A brief introduction to

WAVELET ANALYSIS

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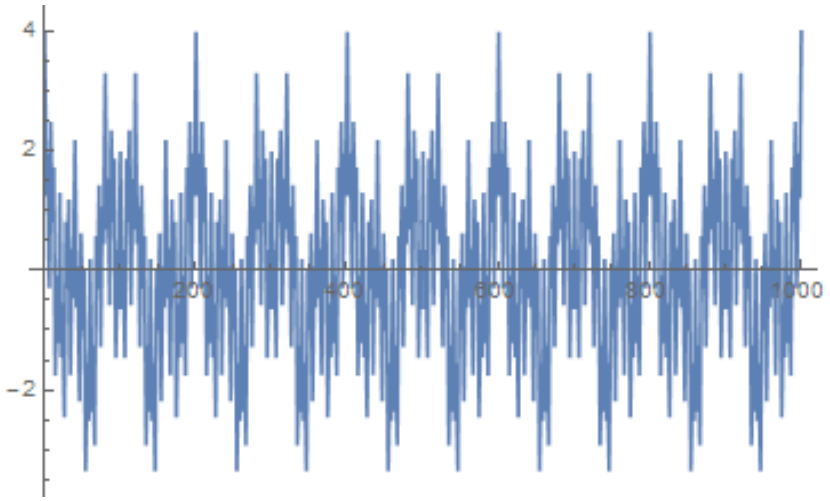
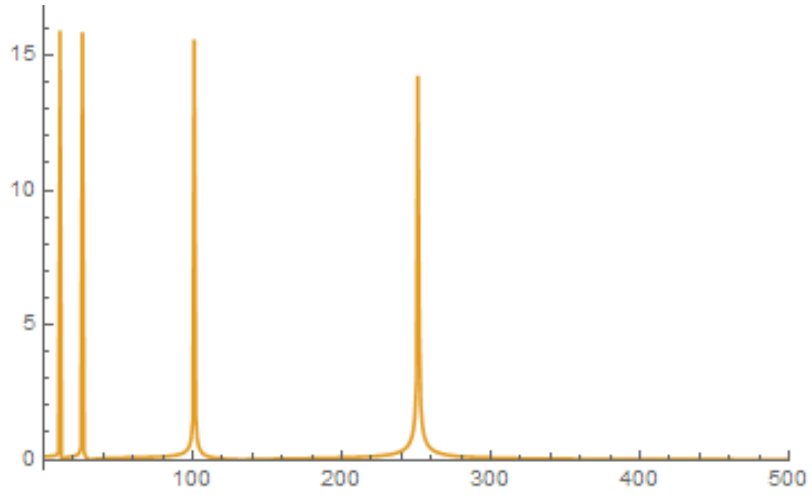
Fourier Transform (FT)

$$F(\omega) = \int_{-\infty}^{\infty} f(t)\, \mathrm{e}^{-\mathrm{i}\omega t}\, \mathrm{d}t$$

Time

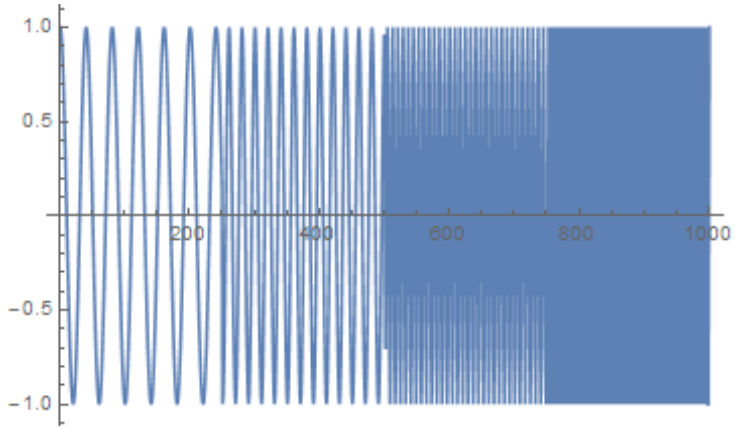


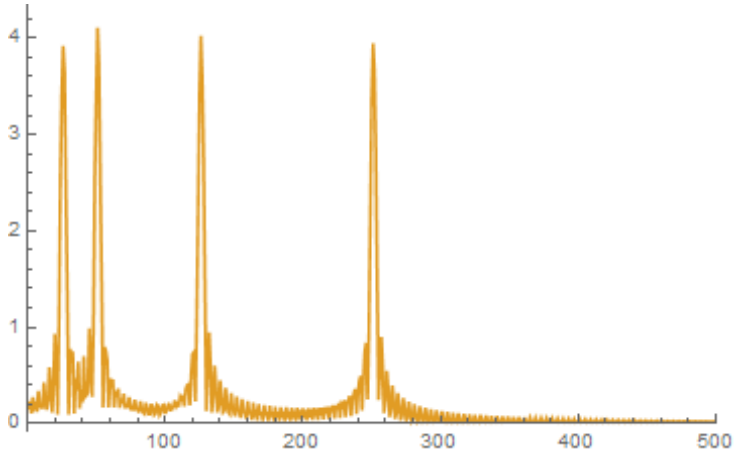
Frequency

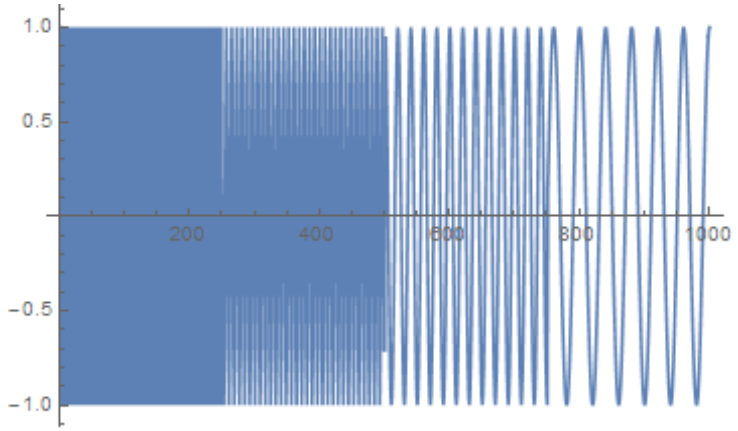


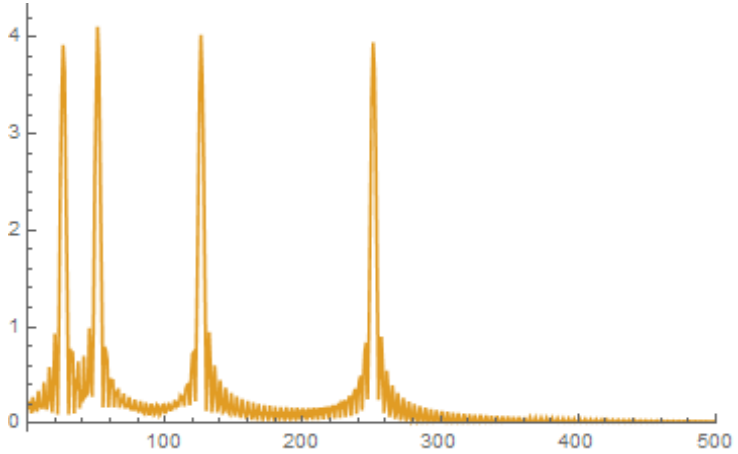
$f(t) = \cos(20\pi t) + \cos(50\pi t) + \cos(200\pi t) + \cos(500\pi t)$

Fourier Transform (FT)









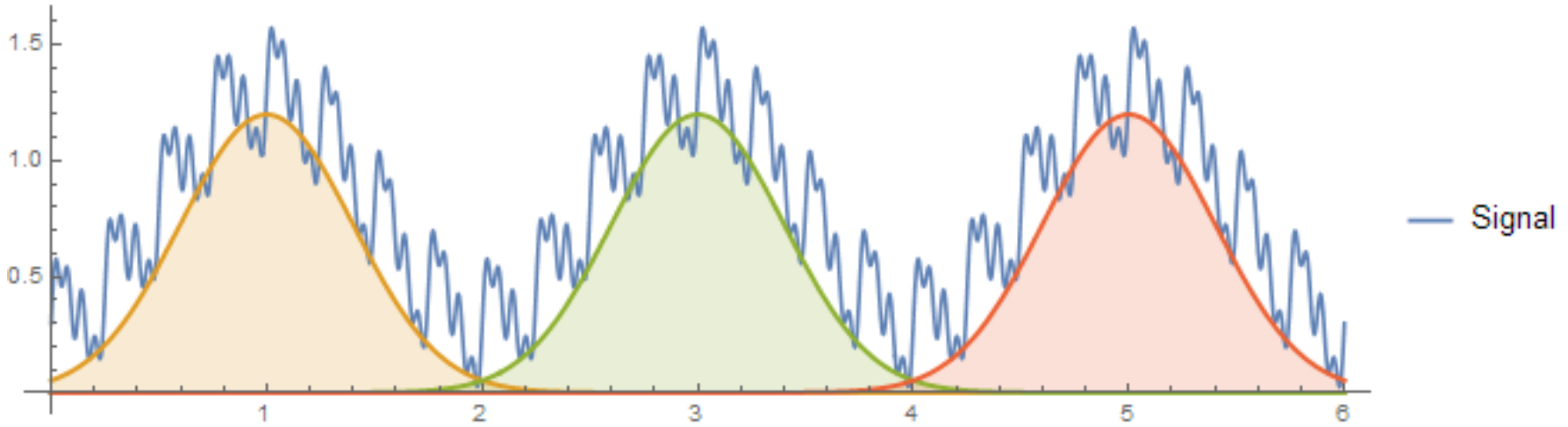
Only for
stationary signal

How to get information from ***both*** time and frequency?

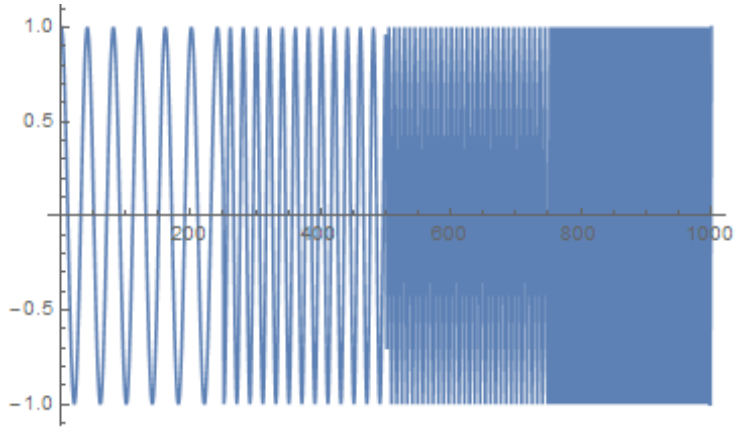
Short-Time Fourier Transform (STFT)

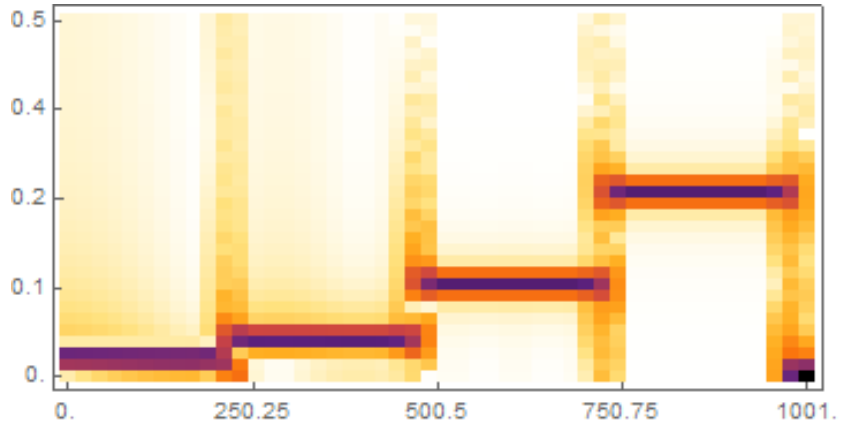
Window function

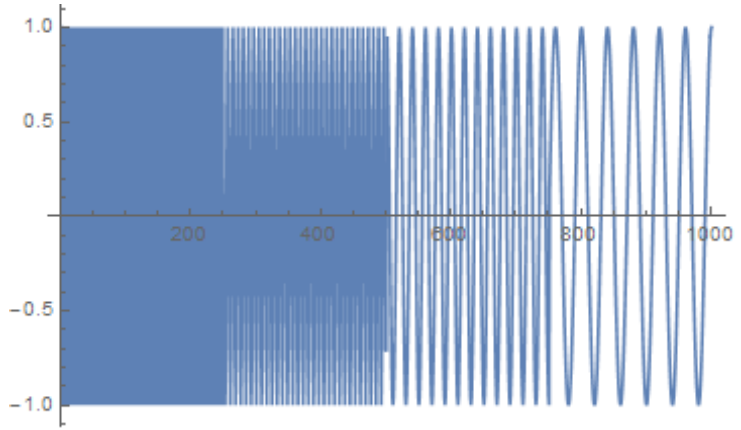
$$F(\omega) = \int_{-\infty}^{\infty} f(t)\, \mathrm{e}^{-\mathrm{i}\omega t}\, \mathrm{d}t \quad \Rightarrow \quad F(\tau, \omega) = \int_{-\infty}^{\infty} f(t)\, w(t-\tau)\, \mathrm{e}^{-\mathrm{i}\omega t}\, \mathrm{d}t$$

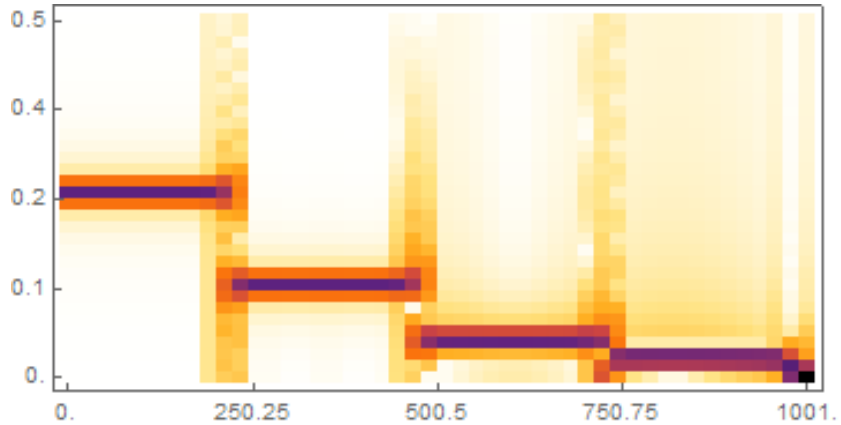


Short-Time Fourier Transform (STFT)









Everything seems OK, right?

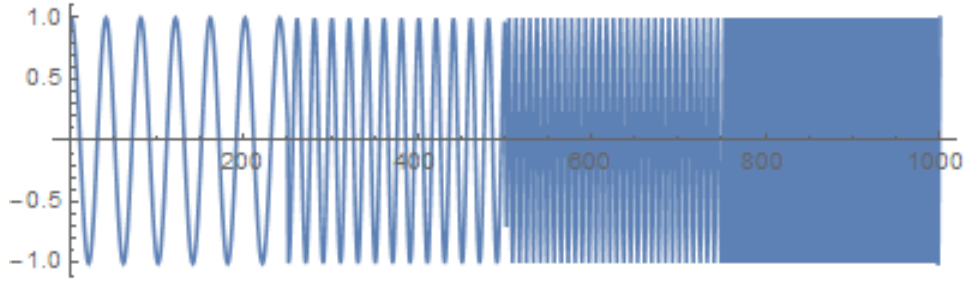
Uncertainty Principle

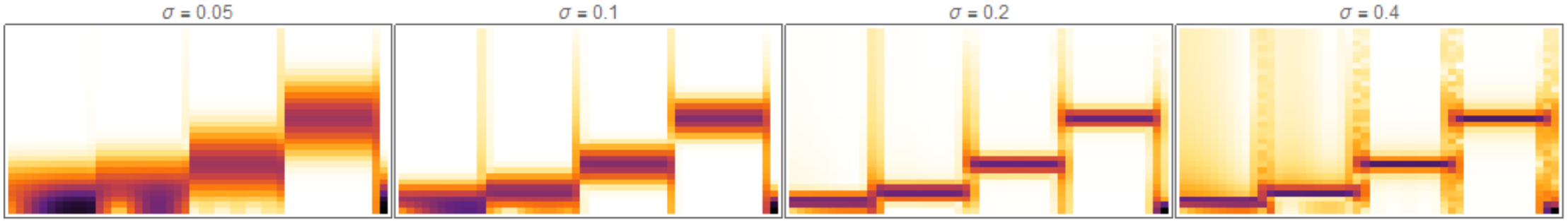
Werner Heisenberg

$$\Delta x \, \Delta p \geq \frac{\hbar}{2}$$

One **CANNOT** know the exact time-frequency representation of a signal!

Resolution Problem





Time resolution

Frequency resolution

Can we use the “*changeable*” window?

Continuous Wavelet Transform (CWT)

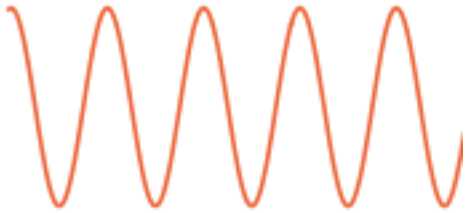
Add window function

$$F(\omega) = \int_{-\infty}^{\infty} f(t)\, \mathrm{e}^{-\mathrm{i}\omega t}\, \mathrm{d}t$$

$$F(\tau, \omega) = \int_{-\infty}^{\infty} f(t)\, w(t-\tau)\, \mathrm{e}^{-\mathrm{i}\omega t}\, \mathrm{d}t$$

Change basis

$$WT(\tau, s) = \frac{1}{\sqrt{s}} \int_{-\infty}^{\infty} f(t)\, \psi\left(\frac{t-\tau}{s}\right) \mathrm{d}t$$



Infinite trigonometric basis

Finite "**wavelet**" basis

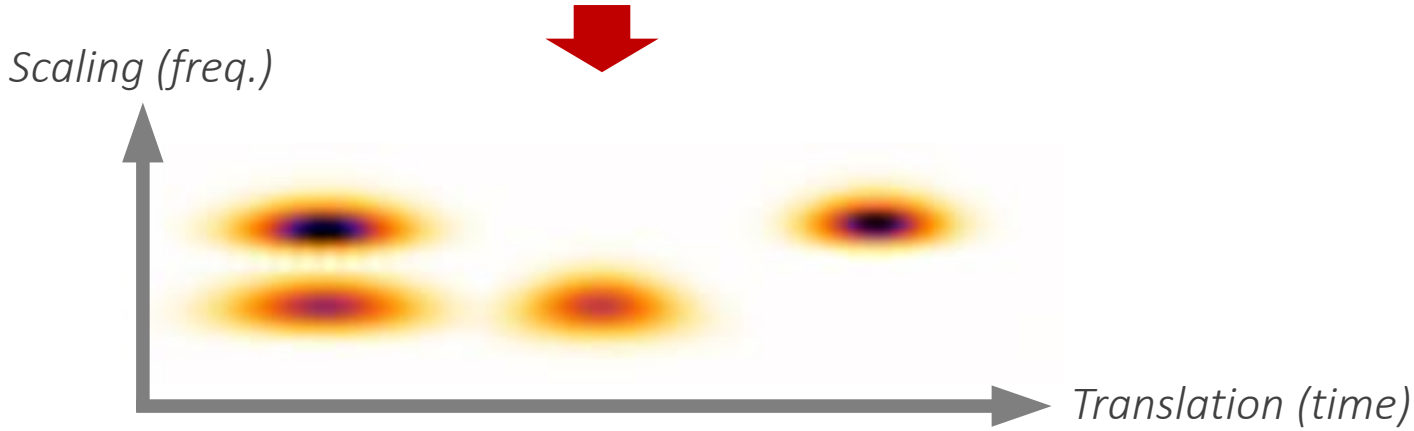
from French: ondelette (means "small wave")

Continuous Wavelet Transform (CWT)

Signal

×

Translation (τ) & scaling (s) of $\psi\left(\frac{t-\tau}{s}\right)$



Scaleogram

$$f(t) = [\sin(50\pi t) + 2\cos(140\pi t)]\mathrm{e}^{-50\pi(t-0.2)^2} + \sin(50\pi t)\,\mathrm{e}^{-100\pi(t-0.5)^2} + 2\sin(150\pi t)\,\mathrm{e}^{-80\pi(t-0.8)^2}$$

However, the horrible computation …

Discrete Wavelet Transform's Family

- Discrete Wavelet Transform (DWT)
- Stationary Wavelet Transform (SWT)
- Lifting Wavelet Transform (LWT)

- Discrete Wavelet Packet Transform (DWPT)
- Stationary Wavelet Packet Transform (SWPT)

- Fast Wavelet Transform (FWT)

References

[1] *Wavelet*. https://en.wikipedia.org/w/index.php?title=Wavelet&oldid=744313663

[2] Robi Polikar. *The Wavelet Tutorial*. http://users.rowan.edu/~polikar/WAVELETS/WTtutorial.html

[3] *Wavelet Analysis*. https://reference.wolfram.com/language/guide/Wavelets.html

[4] *Can you explain the relationship between wavelet analysis and Fourier analysis?* https://www.zhihu.com/question/22864189

Question Time