Holographic strange correlators

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Outline

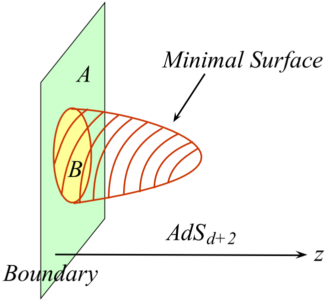
- Motivation & background
  - AdS/CFT correspondence
  - Topological orders
- Ongoing works
  - Build up 2D tensor network from strange correlators
  - Use tensor network to calculate CFT data (e.g. central charge)
- Further research
  - Extract more data from the tensor network
  - Coarse graining and holographic tensor network
  - Understand strange correlators via integrable system

Motivation: AdS/CFT

- Duality between a gravity theory in AdS_{d+1} spacetime (bulk) and a CFT_d (boundary)
- Ryu–Takayanagi formula:

$$S_A = \frac{\mathrm{Area}(\gamma_A)}{4G^{(d+1)}}$$

- MERA tensor network
  - Coarse-graining via isometries & disentanglers
  - Discretized version of AdS geometry & RT formula



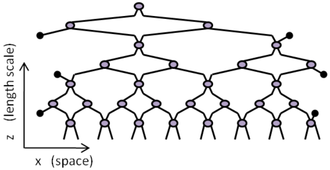
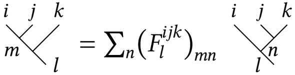


Image credit: arXiv 0905.0932, 1106.1082

Background: topological orders

- Phase transition beyond Landau's theory
  - Ground state degeneracy: topological protected
  - Long-range entanglement: can't be built with direct product state + local unitary transformation
- String-net models
  - Oriented edges on hexagonal lattice
  - Hamiltonian: $H = -\sum_v A_v - \sum_p B_p$ (v: vertices, p: plaquettes)
- Mathematical framework: modular tensor category
  - Fusion rules: $a \otimes b = \sum_c N_{ab}^c c$
  - F-moves: $(a \otimes b) \otimes c = a \otimes (b \otimes c) \implies$ 

  - Coherent condition: pentagon equations

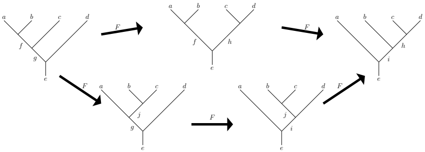
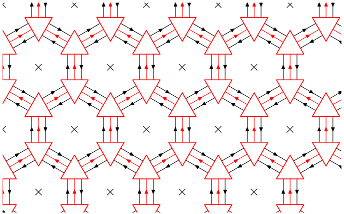
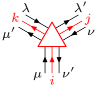


Image credit: arXiv 2007.10562

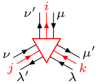
Build up tensor network from strange correlators

- Ground state of string-net can be described by a tensor network
    - Vertices: F-symbols
    - Virtual bonds: summed over (black)
    - Physical bonds: left uncontracted (red)
- Strange correlators
    - Inner product between the string-net ground state and direct product state
    - In tensor network: fix all physical bonds to certain labels
    - Result in a 2D network, whose trace (or contraction) gives the partition function





$$= F^{\mu^* i \nu}_{j^* \lambda k}\,\delta_{\mu\mu'}\delta_{\nu\nu'}\delta_{\lambda\lambda'}$$

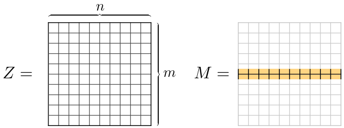


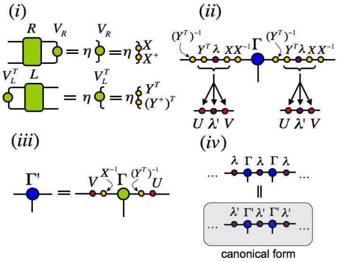
$$= \sqrt{d_j}\, F^{\nu^* j \lambda}_{k \mu i}\,\delta_{\mu\mu'}\delta_{\nu\nu'}\delta_{\lambda\lambda'}$$

Image credit: arXiv 0809.2393

Extact CFT data: iTEBD algorithm (1)

- Motivation: simulate the (imaginary-) time evolution of $|\psi\rangle$ by applying $U = \mathrm{e}^{-\mathrm{i}tH}$ or $U = \mathrm{e}^{-\tau H}$
- The contraction of 2D tensor network can be understood as repeatedly applying MPO on an MPS
- Steps:
  - Written the translationally symmetric 1D system as infinite MPS
  - Canonicalization: maintain the iMPS in its canonical form to reduce truncation error
  - SVD truncation: keep virtual dimension unchanged & discard insignificant data





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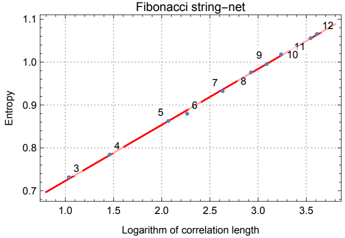
Extact CFT data: iTEBD algorithm (2)

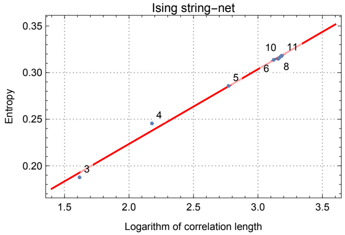
- Build up square lattice from honeycomb lattice by joining triangle tensors
- Von Neumann entropy vs correlation length:

$$S_A \sim \frac{c}{6} \log \xi$$

where $S_A = \sum_i \lambda_i^2 \log \lambda_i^2$, $\xi = -1/\log|\lambda_2/\lambda_1|$

- Results:
  - Fibonacci: $c = 0.783 \pm 0.009$ (exact: 4/5)
  - Ising: $c = 0.481 \pm 0.013$ (exact: 1/2)
- Technical details:
  - Written with a Mathematica TN library
  - Eigensystems solved by Arnoldi method





Code: github.com/stone-zeng/research

Extact CFT data: exact diagonalization

- Transfer matrix: one layer in the network
- Eigenvalues of the transfer matrix:

$$\tilde{\lambda}_\alpha \sim \exp\left[\frac{2\pi}{n}\left(\frac{c}{12} - \Delta_\alpha\right) + \frac{2\pi \mathrm{i}}{n} s_\alpha\right]$$

- Conformal spectrum: scaling dimensions Δ_α vs conformal spins s_α
- Tube algebra basis elements ($\mathcal{T}^c_{ab}$) are used to separate different topological sectors in the spectrum

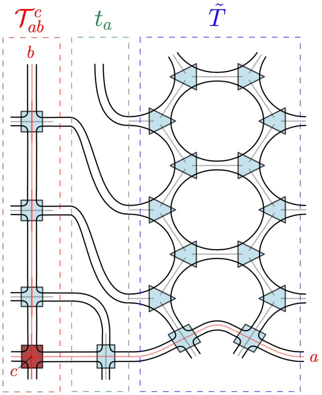


Image credit: Phys. Rev. Lett. **121**, 177203 (supplementary material)

Coarse-graining

- CFT viewpoint: allow us to diagonalize much larger lattice, so we can obtain more accurate data for infinite descendant fields
- Holographic viewpoint: the coarse-graining procedure builds up a holographic network
  - Bulk-boundary propagator:

$$G_{\Delta}(z, \boldsymbol{x}_1, \boldsymbol{x}_2) = \frac{z^{\Delta}}{\left[(\boldsymbol{x}_1 - \boldsymbol{x}_2)^2 + z^2\right]^{\Delta}}$$

  - Strange correlators give certain boundary conditions in p-adic AdS/CFT

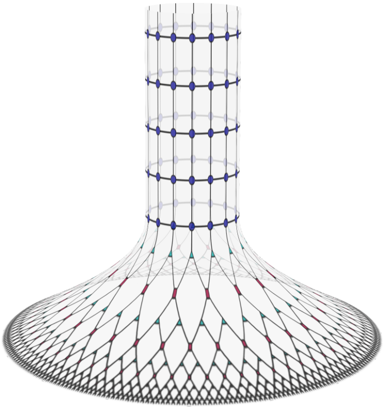
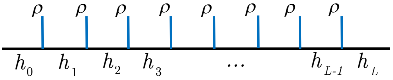
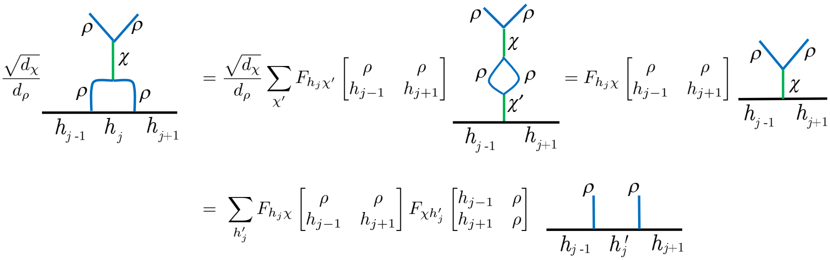


Image credit: glenevenbly.com

Why strange correlators?

- In the height models, one need to assign an energy gain for some specific fusion channels to make it integrable, such that the Boltzmann weights satisfy the Yang–Baxter equation
- This is the same as what we do with a strange correlator





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Summary

- Strange correlators can give the partition function for string-net models in the form of a 2D tensor network
- Numerical methods, such as iTEBD and exact diagonalization can be used to extract CFT data from the above tensor network
- Coarse-graining procedure can be viewed as a holographic tensor network, where we can explicitly calculate the bulk-boundary propagator
- The reason why strange correlators are successful is closely related to the integrable systems

Thank you!