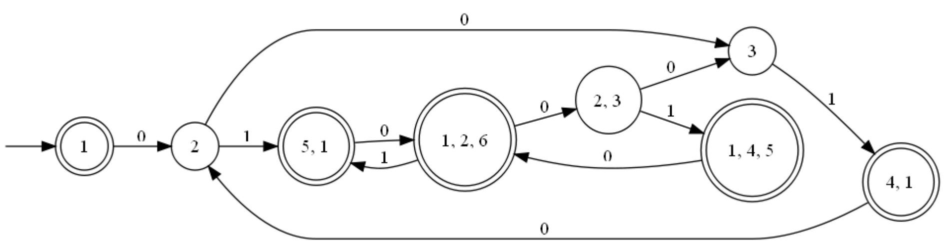


Figure 4: DFA for Exercise 1.17b

5 Exercise 1.19b

Convert the following regular expression to an NFA: $(((00)^{\star}(11)) \bigcup 01)^{\star}$.

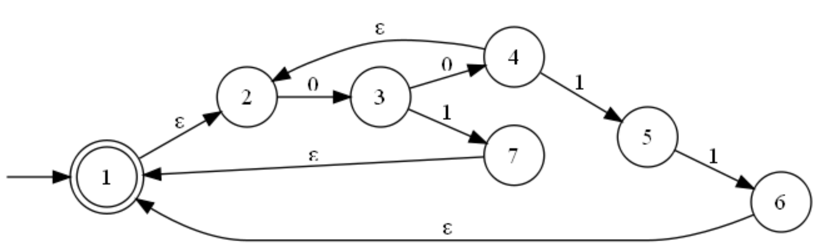
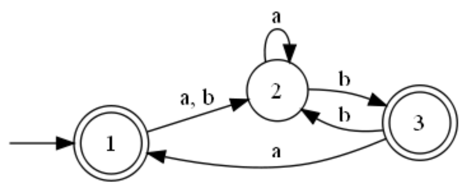


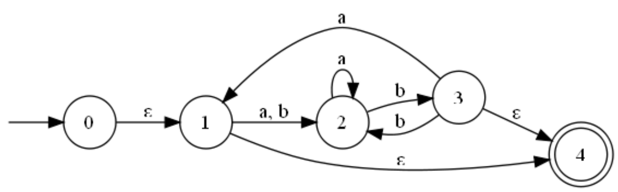
Figure 5: DFA for Exercise 1.19b

6 Exercise 1.21b

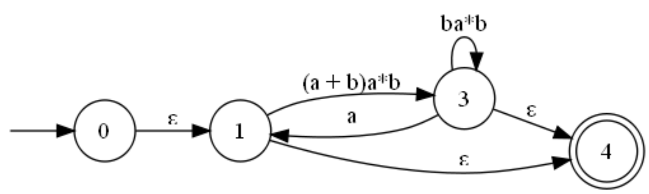
Convert the following NFA to a regular expression.



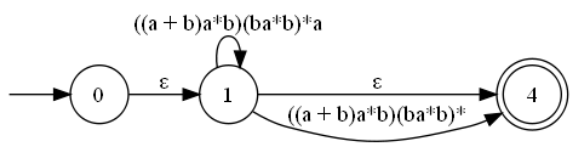
(a) The original NFA.



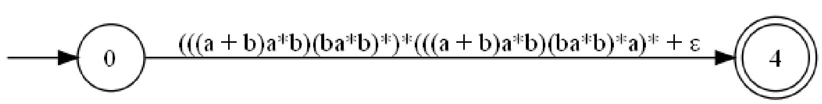
(b) Add a start and end state.



(c) Rip out state 2.



(d) Rip out state 3.



(e) Rip out state 1 to get the final regular expression.

Figure 6: DFA for Exercise 1.21b

7 Exercise 1.28c

Convert the regular expression $(a \bigcup b^+)a^+b^+$ to an NFA, given that $\Sigma = \{a, b\}$.

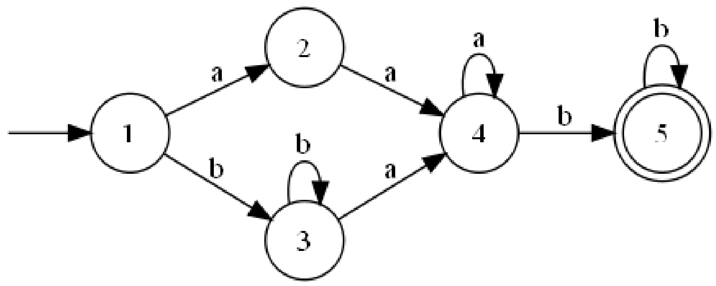


Figure 7: DFA for Exercise 1.28c

8 Exercie 1.29b

Use the pumping lemma to show that the language $A_2 = \{\omega\omega\omega \mid \omega \in \{a, b\}^\star\}$ is not regular.

Assume that A_2 is regular. Let p be the pumping length of A_2. Let $\omega = ab^pa$. So the string $ab^paab^paab^pa$ is in the language A_2. Pumping b^p gives $ab^pb^paab^paab^pa$, which is not in the language. Thus, A_2 is not regular.

9 Exercie 1.32

Show that the language B (omitted) is regular.

Let w_1 be the top row, w_2 be the second row, and w_3 be the bottom row. Assume that the language is regular, which means that it can be pumped. Let p be the pumping length. Let $w = xy^pz$. If B is regular, then xy^py^pz is also in B.

Repeating binary sums requires examination of three cases: $0 + 0 = 0$, $0 + 1 = 1$, and $1 + 1 = 0$. Neither $0 + 0 = 0$ nor $0 + 1 = 1$ create carry-overs, so repeating them does not affect any position to the left of those sums. However, repeating $1 + 1 = 0$ does cause a carry-over that needs to be considered.

TODO

10 Exercise 1.51

TODO

11 Exercise 1.53

Let $\Sigma = \{0, 1, +, =\}$ and $ADD = \{x = y + z \mid x, y, z$ are binary integers and $xisthesumofyandz\}$. Show that the language ADD is not regular.

Assume that ADD is regular, which means it can be pumped. Let p be the pumping length of ADD. Let $x = y^p + z$ be in the language ADD. Pumping y gives $x = y^py^p + z$ which no longer satisfies the condition that y and z sum to x. Therefore, ADD cannot be pumped and is not regular.

12 Exercise 1.55c

What is the minimum pumping length for $001 \bigcup 0^{\star}1^{\star}$?

The minimum pumping length is 2 because at that length, either a 0 or a 1 in the second position could be pumped.

13 Exercise 1.55h

What is the minimum pumping length for $10(11^{\star}0)^{\star}0$?

The minimum pumping length is 5, where the string 10100 can be divided as $x = 10, y = 1, z = 00$.