Evaluating computational methods for modeling off-normal operation of gas centrifuge cascades

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Abstract

This work compares and evaluates different computational approaches for modeling off-normal operation of a gas centrifuge enrichment cascade.

The goal of this work focuses on developing the necessary understanding of potential misuse of enrichment cascades, contributing to more effective international safeguards designs and approaches. While it is straightforward to design a symmetric enrichment cascade under ideal conditions as a function of the theoretical feed, product, and tails assays, it is very difficult to find reliable information about the behavior of a given cascade when the feed assay does not match the design value. Several methods have been developed to assess the behavior of an enrichment cascade in such circumstances. In addition to the cut, (θ) these methods evaluate the feed-to-product, feed-to-tails, and the product-to-tails enrichment ratio, α, β and γ, respectively, as a function of the cascade feed assay. As those four parameters depend on each other, determining two of them fully defines the other. The first approach consists of fixing θ and α, recomputing the corresponding assays at each stages of the cascade. The second one maintains the ideal condition of the cascade (α and β fixed across the whole cascade), modifying θ values at each stage accordingly. Both approaches have been implemented into the Cyclus *fuel cycle simulator[**?** **?**]. The third fixes θ and γ, using both α and β at each stage as free parameters. The third method has been investigated in [**?**].*

Following a description of each method and an evaluation of differences between each approach, this work compares the results produced by these methods within scenarios involving misuse of symmetric enrichment cascades simulated using the dynamic nuclear fuel cycle simulator, Cyclus*.*

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1. Motivation

Gas centrifuge cascades are usually designed to operate in an ideal manner, with no losses in separative work. To achieve such ideal configuration, the cascade is designed to be fed with a specific feed assay and produce the target enrichment while rejecting tails at a fixed assay.

With the current international tensions regarding enrichment capabilities, this work aims to measure the effectiveness of a symmetric enrichment cascade when used outside of its designed scope and to quantify the attractiveness of such ways to build up a significant quantity (SQ) of Highly Enriched Uranium (HEU), defined by the International Atomic Energy Agency (IAEA) as 25 *kilograms*[**?**].

The present work investigates the performance of an enrichment cycle when chaining gas centrifuge enrichment cascades tuned for Low Enriched Uranium (LEU) production from natural uranium to instead produce HEU. Literature on the subject is limited due to its internationally and politically sensitive nature. Three behavior models have been implemented and used to evaluate the response of an enrichment cascade when fed with different assays than originally designed. This work also takes advantage of the CYCLUS [**?**] fuel cycle simulator's capabilities to evaluate the assay values at equilibrium.

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2. Theory

In the following section, equations and algorithms used to model the behavior of a symmetric ideal gas centrifuge cascade from individual centrifuge properties are described. Details will also be provided to model centrifuge behavior when fed with a different feed enrichment than designed.

2.1. Centrifuge properties

2.1.1. Separative power

Räetz equation. As described by Glaser in [?], the separative power of a single centrifuge can be express as an analytical solution [?] of the differential equation for the gas centrifuge:

$$\begin{aligned}
\delta U(L,F,\theta,Z_p) &= \frac{1}{2}F\theta(1-\theta)\left(\frac{\Delta M}{2RT}v_a^2\right)^2\left(\frac{r_2}{a}\right)^4\left[1-\left(\frac{r_1}{r_2}\right)^2\right]^2 && (1)\\
&\left[\left(\frac{1+L/F}{\theta}\right)(1-exp[-A_P(L,F,\theta)Z_p])\right.\\
&\left.+\left(\frac{L/F}{1-\theta}\right)(1-exp[-A_W(L,F,\theta)(Z-Z_p])\right]^2,\\
\text{with}\qquad A_P &= \frac{2\pi D\rho}{ln(r_2/r_1)}\frac{1}{F}\frac{1-\theta}{(1+L/F)(1-\theta+L/F)} && (2)\\
A_W &= \frac{2\pi D\rho}{ln(r_2/r_1)}\frac{1}{F}\frac{1-\theta}{(L/F)(1-\theta+L/F)} && (3)
\end{aligned}$$

In this equation, the parameters of average gas temperature, T, peripheral speed, v_a, height, h, diameter, d, pressure ratio, x, feed flow rate, F, counter-current flow ratio, L/F, are intrinsic to the centrifuge design. To match the cascade design describe in [?] and [?], P1-type centrifuge properties have been chosen (Table 1).

Table 1: Summary of the centrifuge parameters.

T[K]	v[m/s]	h[m]	d[m]	x	F[mg/s]	L/F
320	320	1.8	0.105	10^3	13	2

The variable Z_p is the rectifier length, or the location of the feed point, and has an optimal axial location as defined by [**?**]:

$$Z_p = \frac{(1-\theta)(1+L/F)}{1-\theta+L/F} Z \tag{4}$$

This optimizes the recitifer length based on the cut, θ, which is an expression of the fraction of the centrifuge feed that is output as product, and the counter-current flow, L/F. In practice, this value is a design parameter that is defined in the model during the design of the centrifuge cascade.

The parameters r_1 and r_2 are the separation radii of the enriched material (here ^{235}U vs. ^{238}U), r_1 being the withdrawal radius for the lighter isotope and r_2 for the heavier isotope. Räetz's two-shell model looks for optimal values between these two, as described by the hydrodynamic equations. The radii ratio is optimized using the following relationship:

$$\max\left\{\left[1-\left(\frac{r_1}{r_2}\right)^2\right]^2 \times \left[\ln\left(\frac{r_2}{r_1}\right)\right]^{-1}\right\} \tag{5}$$

This ratio can further be constrained by approximating r_2 as being very close to the inner centrifuge wall with radius a:

$$\left(\frac{r_1}{r_2}\right) \approx \left(\frac{r_1}{a}\right) = \sqrt{1-\frac{2RT}{M}(\ln x)\frac{1}{v_a^2}} \tag{6}$$

Here the gas constants are molar mass, M, temperature T, universal gas constant, R, and the pressure ratio, x (typically $1000:1$) []. This relationship is valid when $v_a > 380\frac{\text{m}}{\text{s}}$. Otherwise, the relationship can be approximated $\{\frac{r_1}{r_2} \approx 0.534 \mid v_a \leq 380\frac{\text{m}}{\text{s}}\}$. In order to decompose the ratio into each individual radius, knowledge on one of them is required. Glaser [**?**] states that r_2 typically ranges from 96% to 99% the value of the inner centrifuge wall radius a. The exact behavior of r_2 between these two values in operational conditions is unknown, but a linear approximation can be made. It is assumed that r_2 is always at the midpoint of these two extremes (i.e. $r_2 = 0.975a$). Then, r_1 can be found by simply multiplying this value by the ratio.

With this, all parameters of the separative power equation 1 are defined and a value can be calculated. Separative power, δU, has units similar to the feed flow, that of $[mg/s]$.

First principle. Separative power can be written as a function of the change in entropy performed [**?**]:

$$\delta U = \frac{L}{RN}\frac{\Delta S}{(1-N)} \tag{7}$$

Here, L is the feed flow expressed as moles per second and could be interchangeably expressed as F above. Using the Boltzmann definition of entropy, the change in entropy for a mixture of two isotopes is:

$$\Delta S(N) = R\left(N\ln(N) + (1-N)\ln(1-N)\right) \tag{8}$$

Using this in the expression for free energy [**?**] and normalizing for temperature and gas constant, a value function can be constructed [**?**]:

$$V(x) = (2x-1)\ln\left(\frac{x}{1-x}\right) \tag{9}$$

Where x here is a concentration for a mass (e.g. N). The separative work applied on a feed is then an expression of this value function:

$$\delta U = \theta F V(N') + (1-\theta) F V(N'') - F V(N) \tag{10}$$

Conducting a Taylor expansion of $V(N)$ around N and utilizing the definition $N = \theta N' + (1-\theta)N''$ lends a simplification:

$$\delta U = \frac{\theta}{1-\theta}\frac{F(\alpha-1)^2}{2}\frac{d^2V(N)}{dN^2}\left[N(1-N)\right]^2 \tag{11}$$

Finally, specifying the change in value function to be independent of N (?) concludes in an expression for the separative power of a single centrifuge in terms of the feed-to-product enrichment factor, α, the cut, θ, and the feed rate, F [**?**]:

$$\delta U = \frac{\theta}{1-\theta}\frac{F}{2}(\alpha-1)^2 \tag{12}$$

2.2. Centrifuges basic properties and definition

The outputs of a centrifuge relative to its input can be described with ratios of the abundance ($R = \frac{N}{1-N}$) where the feed, product, and tail enrichment N, N', N'' respectively. Enrichment factors of α (feed-to-product), β (feed-to-tail), and γ (tail-to-product) can then be defined:

$$\alpha = \frac{1-N}{N}\frac{N'}{1-N'} \tag{13a}$$

$$\beta = \frac{1-N''}{N''}\frac{N}{1-N} \tag{13b}$$

$$\gamma = \alpha\,\beta \tag{13c}$$

2.3. Cascade Design

2.3.1. Symmetric Cascade

A symmetric cascade is a cascade where a stage is fed using the tail, T, of the next stage and the product, P, of the previous one. The the cascade feeding stage, F_i flow, with external feed, F_{ext}, is given by:

$$F_i = T_{i+1} + P_{i-1}\ (+F_{ext}) \tag{14}$$

2.3.2. Symmetric Ideal Cascade

This model constructs cascades as symmetrical and ideal, with no losses in separative work. This means that the tail assay of the next stage (N''_{i+1}) is the product assay of the previous stage (N'_{i-1}), which can be expressed as:

$$\forall i\ N_i = N'_{i-1} = N''_{i+1} \quad\Leftrightarrow\quad \forall(i,j)\ \alpha_i = \beta_j \tag{15}$$

2.4. Building the cascade

Designing a symmetric ideal cascade beings with the feeding stage. As all the enrichment factors are equal across all the cascade, the feeding stage is used to determine all subsequent stages. The feed assay of the feeding stage, N_0, is fixed by the external feed assay provided as an input.

From equation (1) and (12) it is possible to express the feed-to-product enrichment factor α as a function of the feed rate F, the separative performance δU (a function of θ), and the cut θ:

$$\alpha = \sqrt{\frac{2\delta U(\theta)}{F}\frac{1-\theta}{\theta}} + 1 \tag{16}$$

From equations (13), the product assay can be expressed as:

$$N' = \frac{\alpha\frac{N}{1-N}}{1+\alpha\frac{N}{1-N}} = \frac{\alpha R}{1+\alpha R} \tag{17}$$

Then, from mass conservation, $N = \theta N' + (1-\theta)N''$ and equation (17), it is possible to express the feed-to-tail enrichment factor β as a function of only the feed abundance, R, the cut θ, and the feed-to-product enrichment factor, α:

$$\beta = \left(1 - \frac{N-\theta N'}{1-\theta}\right)\left(\frac{R}{\frac{N-\theta N'}{1-\theta}}\right) \tag{18a}$$

$$\beta = R\left(\frac{1-\theta}{\frac{R}{R+1} - \theta\frac{\alpha R}{1+\alpha R}} - 1\right) \tag{18b}$$

Finally from equation (16) and (18b) it is possible to determine the cut, θ, required to build an ideal cascade:

$$\theta_i = \frac{N_i - \frac{1}{1+\beta/R_i}}{\frac{\alpha R_i}{1+\alpha R_i} - \frac{1}{1+\beta/R_i}} \tag{19}$$

To construct an ideal stage, a cut, θ, must first be computed. This is found iteratively, searching for the optimal cut value where $\alpha = \beta$ for given feed and centrifuge parameters. With a separative power calculated, an α and β value can be determined for an initial cut guess. This model assumes that the ideal cut for a stage should be between 0.1 and 0.9. The two enrichment factors are compared, and the higher or lower cut value is chosen by which pair of factors are closer. A new cut is determined from the chosen factors, a new separative power is computed, and new α and β values are compared. This process continues until, to some precision, the resulting enrichment factors are approximately equal. As illustrated in Fig. 1, for a given input feed assay, the ideal feeding stage typically has a θ value between 0.45 and 0.525 when $\alpha = \beta$.

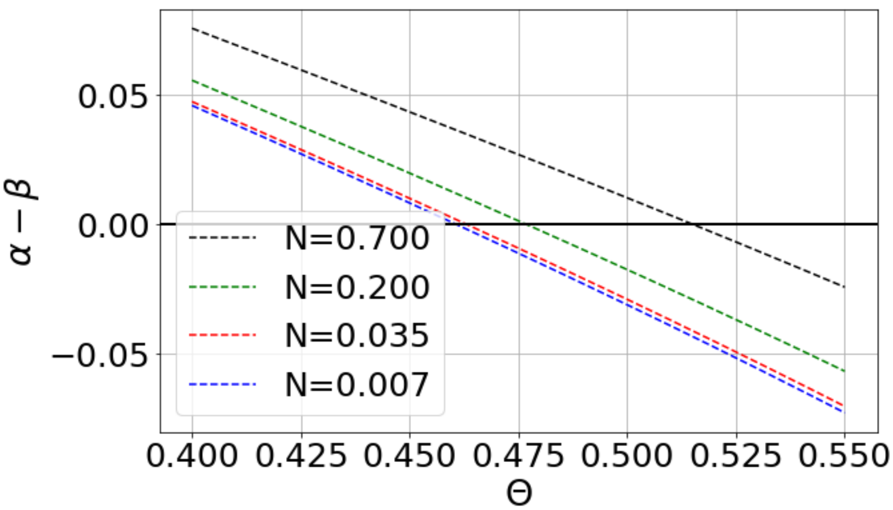


Figure 1: Evolution of the difference between α and β as a function of the cut value for different of feed assays, 0.007 (black), 0.035 (red), 0.2 (green), 0.7 (black).

The number of machines required to construct a stage can then be computed using equation (12) to solve for the centrifuge feed flow:

$$F_c = \frac{2\delta U}{(\alpha - 1)^2} \frac{M}{M_{238}} \frac{1 - \theta}{\theta} \tag{20}$$

Where the molar mass ratio $\frac{M}{M_{238}}$ accounts for the molar mass differences between the feed gas, UF_6, and the individual uranium isotopes being separated. The stage feed flow can then be divided by the individual centrifuge feed flow, equation (20), to find the exact number of machines needed for the ideal stage. In practice, this number is rounded up to account for fractional machines required.

In a cascade, as the stage feed-to-product enrichment factor α_i and stage feed-to-tail enrichment factor β_i remain constant, only the value of the cut, θ_i, changes across the different stages of a cascade. This algorithm assumes that the corresponding separative power δU (not re-computed) can be achieved with the chosen centrifuge design by tuning other operational parameters such as the rotation speed, counter-current flow ratio, etc. Once θ_i is determined, it is possible to compute the product and the tail assays.

The design of the ideal symmetric cascade is performed through 2 steps. First, the configuration and number of stages is determined, adding stages with product assay N_i' calculated using equation 17, until the product assay of the final stage is greater than or equal to the product targeted assay. The stage tails assay N_i'' is calculated similarly, until it is less than or equal to the tails desired assay. This determines the number of enriching and stripping stages as well as their enrichment properties (N_i, N_i', N_i'',θ_i).

The second step determines the relative flows at each stage, solving the linear flow equation, (21). The cascade can then be populated with actual machines until the maximum number of available machines is reached.

$$\begin{bmatrix} -1 & 1-\theta_{S+1} & 0 & ... & 0 & 0 & 0 & 0 & 0 & ... & 0 & 0 & 0 \\ \theta_S & -1 & 1-\theta_{S+2} & ... & 0 & 0 & 0 & 0 & 0 & ... & 0 & 0 & 0 \\ & & & & & & ... & & & & & & \\ 0 & 0 & 0 & ... & \theta_{-2} & -1 & 1-\theta_0 & 0 & 0 & ... & 0 & 0 & 0 \\ 0 & 0 & 0 & ... & 0 & \theta_{-1} & -1 & 1-\theta_1 & 0 & ... & 0 & 0 & 0 \\ 0 & 0 & 0 & ... & 0 & 0 & \theta_0 & -1 & 1-\theta_2 & ... & 0 & 0 & 0 \\ & & & & & & ... & & & & & & \\ 0 & 0 & 0 & ... & 0 & 0 & 0 & 0 & 0 & ... & \theta_{E-2} & -1 & 1-\theta_E \\ 0 & 0 & 0 & ... & 0 & 0 & 0 & 0 & 0 & ... & 0 & \theta_{E-1} & -1 \end{bmatrix} \times \begin{bmatrix} F_S \\ F_{S+1} \\ \cdots \\ F_{-1} \\ F_0 \\ F_1 \\ \cdots \\ F_{E-1} \\ F_E \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \cdots \\ 0 \\ F \\ 0 \\ \cdots \\ 0 \\ 0 \end{bmatrix} \tag{21}$$

2.5. Misuse models

Little information is available about optimising an existing enrichment cascade that is being fed with a feed enrichment that does not match the design enrichment. Here, 3 different methods will be investigated.

The first method, A, assumes that no changes are being made to the cascade, i.e δU, F and θ are fixed across all stages. The second method, B, assumes the cut value θ is re-tuned at each stage to maintain the ideal state of the cascade, while α and β remain fixed. The last method, C, described in [**?**] assumes the tails-to-product enrichment factor γ and the cut θ remain constants (eq. (13c): $\gamma = \alpha\beta$). Model behaviors and assumptions are summarized in Tab. 2.

Table 2: Summary of misuse model properties.

Model	A	B	C
Constant parameters	α_i, θ_i	$\alpha_i = \beta_i$	$\gamma_i = \alpha_i\beta_i, \theta_i$
Varying parameters	β_i	θ_i	α_i, β_i
Assays determination	blended	ideal	blended
Flow	unchanged	reduced	unchanged

2.5.1. Model A

The tuning method A does not re-optimize θ_i, keeping the same flow as the ideal configuration. From equation (16), maintaining δU and F while θ is unchanged implies α remains unchanged as well. According to equation (18b), when α and θ are fixed, if the feed assay (N) changes, β will change accordingly. This breaks the ideal status of the cascade, i.e. $N_i \neq N'_{i-1} \neq N''_{i+1}$.

In order to compute the proper product and tails assay at each stage, the tail and the product from the next and the previous stage respectively must be blended in order to determine the correct stage feed assay. All feed assays are iteratively updated, blending the proper product and tails, then using the updated feed assay, the new product and tails assays are recomputed. This process is repeated until the sum of the square difference in assays is smaller than 10^{-8}. As the cut remains fixed at each stage, the flows do not need to be recomputed.

This model assumes that it is possible to maintain the separative power of a centrifuges, δU, for any feed assays N while maintaining its cut θ and feed flow F.

2.5.2. Model B

Using the second method, the cut value at each stage, θ_i, is retuned in order to maintain the α_i and β_i at their original values (equation (19)). Since the cascade remains ideal, the product and tails assay at each stage can easily be determined using equations (13).

As the cut values change, the relative flow rates between the different stages are recomputed using equation (21). Under this model, the flow at each stage of the original ideal cascade is assumed to be the maximum flow allowed at that stage. Therefore, all of the recomputed flow rates are scaled together to ensure that no stage experiences a flow rate larger than that of the original cascade. Some stages may now experience flow rates much lower than the original cascade.

This model assumes that it is possible to tune a centrifuge separative power δU, for any feed assay N, cut θ and feed flow F, in order to maintain its constant feed to product enrichment factor α.

2.5.3. Model C

The last model assumes that the tails to product enrichment factor remains constant regardless to the feed assays. To compute the response of the cascade one need to determine α and β such that their product and θ remain fixed. From equations (13) and the assay conservation equation $N = \theta N' + (1-\theta)N''$ it is possible to express the product, N', dependent on the feed assay N, γ, and the cut, θ, as one solution of the second order equation (22):

$$\theta(\gamma-1)N'^2 + ((N+\theta)(\gamma-1)+1)N' - N\gamma = 0 \tag{22}$$

The only solution allowing product assay to range between 0 and 1 is the following :

$$N' = \frac{N+\theta}{2\theta} + \frac{1-\sqrt{\gamma^2(N-\theta)^2 + 2\gamma(N^2+N-\theta^2+\theta) + (N+\theta+1)^2}}{2\theta(\gamma-1)} \tag{23}$$

Once the product assay is known, one can trivially determine the tails assay, α and β, using equations (13) and mass conservation.

Similar to model A, because the cut values remain constant, the flows do not need to be recomputed and the correct assays, α and β, are determined through iterative blending of the product assays of the previous stage and the tails assay of the next stage using equation (23).

This model assumes that it is possible to tune the centrifuge separative power δU in order to maintain for any feed assay N and its tails to product enrichment factor γ while maintaining its cut θ and feed flow F.

3. The experiment

This work focuses on comparing the different misuse models to a reference calculation in which a single large cascade is designed and built to directly produce HEU from natural uranium. This works uses the Cyclus fuel cycle simulator to allow material exchange between facilities. The enrichment cascade algorithm has been implemented in the *mbmore* package [**?**]. In each cases, 5060 centrifuges have been used and spread across up to 30 different gas centrifuge enrichment cascades. This is inspired by limitations placed on the Iranian uranium enrichment program via the Joint Comprehensive Plan of Action (JCPoA) [**?**].

3.1. The cascade configuration

3.1.1. Reference

As mentioned, all the further calculations will be compared to the most favorable configuration to produce HEU, where all the available centrifuges are used in a single large ideal symmetric cascade designed to directly produce HEU from natural uranium, with a tails assay close to $0.3w\%$. The design characteristic of the reference cascade are summarized in Table 3.

3.1.2. Default cascade

The default cascade is the ideal symmetric cascade designed for normal civilian enrichment operation, enriching natural uranium to about $3.5w\%$, with a tails assay close to $0.3w\%$. This cascade will be layered and fed with uranium at higher enrichment to evaluate the possibility to use them, with little or no tuning, to produce HEU. The characteristics of the default cascade are summarized in Table 3.

Table 3: Summary of cascade design.

Cascade Design		Reference	Default
Targeted Assays	Feed	$0.71w\%$	$0.71w\%$
	Product	$90w\%$	$3.5w\%$
	Tails	$0.3w\%$	$0.3w\%$
Effective Assays	Product	$90.35w\%$	$4.13w\%$
	Tails	$0.29w\%$	$0.29w\%$
Stages Number	Stripping	4	4
	Enriching	39	10

3.2. Scenarios

In the following, cascades can be connected in tandem, where each set of cascade in parallel is called a “level“, as illustrated in Figure 2. The reseults from seven different simulations have been compared, to evaluate the effectiveness of an enrichment cascade when used outside of its designed scope :

- one as the reference calculation, with a single cascade designed to directly produce HEU from natural uranium,
- three calculations (one per misuse model) where default cascade are chained to produce HEU, without recycling the tails of each cascade sending their tails to the waste,
- three calculations (one per misuse model) where default cascade are chained to produce HEU, and the tails of each cascade are recycled, blending the tails of one level in the feed of the previous level of cascades (see Figure 2).

3.3. Level population

In order to assign the optimum number of cascades to each level, a “level cut” as been computed as:

$$\Omega_j = \frac{N_j - N_j''}{N_j' - N_j''}, \tag{24}$$

where, j represents a level of cascade and N_j, N_j' and N_j'' the feed, product and tails assay, respectively, of the cascades at this level.

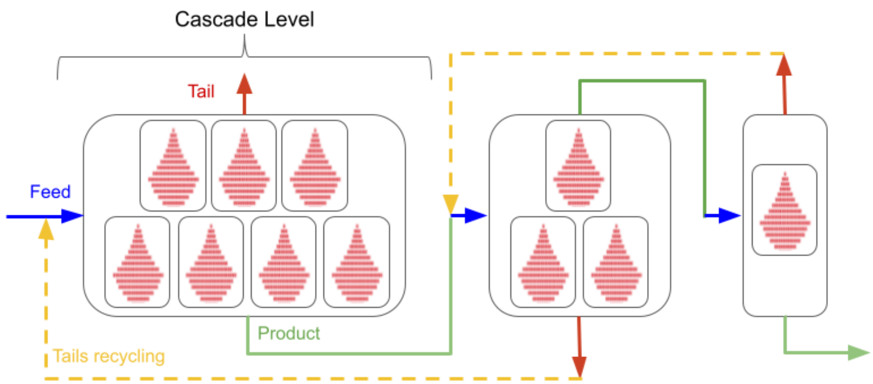


Figure 2: Schematic representation of the chained cascades with three levels, with the feed, product and the tails flows, in blue, green and red, respectively. The dashed orange line represent the alternative tails flow when tails recycling is considered.

A flow equation similar to (21) is then solved to obtain the optimum number of cascade per level. When the tails are not recycled, the $(1-\theta)$ terms are removed from the flow equation. The results of the level population are summarized in Table 4.

As it is not possible to assign a fraction of an enrichment cascade, cascade per level must be rounded and assigned for each level. Again, per the JCPoA, 30 cascades are distributed to the appropriate number of levels to achieve an HEU threshold with the least amount of levels. This optimal assignment is manually tested by running many different configurations. In generally, each level besides the first is rounded up from its optimal fractional value, with the remaining cascades being assigned to the first level.

4. Results

4.1. Miss-use modeling

As illustrated in Figures 3 and summarized on Table 4, the different models don't have the same effect on the cascade behavior. While models A and C achieve a quick enrichment gain with the cascades chaining, 4/21/76/98 and 4/21/79/99, respectively, model B only achieves an enrichment gain of

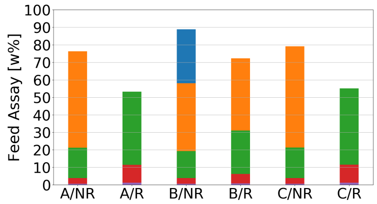
Table 4: Summary of cascades level population.

Model			A/NR	A/R	B/NR	B/R	C/NR	C/R
		Feed $w\%$	0.71	1.31	0.71	0.92	0.71	1.30
Level 0	Assay	Product $w\%$	3.97	7.27	3.97	5.10	3.97	7.24
		Tails $w\%$	0.28	0.52	0.28	0.36	0.28	0.51
	Cascades	Impl. (Ideal)	25(26.5)	25(26.5)	24(26.4)	25(26.3)	25(26.5)	25(26.2)
		Feed $w\%$	3.97	11.48	3.97	6.29	3.97	11.64
Level 1	Assay	Product $w\%$	21.29	53.30	19.33	27.98	21.41	55.12
		Tails $w\%$	1.68	5.93	1.58	2.54	1.66	5.88
	Cascades	Impl. (Ideal)	3(3.1)	4(3.1)	3(3.1)	3(3.1)	3(3.1)	4(3.4)
		Feed $w\%$	21.29	53.3	19.33	31.09	21.41	55.12
Level 2	Assay	Product $w\%$	76.39	94.70	58.10	72.31	79.21	96.37
		Tails $w\%$	13.98	47.81	8.51	14.91	13.75	49.65
	Cascades	Impl. (Ideal)	1(0.4)	1(0.4)	1(0.4)	1(0.5)	1(0.4)	1(0.4)
		Feed $w\%$	76.39	N.A.	58.10	72.31	79.21	N.A.
Level 3	Assay	Product $w\%$	98.09	N.A.	88.92	93.79	98.86	N.A.
		Tails $w\%$	73.51	N.A.	35.01	50.36	76.60	N.A.
	Cascades	Impl. (Ideal)	1(0.04)	N.A.	1(0.09)	1(0.1)	1(0.04)	N.A.
		Feed $w\%$	N.A.	N.A.	88.92	N.A.	N.A.	N.A.
Level 4	Assay	Product $w\%$	N.A.	N.A.	97.89	N.A.	N.A.	N.A.
		Tails $w\%$	N.A.	N.A.	75.71	N.A.	N.A.	N.A.
	Cascades	Impl. (Ideal)	N.A.	N.A.	1(0.04)	N.A.	N.A.	N.A.

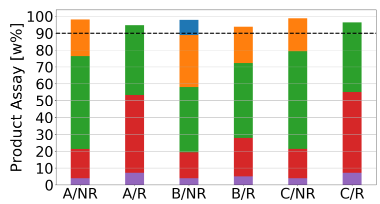
4/19/58/89/98, requiring more levels to even achieve HEU enrichment. This table also shows the integer number of cascades implemented (Impl.) at each level in the simulation, compared to the non-integer number of cascades that would achieve an ideal configuration (Ideal).

4.2. Tails recycling

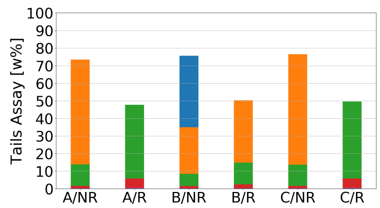
As shown in Figures 3, recycling the tails increases the overall product assay at all the different levels, requiring less levels in the configuration to achieve the desired enrichment. As the tails assay of a level $n+1$ is always higher than the product assay of the level $n-1$, recycling the tails of level $n+1$ will consequently increase the feed assay of level n (see Table 4). Moreover, with an increased feed assay, tails and product assays increase as well, increasing de facto the feed assays of respective cascade levels $n-1$ and $n+1$, etc. This effect reduces the number of cascade levels required to reach HEU in all cases.



(a) Feed Assays



(b) Product Assays



(c) Tails Assays

Figure 3: Feed (a), product (b) and tails assays (c) in $w\%$ of ^{235}U, per cascade level from 0 to 4 (purple, red, green, orange, blue), per model (A/B/C) and without/with tails recycling (NR/R). The black dashed line represents the $90w\%$ enrichment threshold.

4.3. HEU Production Rate

As shown in Figure 4, recycling increases the final HEU production rate, from 5 to almost 45 kg/y when using models A and C, and from 12 to 45 kg/y with the model B. For the reference calculation, where all the available cascades are used within a single large cascade design for direct HEU production, the HEU production rate is slightly over 60 kg/y.

As models A and C rely on maintaining the cut values at each stage of the cascade and share the same number of levels, they have the exact same cascade repartition across the different levels and the same HEU production rate.

5. Discussion

We can observe that when the cascade is left completely untouched (Model A) or when it is slightly retuned to maintain the tails to product enrichment factor as well as the cut of each centrifuges (Model C), chaining the cascade can

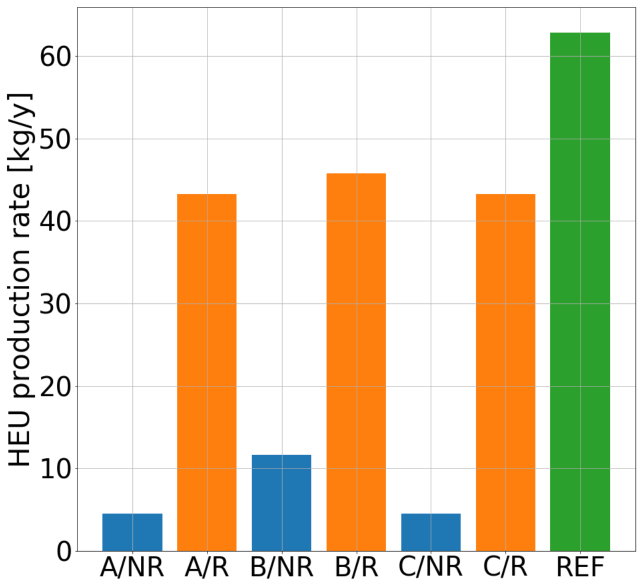


Figure 4: Production rate at equilibrium for the different model configurations, the case without tails recycling (blue), with tails recycling (orange), and the reference one (green). A-B-C represent the model used, and NR-R the case without tails recycling and the case with tails recycling, respectively.

achieve large increase of the enrichment at each level. On the contrary, when retuning the cut of each centrifuge to maintain the ideal state of the cascades (Model B) while chaining them, the HEU production rate is favored over the enrichment gain.

The tails recycling allows each model to achieve a large gain in productivity. Even if no cascade chaining options achieves the same production rate as direct enrichment, all models with tail recycling reached close to 73% of the optimum production rate. Such a production rate would allow the accumulation of a Significant Quantity of HEU in less than 8 months...

6. Conclusion and future work

This work has investigated the possibility to chain centrifuge enrichment cascades that are designed to enrich uranium for commercial reactors in order to produce HEU instead. Three methods have been implemented to model symmetric enrichment cascade behavior when fed with different uranium enrichment than designed.

Each method achieves up to 73% of the production rate of a single large enrichment cascade designed specifically for HEU production using the same number of centrifuges.

This work will be extended to the near future with additional misuse methods, allowing for example, the reconfiguration of the centrifuges in the cascades.

For this study, the use of the Cyclus fuel cycle simulator was not required; it only allows a quick determination of the blending equilibrium. Future studies will make use of the full capability of Cyclus Dynamic Resource Exchange in order to automatically assign the different cascades to the different level as function of the resources availability, optimising the productions rates in each cases.

While mathematically correct, the authors do not guaranty the feasibility of the different misuse tuning methods implemented and are welcoming any insight on the matter.

7. Acknowledgments

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