CIT 596 Homework 4

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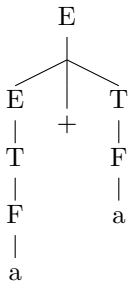
1 Exercise 2.1

Given the following CFG, provide parse trees for the string in each part:

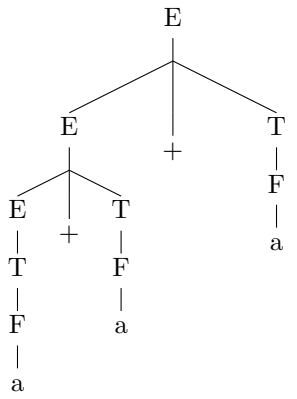
$$E \to E + T \mid T$$
$$T \to T \times F \mid F$$
$$F \to (E) \mid a$$

1.1 Part a: a

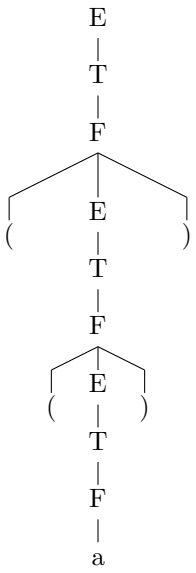
1.2 Part b: $a + a$



1.3 Part c: $a + a + a$



1.4 Part d : $((a))$



2 Exercise 2.2

TODO

3 Exercise 2.4b

Given $\Sigma = \{0, 1\}$, give a CFG that generates the language $\{w \mid w$ starts and ends with the same symbol$\}$.

$$S \rightarrow 0A0 \mid 1A1$$
$$A \rightarrow 0 \mid 1 \mid A \mid \epsilon$$

4 Exercise 2.4c

Given $\Sigma = \{0, 1\}$, give a CFG that generates the language $\{w \mid w$ the length of w is odd$\}$.

$$S \rightarrow 0A \mid 1A$$
$$A \rightarrow 00 \mid 01 \mid 10 \mid 11 \mid A \mid \epsilon$$

5 Exercise 2.4e

Given $\Sigma = \{0, 1\}$, give a CFG that generates the language $\{w \mid w = w^R$, that is$w$is a palindrome$\}$.

$$S \rightarrow 0A0 \mid 1A1 \mid \epsilon$$
$$A \rightarrow S$$

6 Exercise 2.5b

Give an informal description and state diagram for the language describe by Exercise 2.4b.

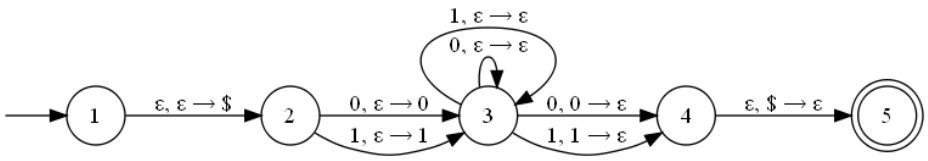


Figure 1: PDA for Exercise 2.5b

7 Exercise 2.5c

Give an informal description and state diagram for the language describe by Exercise 2.4c.

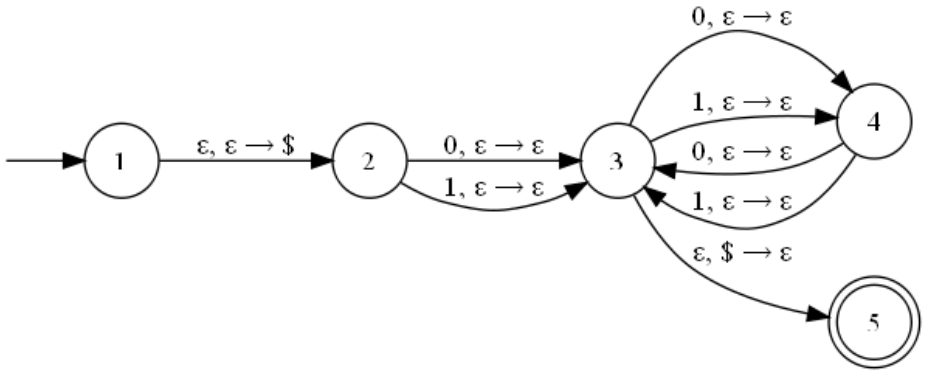


Figure 2: PDA for Exercise 2.5c

8 Exercise 2.5e

Give an informal description and state diagram for the language describe by Exercise 2.4e.

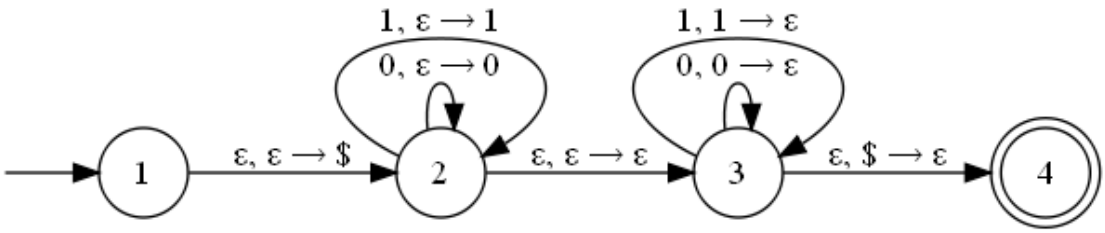


Figure 3: PDA for Exercise 2.5e

9 Exercise 2.9

Give a CFG that generates the langage $A = \{a^i b^j c^k \mid i = j \text{ or } j = k \text{ where } i, j, k \geq 0\}$. Is this CFG ambiguous?

9.1 Part a

$$S \to Wc \mid aX$$
$$W \to aWbY$$
$$X \to bXcZ$$
$$Y \to W \mid \epsilon$$
$$Z \to X \mid \epsilon$$

9.2 Part b

No, this CFG is not ambiguous because the leftmost derivation of any string only generates one parse tree. The productions progress linearally and the only loops in the CFG always loop back to the same location. There is never an option for a loop to have a choice of where to return to.

10 Exercise 2.13

10.1 Part a

$L(G)$ generates a string of zeros with one or two hash marks in the string. If there are two hash marks in the string, the hash marks can be at the beginning, end, or anywhere in the middle. If there is only one hash mark, the number of zeros after the hash mark is twice the number of zeros before the hash mark.

10.2 Part b

TODO

11 Exercise 2.14

Convert the following CFG to Chomsky Normal Form:

$$A \to BAB \mid B \mid \epsilon$$
$$B \to 00 \mid \epsilon$$

Step 1

Add a new start variable

$$S \to A$$
$$A \to BAB \mid B \mid \epsilon$$
$$B \to 00 \mid \epsilon$$

Step 2

Eliminate $B \to \epsilon$

$$S \to A$$
$$A \to BAB \mid B \mid \epsilon \mid BA \mid AB$$
$$B \to 00$$

Step 3

Eliminate $A \to \epsilon$

$$S \to A \mid \epsilon$$
$$A \to BAB \mid B \mid BA \mid AB \mid BB$$
$$B \to 00$$

Step 4

Remove unit rules

$$S \to BAB \mid 00 \mid BA \mid AB \mid BB \mid \epsilon$$
$$A \to BAB \mid 00 \mid BA \mid AB \mid BB$$
$$B \to 00$$

12 Exercise 2.20

Let $A/B = \{w|wx \in A$ for some x in B. Show that, if A is context free and B is regular, than A/B is context free.

Proof.

1. Given that A/B is context free.
2. Given that $wx \in A$ for some $x \in B$.
3. Then B is contained in the language A.
4. Therefore, B is context free. □

13 Exercise 2.26

Show that, if G is a CFG in Chomsky normal form, then for any string $w \in L(G)$ of length $n \geq 1$, exactly $2n - 1$ steps are required for any derivation of w.

Proof.

1. Given that CNF only allows productions of the form $A \to BC$ and $A \to a$.
2. If $n = 1$, then the only production required to generate w is of the form $A \to a$, thus the length is $1 = 2n - 1$.
3. If $n = 2$, then three productions are required, one of the form $A \to BC$ and two of the form $A \to a$, thus the number of steps in the derivation is $3 = 2n - 1$.
4. If $n > 2$, then we can break w down into units of size 2 or 1 and combine those units to form the larger derivation, which will have $(2n_i - 1) + (2n_j - 1) + \ldots + (2n - 1)$ steps.
5. Therefore, the number of steps required to derive w is $2n - 1$. □

14 Exercise 2.30a

Use the pumping lemma to show the langauge $\{0^n1^n0^n1^n \mid n \geq 0\}$ is not context free.
TODO