Tensor network representation for topological phases

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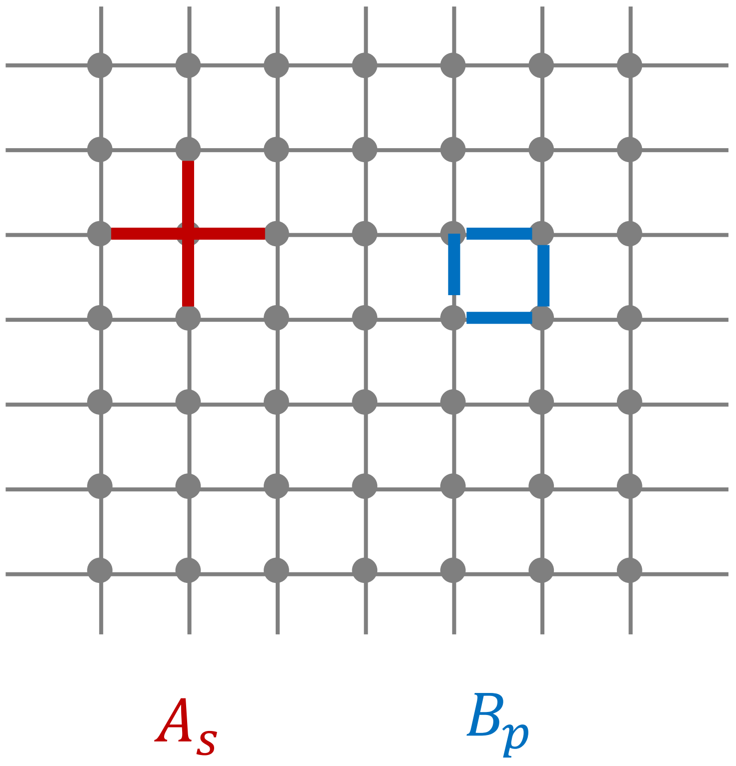
Outline

- Kitaev's toric code model
- Levin–Wen model (string-nets)
- Basic ideas of tensor network
- Tensor network for the above models

Toric code model

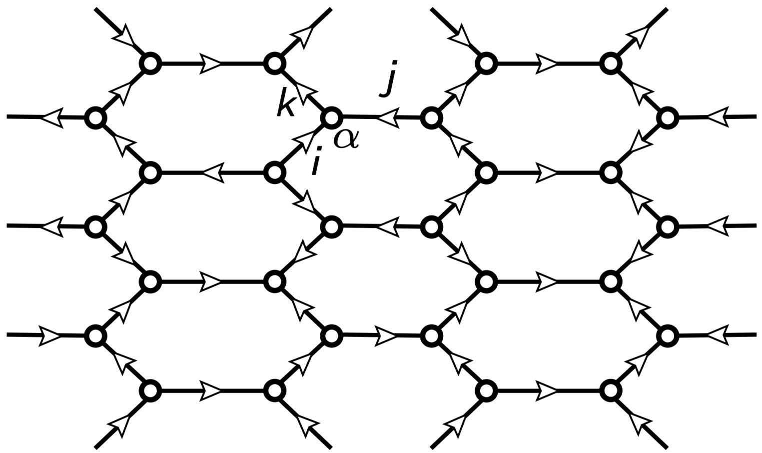
- Hamiltonian: $H = -\sum_s A_s - \sum_p B_p$
- $A_s = \prod_j \sigma_j^x, B_p = \prod_j \sigma_j^z, [A_s, B_p] = 0$
- Ground state: $A_s|0\rangle = B_p|0\rangle = |0\rangle$
- String operator can generate e and m
- 4 anyons: $\mathbf{1}$, e, m and ϵ



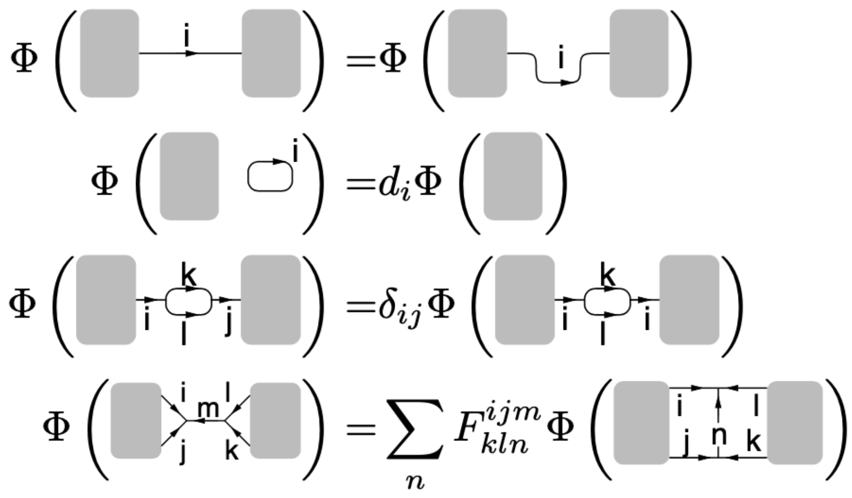
Kitaev, A. Y. (2003). *Annals of Physics*, *303*(1), 2-30.

String-net models

- Oriented edges @ hexagonal lattice
- Labels + fusion rules → fusion category
- Hamiltonian: $H = -\sum_s A_s - \sum_p B_p$



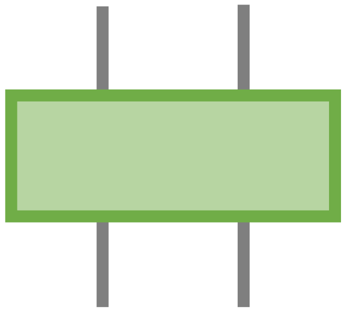
- Fixed-point wave function can be speicified uniquely by ☞



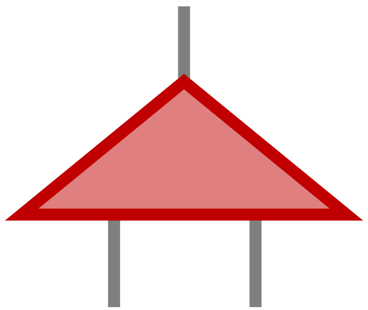
Levin, M. A., & Wen, X. G. (2005). *Physical Review B, 71*(4), 045110.

Tensor network

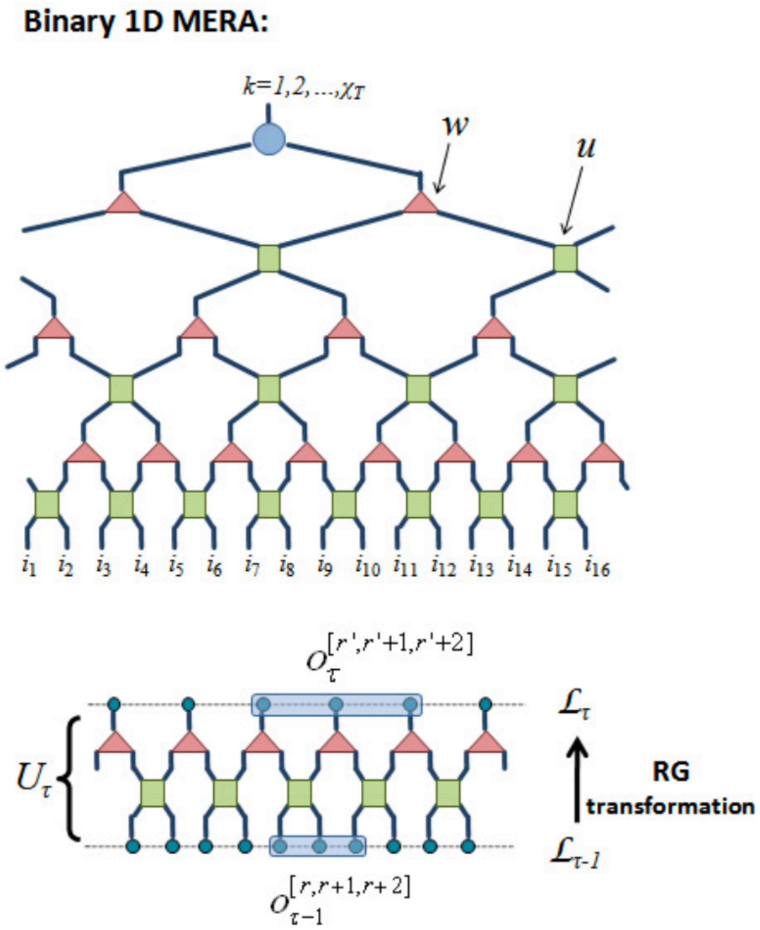
- Efficient representation of wave function
- Can be interpreted as quantum circuits
- Some important “bricks”:



Disentangler u:
Remove short-range correlations

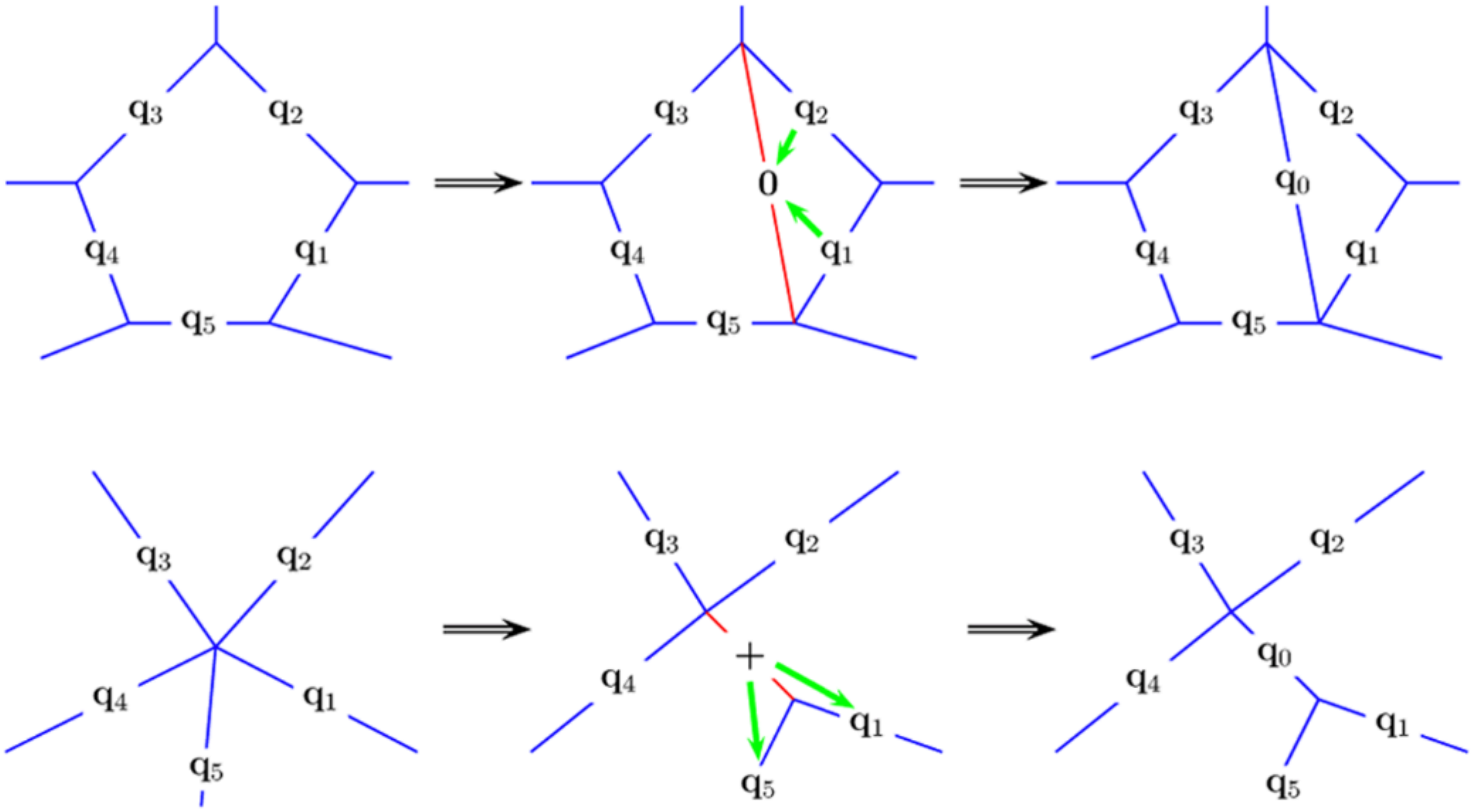


Isometry w:
Coarse-grain, introduce a truncation error



Evenbly, G., & Vidal, G. (2009). *Physical Review B*, 79(14), 144108.

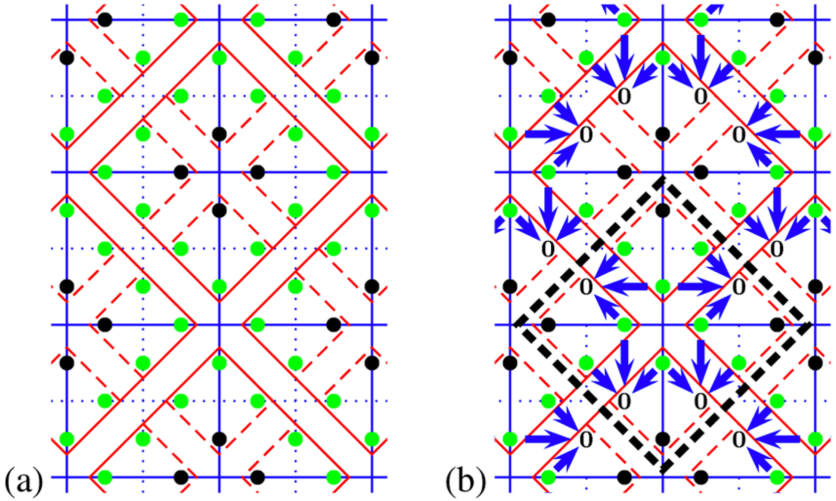
Tensor network for toric code (1)



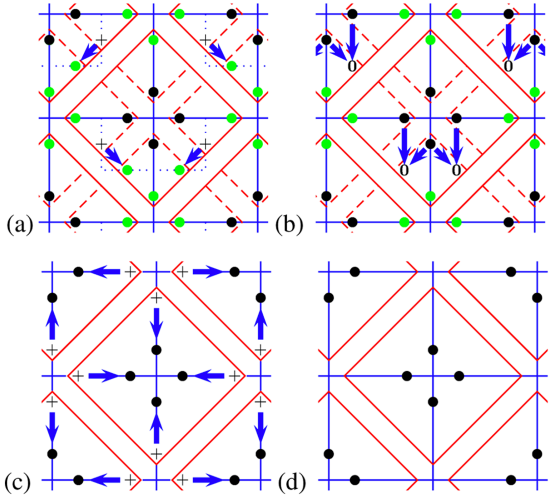
Elementary moves can add plaquette and vertices

Tensor network for toric code (2)

- Use 2 elementary moves to form RG transformation
- Disentanglers & isometries compose MERA



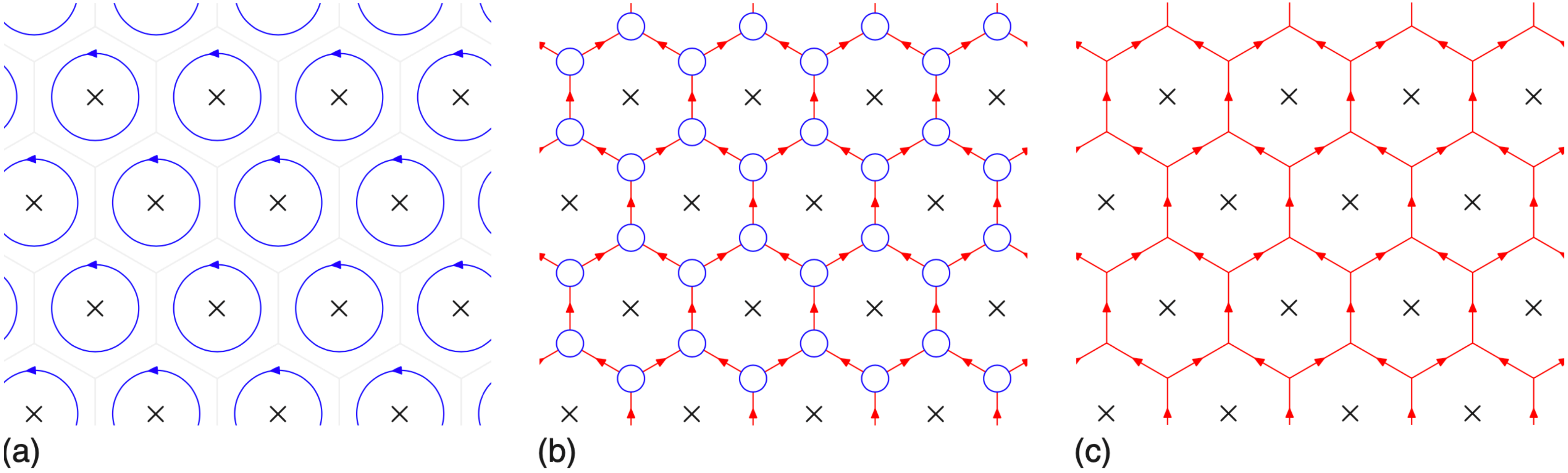
Step 1: disentanglers



Step 2: isometries

Aguado, M., & Vidal, G. (2008). *Physical review letters*, 100(7), 070404.

Tensor network for string-nets (1)

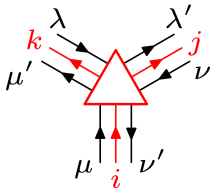


$$|\Psi_0\rangle = \sum_{\{\alpha_p\}} \prod_p d_{\alpha_p} \left|\{\alpha_p\}\right\rangle$$

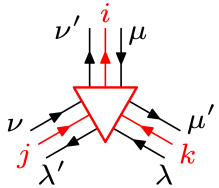
$$= \sum_{\{\alpha_p\}} \prod_p d_{\alpha_p} \left|\{\alpha_p\}\right\rangle \cdot \sum_{\{i_p, j_p, k_p\}} \prod_{p,q \in E_1, E_2, E_3} F^{\cdots}_{\cdots} \cdot \left|\{\alpha_p, i_p, j_p, k_p\}\right\rangle$$

$$= \sum_{\{i_p, j_p, k_p\}} \prod_p \underbrace{\sqrt{d_{j_p}} \cdot \sum_{\{\alpha_p\}} \prod_{v \in \Lambda_1} f(v) \prod_{w \in \Lambda_2} g(w)}_{\lambda_{\{i_p, j_p, k_p\}}} \cdot \left|\{i_p, j_p, k_p\}\right\rangle$$

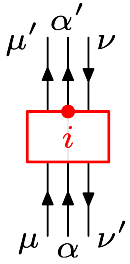
Tensor network for string-nets (2)

- 

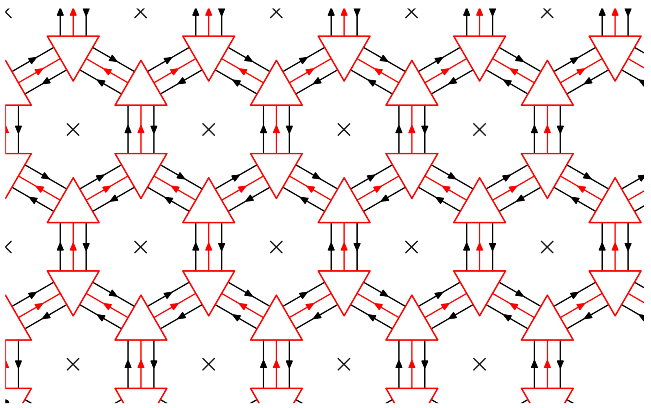
$= T^{[ijk]}_{\mu\mu'\nu\nu'\lambda\lambda'} = \sqrt{d_j} F^{\cdots}_{\cdots} \delta_{..}\delta_{..}\delta_{..}$

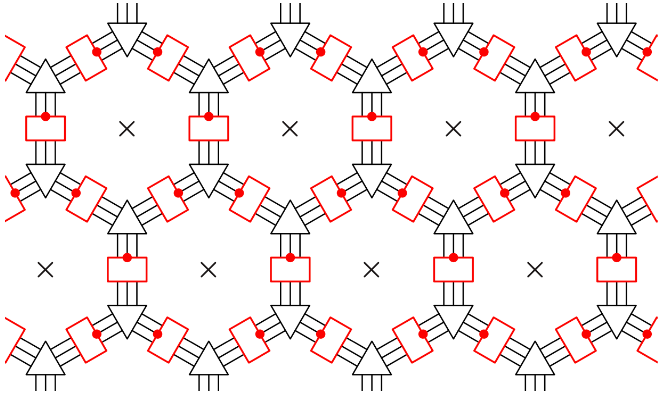
- 

$= \tilde{T}^{[ijk]}_{\mu\mu'\nu\nu'\lambda\lambda'} = F^{\cdots}_{\cdots} \delta_{..}\delta_{..}\delta_{..}$

- 

$= A^{[i]}_{\alpha\alpha'\mu\mu'\nu\nu'} = \delta_{..}\delta_{..}\delta_{..}\delta_{..}$





Buerschaper, O., Aguado, M., & Vidal, G. (2009). *Physical Review B*, 79(19), 085119.

Summary & outlook

- Topological phases appear in toric code & string-net models
- Tensor network is an efficient representation
- Provide methods for calculating entanglement entropy etc.

Thank you for listening!