Holographic strange correlator

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May 13, 2021 @ Futian International Quantum Institute, Shenzhen

Basic ideas

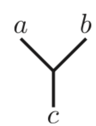
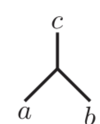
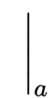
- String-net wavefunction + certain boundary conditions → partition function
- Application layer:
  - Coarse-graining procedure → holographic tensor network
  - Calculate the bulk-boundary propagator
- Theoretical layer:
  - MPO symmetries / pulling-through conditions → boundary conditions

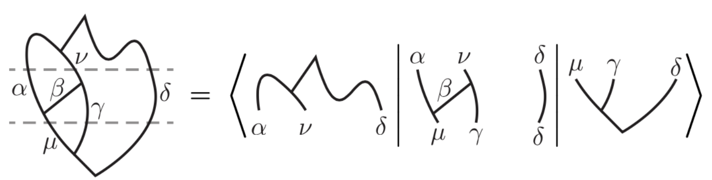
Backgrounds

Fusion categories (1)

- Simple objects and their fusion:
  - $a \otimes b = \bigoplus_c N_{ab}^c c$
  - Fusion coefficients N_{ab}^c: non-negative integers
  - Simple objects
    - Different species of anyon
    - Trivial object: $\mathbf{1} \otimes a = a$
    - Fusion: can't be distinguished at long distance
  - Examples
    - Decomposition of direct product of group representations
    - Operator product expansion (OPE) in CFT

Fusion categories (2)

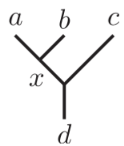
- Fusion diagrams
  - Notations:
    - Terminals: (simple) objects
    - Lines: identity operators
    - Vertices: fusion
  - Building blocks:  $\in V_c^{ab}$,  $\in V_{ab}^{c}$,  $\in V_a^a$.
  - Bra, ket and operator contract to a number:

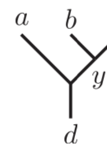


  - Isotopic moves can be performed arbitrarily

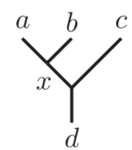
Fusion categories (3)

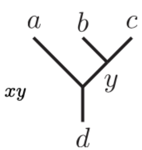
- F-symbols F_d^{abc}
  - Basis changes for the vector space:

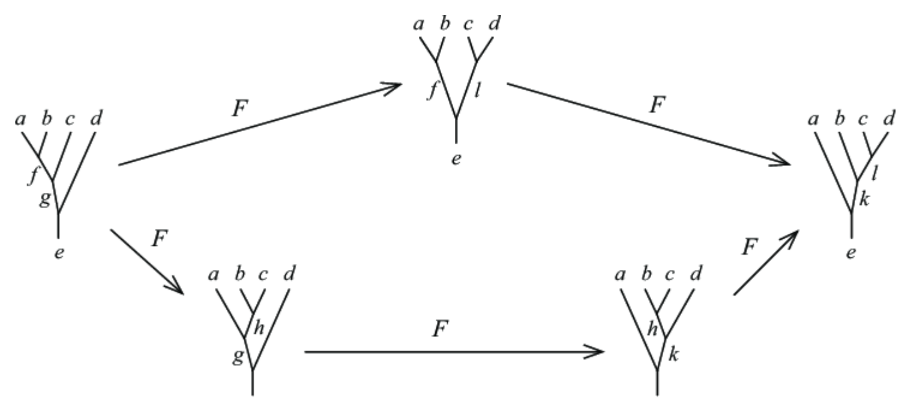
    $\in \bigoplus_x V_x^{ab} \otimes V_d^{xc} \cong V_d^{abc}$,
    

    $\in \bigoplus_y V_d^{ay} \otimes V_y^{bc} \cong V_d^{abc}$,

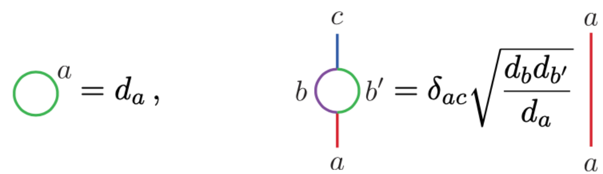
  - Transformation coefficients:
    

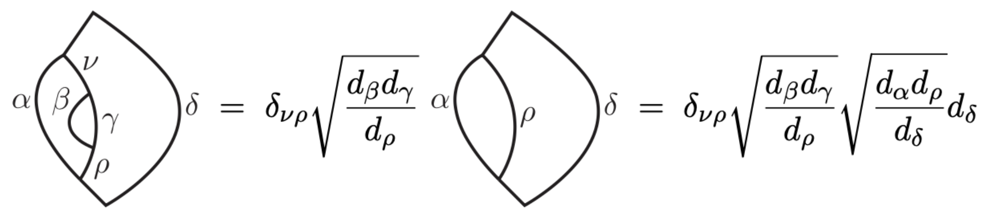
    $= \sum_y [F_d^{abc}]_{xy}$
    

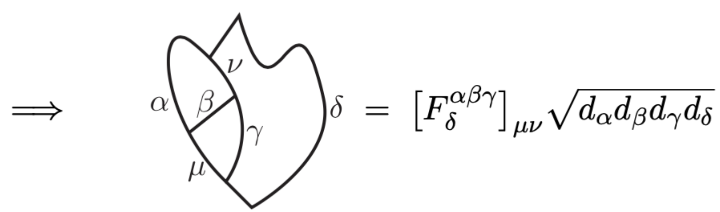
    .

  - Pentagon equations:
    

Fusion categories (4)

- Quantum dimensions d_a
  - Maximal eigenvalue of matrix $[N_a]$
  - Bubble removal: 

- With F-symbols and quantum dimensions we can evaluate an arbitrary diagram:
  - Example: 



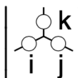
Fusion categories (5)

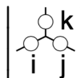
- Fibonacci:
  - Anyon types: $\mathbf{1}$, τ
  - Fusion rules: $\tau \otimes \tau = \mathbf{1} \oplus \tau$
  - Quantum dimensions: $d_{\mathbf{1}} = 1$, $d_\tau = \phi = \frac{1+\sqrt{5}}{2}$
  - F-symbols: $F^{\tau\tau\tau}_{\tau} = \begin{bmatrix} \phi^{-1} & \phi^{-1/2} \\ \phi^{-1/2} & -\phi^{-1} \end{bmatrix}$
- Ising:
  - Anyon types: $\mathbf{1}$, σ, ψ
  - Fusion rules: $\psi \otimes \psi = \mathbf{1}$, $\sigma \otimes \sigma = \mathbf{1} \oplus \psi$, $\psi \otimes \sigma = \sigma$
  - Quantum dimensions: $d_{\mathbf{1}} = d_\psi = 1$, $d_\sigma = \sqrt{2}$
  - F-symbols: $F^{\sigma\sigma\sigma}_{\sigma} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$, $F^{\psi\sigma\psi}_{\sigma} = F^{\sigma\psi\sigma}_{\psi} = -1$

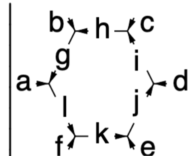
String-net models (1)

- Defined on a trivalent lattice (e.g. honeycomb)
- Edge labels: simple objects in a fusion category $\mathcal{C}$
- Vertex labels: morphism space $V_{ij}^k = \mathrm{Hom}_{\mathcal{C}}(i \otimes j, k)$
- Hilbert space: $\mathcal{H} = \bigotimes_v \mathcal{H}_v$ where $\mathcal{H}_v = \bigoplus_{i,j,k} V_{ij}^k$

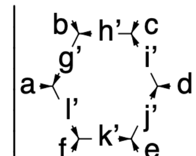
String-net models (2)

- Hamiltonian: $H = -\sum_{v \in \text{vertices}} A_v - \sum_{p \in \text{plaquattes}} B_p$
  - A_v (charge operators:) $Q_{\boldsymbol{I}} \Big|$ 

$\Big\rangle = \delta_{ijk} \Big|$ 

$\Big\rangle$
  - B_p (magnetic flux operators): $B_{\boldsymbol{p}}^{s} \Big|$ 

$\Big\rangle$

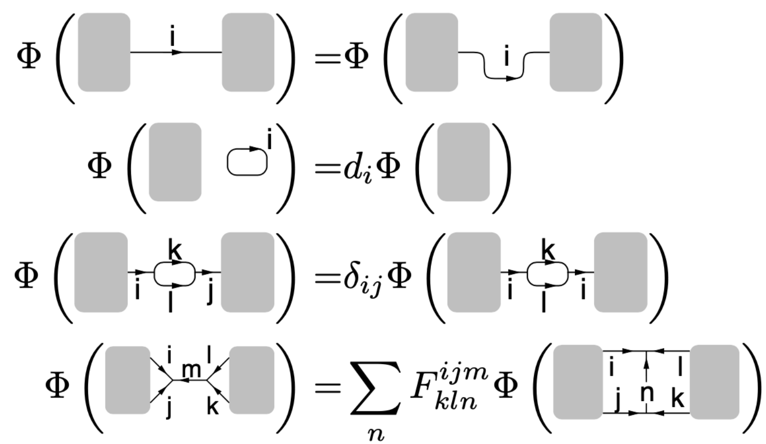
    $= \sum_{m,\ldots,r} B_{\boldsymbol{p},ghijkl}^{s,g'h'i'j'k'l'}(abcdef) \Big|$ 

$\Big\rangle$

    - $B_{\ldots}$ is a product of *F*-symbols

String-net models (3)

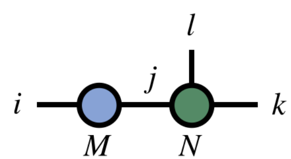
- Ground state:
  - Vertices: fusion rules
  - For a quantum state $|\Psi\rangle = \sum_X \Psi(X)|X\rangle$
    - $|X\rangle$: basis
    - Amplitude $\Psi(X)$:



Tensor network for string-net

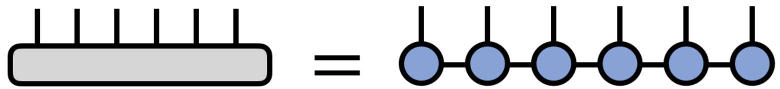
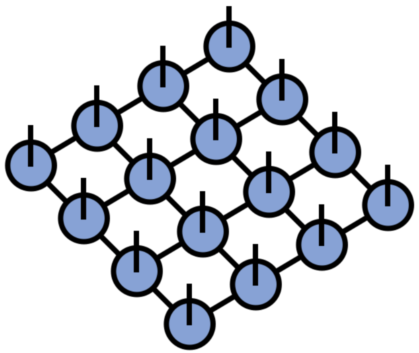
Ref: 1306.2164 (TN review), 0809.2393 (TN for string-net)

Tensor networks (1)

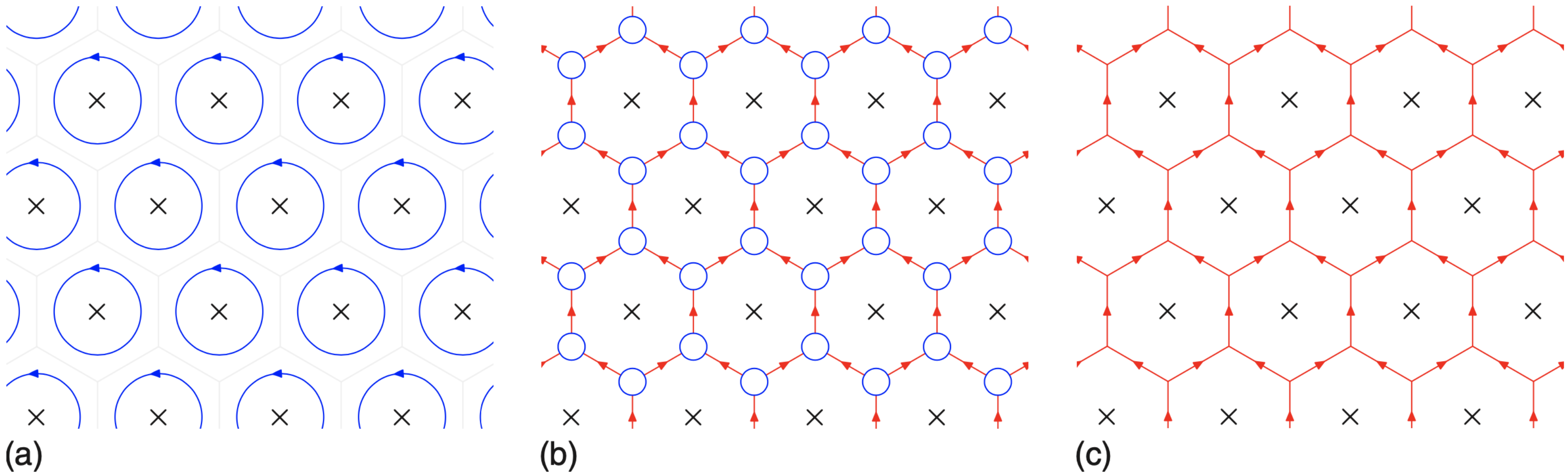
- A “network” constructed with tensors
- Notations:
  - Solid shapes: tensors
  - Bonds or “legs”: indices
  - Connected bonds: contraction
- Example: 

$= \sum_j M_{ij} N_{jkl}$
- Most of the data are irrelevant and can be truncated (most interactions are local)
  - Area-law: $S \sim \partial A$
  - Time/space complexity $\sim \exp L$ (i.e. Hilbert space is too large)

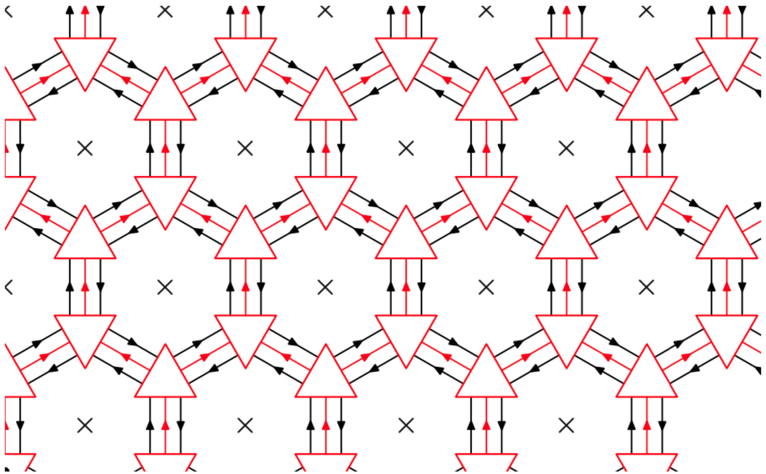
Tensor networks (2)

- Matrix product state (MPS): 
- Projected entangled pair states (PEPS): 
- MPO and PEPO: state → operator
- Terminologies:
  - Physical legs: the original indices of the state
  - Virtual legs: the indices between the tensor units

PEPS for string-net (1)

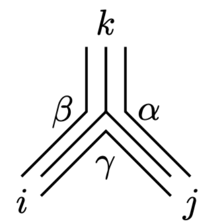


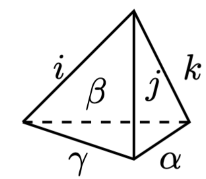
PEPS for string-net (2)

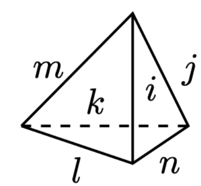


- **Virtual** bonds (black): summed over
- **Physical** bonds (red): left uncontracted, therefore build up a PEPS structure

PEPS for string-net (3)

- Notes on the conventions:
  - Triangle to tetrahedron: 

$= (d_i d_j d_k)^{-\frac{1}{4}} (d_\alpha d_\beta d_\gamma)^{-\frac{1}{3}}$ 

  - Tetrahedron to *F*-symbol: $[F_l^{ijk}]_{mn} = \frac{1}{\sqrt{d_i d_j d_k d_l}}$ 

$= [F_{jk}^{il}]_{mn}$

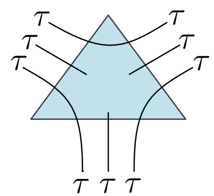
Strange correlators

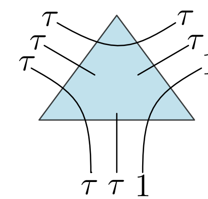
Ref: 1801.05959

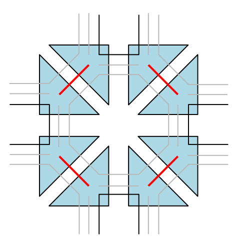
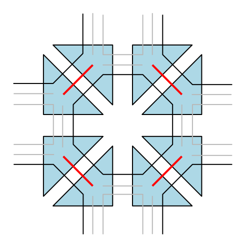
Strange correlators (1)

- Original definition: $C(r,r') = \langle \Omega | \phi(r)\phi(r') | \Psi \rangle / \langle \Omega | \Psi \rangle$
  - $|\Psi\rangle$: a non-trivial short-range entangled state
  - $|\Omega\rangle$: a direct product state
- In the string-net case:
  - $|\Psi_{\mathrm{SN}}\rangle$: PEPS wave function for string-net (the tensor network above)
  - $|\Omega\rangle$: some specific product state $|\omega\rangle^{\otimes N}$
  - Strange correlator: **inner product / overlap** between $|\Psi_{\mathrm{SN}}\rangle$ and $|\Omega\rangle$, or $\langle \Omega | \Psi_{\mathrm{SN}} \rangle$
  - Equivalent to fix all the physical legs to some certain values (labels)
  - After contraction the tensor network will have no free bonds, or simply become a number
  - The result gives the **partition function**

Strange correlators (2)

- Example: Fibonacci string-net
  - Strange correlator: project all physical degree of freedom to τ-label (i.e. $|\Omega\rangle = |\tau\rangle^{\otimes N}$)
  - Building blocks: 

$= -\phi^{-3/4}$ and 

$= \phi^{1/12}$
- Example: critical Ising
  - The product state is given by $|\Omega\rangle = \left[\sqrt{2}\left(\cosh\beta|\mathbf{1}\rangle + \sinh\beta|\psi\rangle\right) \otimes |\boldsymbol{\sigma}\rangle\right]^{\otimes N}$
  - The same procedure leads to the classical 2D Ising tensor $\mathrm{e}^{\beta(\sigma_i\sigma_j+\sigma_j\sigma_k+\sigma_k\sigma_l+\sigma_l\sigma_i)}$
  - Building blocks:  $\oplus$ 
    - Black: $\mathbf{1}$ or ψ, gray: σ, red: $\cosh\beta|\mathbf{1}\rangle + \sinh\beta|\psi\rangle$

Tensor networks & CFT data

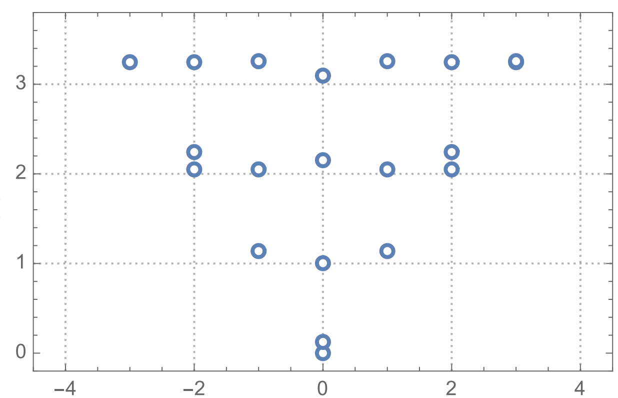
Ref: 0711.3960 (iTEBD), cond-mat/0611687 (TRG), 1412.0732 (TNR), 1512.03846 (spectra/defect)

Verify SC gives the correct CFT partition function

- Transfer matrix → CFT spectra
- Partition function → entanglement entropy, correlation length, central charge, etc

CFT spectra

- CFT partition function on torus: $Z = \sum_\alpha \exp\left[2\pi \frac{m}{n}\left(\frac{c}{12} - \Delta_\alpha\right) + mnf + \cdots\right]$
- Eigenvalues of transfer matrix M ($Z = \mathrm{Tr}\, M^{m/l}$): $\lambda_\alpha \approx \exp\left[2\pi \frac{l}{n}\left(\frac{c}{12} - \Delta_\alpha\right)\right]$
- Add translation operator $T = \exp\left(\frac{2\pi \mathrm{i}}{n} P\right)$ where $P = L_0 - \bar{L}_0$
- Eigenvalues of $\tilde{M} = T \cdot M$: $\tilde{\lambda}_\alpha = \exp\left[2\pi \frac{l}{n}\left(\frac{c}{12} - \Delta_\alpha\right) + \frac{2\pi \mathrm{i}}{n} s_\alpha\right]$
    - Real part: scaling dimension Δ_α
    - Imaginary part: conformal spin s_α
- Example (Ising):



Data from the partition function

- Algorithms:
  - iTEBD: row-by-row
  - TRG/TNR: coarse-graining
- Central charge:
  - Entanglement entropy vs correlation length: $S_A = \frac{c}{6} \log \xi$
  - From fitting with different bond dimensions
- Two-point functions:
  - Inserting operators in the tensor network
  - e.g. $\mathrm{e}^{\beta(\sigma_i \sigma_j + \cdots)} \to \sigma_i \mathrm{e}^{\beta(\sigma_i \sigma_j + \cdots)}$

Strange correlator TRG

Coarse-graining procedure

- Strange correlator: $\langle \Omega | \Psi_{\mathrm{SN}} \rangle$ → partition function
- Reinterpret the TRG/TNR (for partition functions) at the level of quantum states
    - String-net is at RG fixed point
- General picture:
    - Keep the value of $\langle \Omega | \Psi_{\mathrm{SN}} \rangle$ unchanged
    - Apply a PEPO operator U on $| \Psi_{\mathrm{SN}} \rangle$, then put the entanglement into $|\Omega\rangle$ part
    - $\langle \Omega^{(i)} | \Psi_{\mathrm{SN}}^{(i)} \rangle = \langle \Omega^{(i)} | U^\dagger U | \Psi_{\mathrm{SN}}^{(i)} \rangle = \langle \Omega^{(i)} | U^\dagger | \Psi_{\mathrm{SN}}^{(i+1)} \rangle \approx \langle \Omega^{(i+1)} | \Psi_{\mathrm{SN}}^{(i+1)} \rangle$

Bulk-boundary propagator

- To check that the coarse-graining procedure builds up a holographic network
- Bulk field: $\phi(x,z) = \int \mathrm{d}^d y\, K(x,z|y) O(y)$
  - $O(y)$ Is the boundary operator: $\lim_{z\to 0} \phi(x,z) = z^{-\Delta} O(x)$
  - Δ: conformal dimension
- Bulk-boundary propagator: $K(x,z|y) = \left[\frac{z}{z^2-(x-y)^2}\right]^{d-\Delta} \Theta\left(z^2-(x-y)^2\right)$

Open problems

- How to choose the boundary conditions?
  - MPO symmetries and pulling-through conditions
  - From integrable models
- What's the correct bulk operator?
  - Use some state to "represent" the operator