PH3406 Assignment (20%)

Matric No.	Name	Grade

Deadline: Week 12, 11 Apr 2025, 11:59pm. Submit in NTULearn - Assignments.

This homework set is computational in nature. Any coding language is allowed. If you had help from a source, e.g. a website, a forum like stackoverflow, or AI like DeepSeek or ChatGPT, or a human, you will need to acknowledge them and briefly state what help was given.

Submission is in NTULearn. You are to submit the following:

(i) PDF showing your solution, brief explanation of your approach and acknowledgement or citation (if necessary).

(ii) Code with annotations, and attribution where applicable.

Alternatively, a Jupyter notebook with explanation and code is also accepted.

Problem 1 – Quantum State Tomography

You are given data from single qubit measurements, stored in a text file "HW_Prob1_data.txt" and uploaded into NTULearn.

In the data are individual experiments where measurements of $\hat{\sigma}_x, \hat{\sigma}_y, \hat{\sigma}_z$ were performed, and the results saved in column format.

Perform quantum state tomography on the given data and reconstruct the quantum state with your choice of maximum likelihood estimate approach.

Problem 2 – Quantum Process Tomography

(This is Prob. 5.7 in the lecture notes.)

Consider a one qubit black box of unknown dynamics $\mathcal{E}_1$. Suppose that the following four density matrices are obtained from experimental measurements, performed according to the rules of Example 5.9:

$$\hat{\rho}_1' = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \tag{1}$$

$$\hat{\rho}_2' = \begin{pmatrix} 0 & \sqrt{1-\gamma} \\ 0 & 0 \end{pmatrix} \tag{2}$$

$$\hat{\rho}_3' = \begin{pmatrix} 0 & 0 \\ \sqrt{1-\gamma} & 0 \end{pmatrix} \tag{3}$$

$$\hat{\rho}_4' = \begin{pmatrix} \gamma & 0 \\ 0 & 1-\gamma \end{pmatrix} \tag{4}$$

where γ is a numerical parameter. From the input-output relations, one could make several important observations: the ground state $|0\rangle$ is left invariant by $\mathcal{E}_1$, the excited state $|1\rangle$ partially decays to the ground state, and superposition states are damped. Determine the χ matrix for this process.

For this problem, consider these two cases separately: $\gamma = 0$ and $\gamma = 0.15$.

Note: You will get *bonus* marks if the χ matrix that you derive is analytical, i.e. written in terms of γ. If you are unable to do this, finding the χ matrix numerically is also fine!