

非线性物理——混沌

Non-linear Systems & Chaotic Circuit

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图 1 织锦芋螺 (*Conus textile*)

来源: https://en.wikipedia.org/wiki/File:Textile_cone.JPG

实验原理



非线性物理简介

- ▶ 混沌
 - ▶ 对初始条件极其敏感
 - ▶ 行为具有不可预测性
- ▶ 分形
 - ▶ 标度不变性
 - ▶ 奇异吸引子
- ▶ 模式形成
- ▶ 孤立子
- ▶ 元胞自动机
 - ▶ *A New Kind of Science*
- ▶ 复杂系统

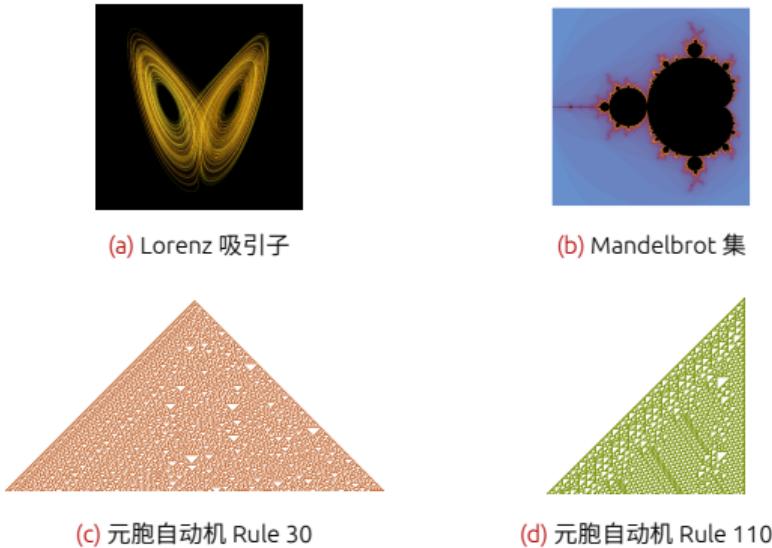


图 2 一些非线性系统

图 (a) 来源: https://en.wikipedia.org/wiki/File:Lorenz_attractor_yb.svg



Chua 电路 (I)



图 3 Prof. Leon O. Chua

来源: <https://people.eecs.berkeley.edu/~chua/>

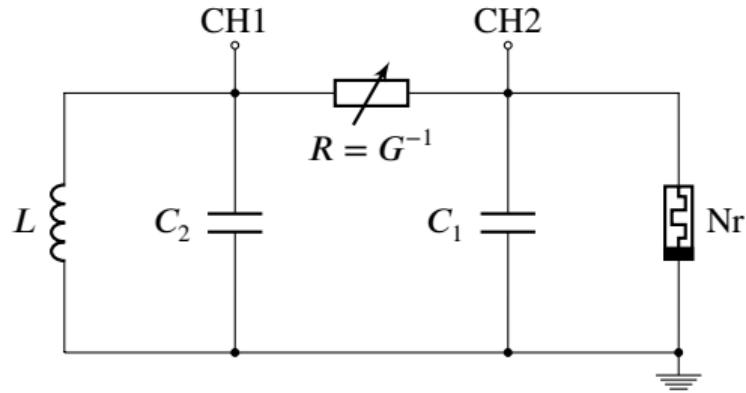


图 4 Chua 电路示意图

非线性负阻 $\times 1$, 电阻 $\times 1$, 储能元件 $\times 3$



Chua 电路 (II)

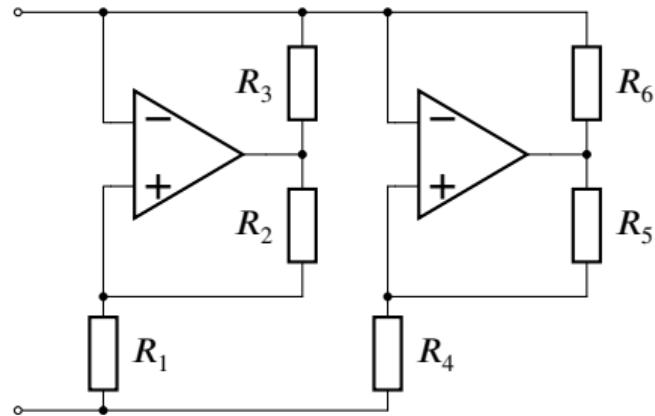


图 5 非线性负阻的内部结构

- ▶ 双运算放大器集成电路 FL353N(P)
- ▶ 定值电阻：
 - ▶ $R_1 = 3.3 \text{ k}\Omega$
 - ▶ $R_2 = R_3 = 22 \text{ k}\Omega$
 - ▶ $R_4 = 2.2 \text{ k}\Omega$
 - ▶ $R_5 = R_6 = 220 \Omega$
- ▶ 输入电压： $\pm 15 \text{ V}$



Chua 电路 (III)

状态方程：

$$\begin{cases} C_1 \frac{dU_1}{dt} = G \cdot (U_2 - U_1) - g(U_1) \\ C_2 \frac{dU_2}{dt} = G \cdot (U_1 - U_2) + I_L \\ L \frac{dI_L}{dt} = -U_2 \end{cases}$$

非线性负阻 $I-U$ 特性：

$$g(U) = G_b \cdot U + \frac{G_b - G_a}{2} \cdot (|U - E| + |U + E|)$$

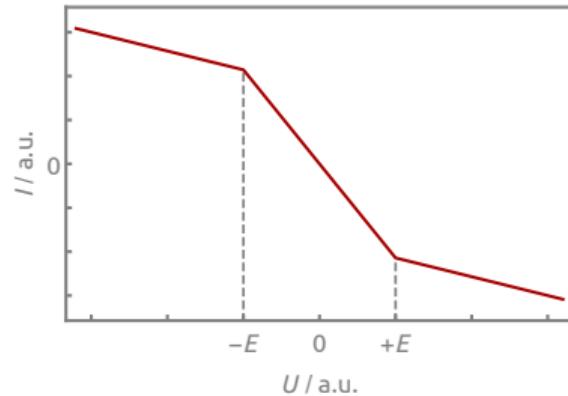


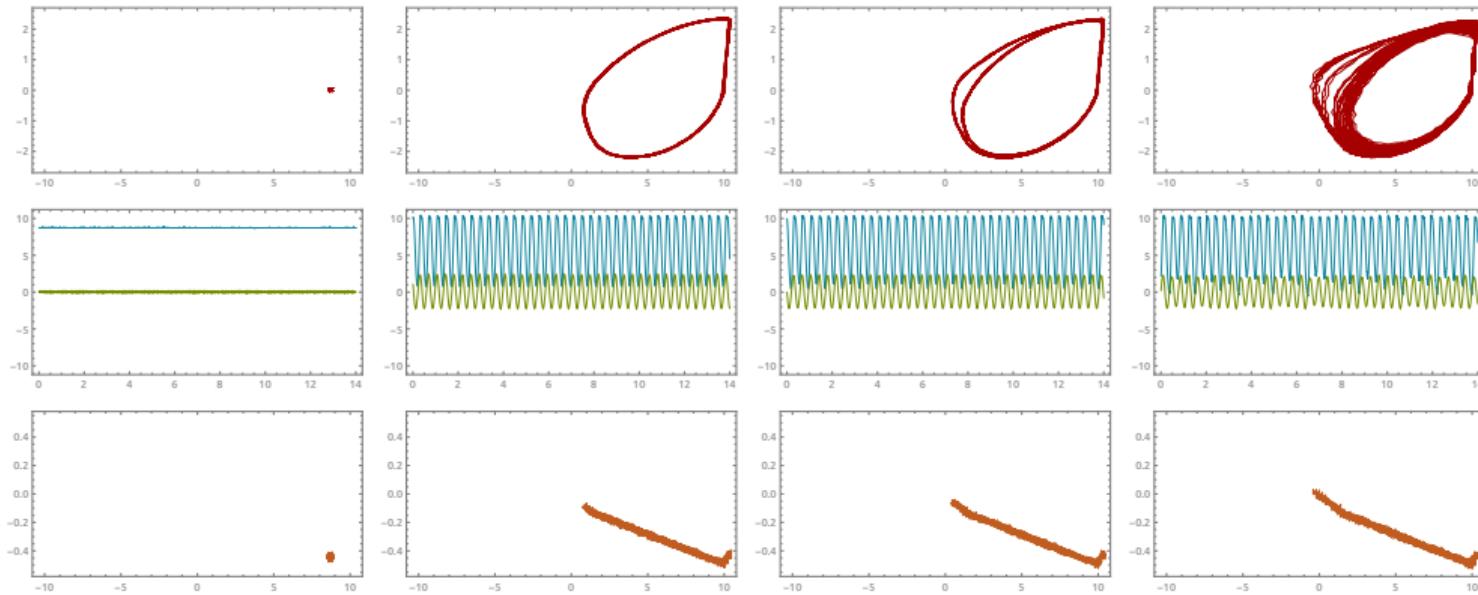
图 6 非线性负阻 $I-U$ 特性曲线



混沌现象



混沌现象 & I-U 特性 (I)



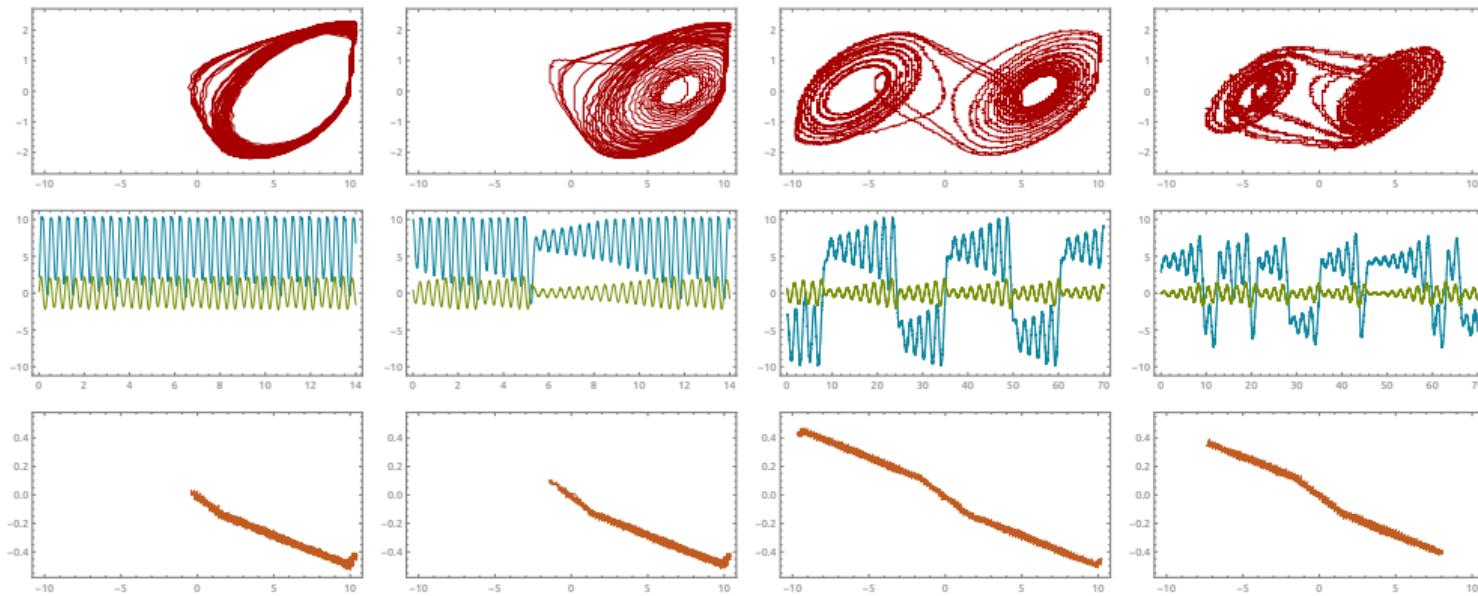
(a) 不动点

(b) 单周期

(c) 双周期

(d) 阵发混沌

混沌现象 & I-U 特性 (II)



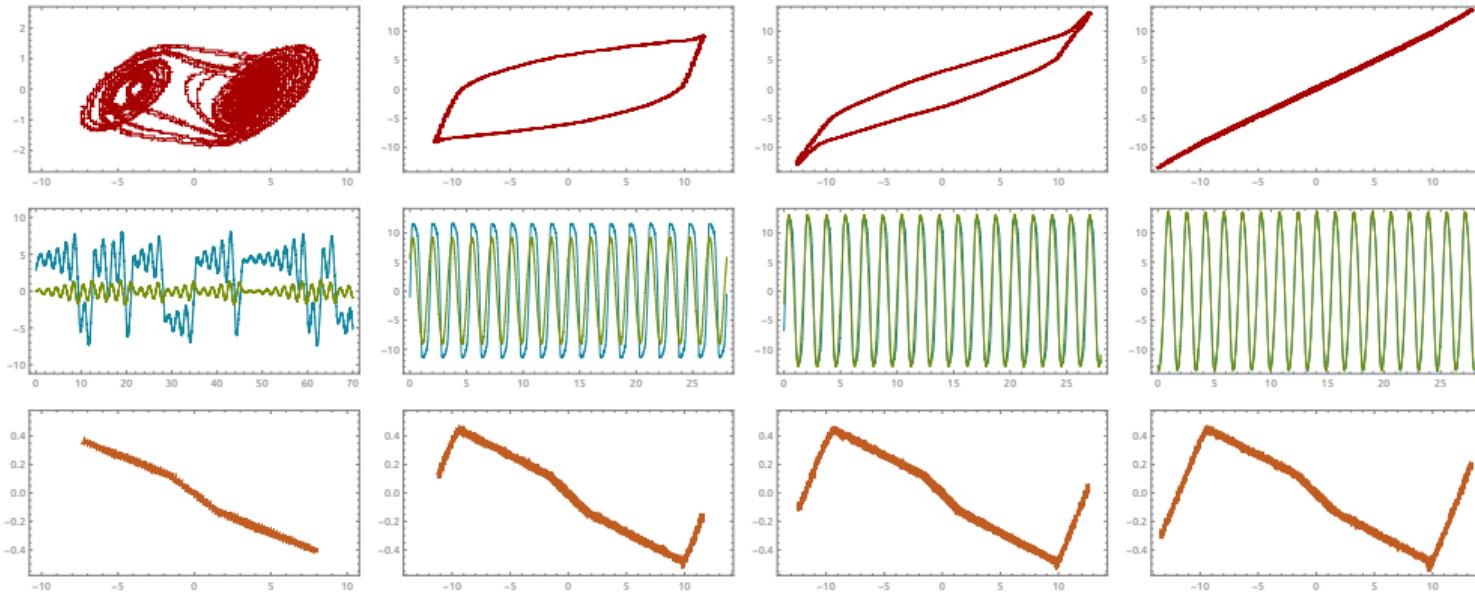
(d) 阵发混沌

(e) 单吸引子

(f) 双吸引子 I

(g) 双吸引子 II

混沌现象 & I-U 特性 (III)



(g) 双吸引子 II

(h) 初始状态 I

(i) 初始状态 II

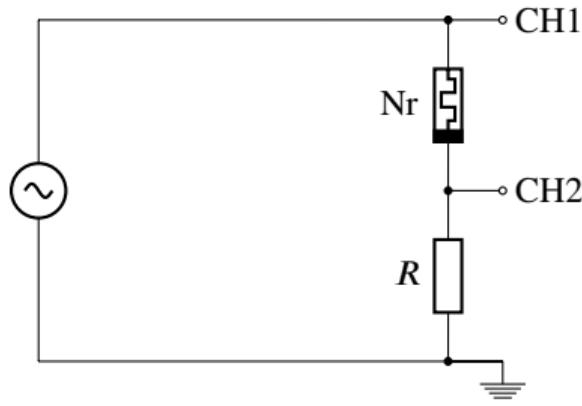
(j) 初始状态 III



非线性负阻 $I-U$ 特性

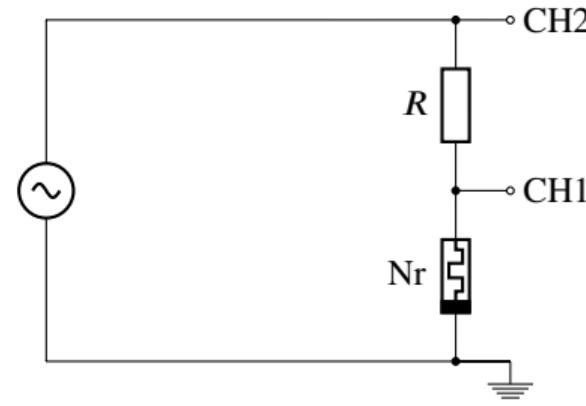


外部信号扫描法 (I)



$$(a) U_R \lesssim U_{Nr}$$

$$U = U_1 - U_2, \quad I = U_2 / R$$

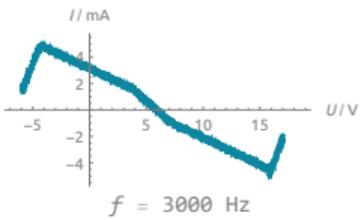
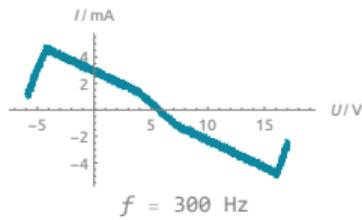
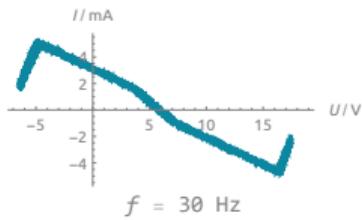
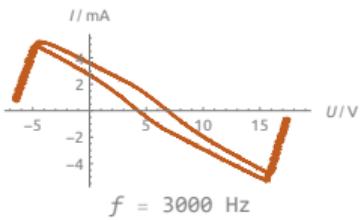
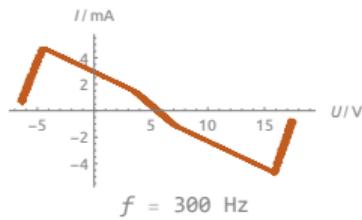
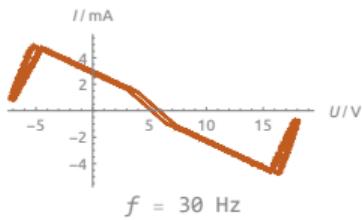
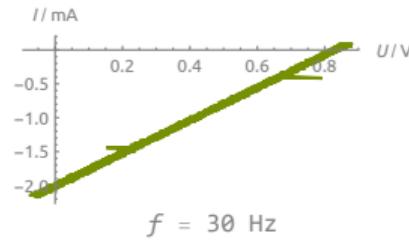


$$(b) U_R \gg U_{Nr}$$

$$U = U_1, \quad I = (U_2 - U_1) / R$$

图 7 外部信号扫描法电路图

外部信号扫描法 (II)

(a) $R = 100 \Omega$ (b) $R = 1 \text{ k}\Omega$ (c) $R = 10 \text{ k}\Omega$

阻值相差过大，导致非线性负阻
上的分压变化很小



伏安法

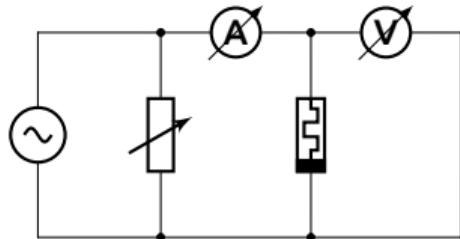


图 8 伏安法电路图

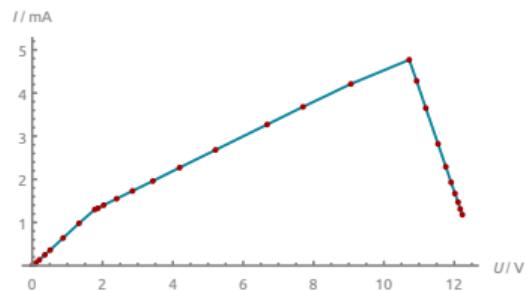


图 9 特性曲线

▶ 拟合值：

- ▶ $G_a = (7.40 \pm 0.04) \times 10^{-4} \Omega^{-1}$
- ▶ $G_b = (3.94 \pm 0.03) \times 10^{-4} \Omega^{-1}$

▶ 理论值：

- ▶ $G_a = (-7.6 \pm 0.1) \times 10^{-4} \Omega^{-1}$
- ▶ $G_b = (-4.09 \pm 0.06) \times 10^{-4} \Omega^{-1}$

数值模拟



准备

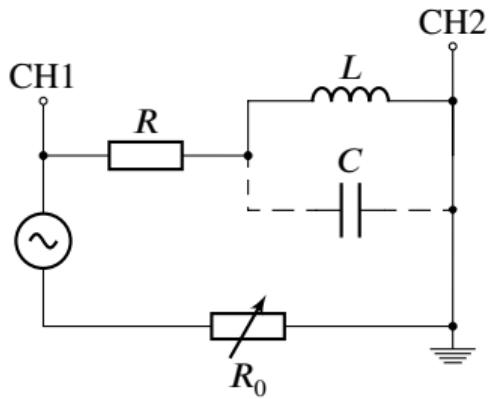


图 10 电容电感测量：串联谐振电路

测量值： $C_1 = 10.04 \text{ nF}$, $C_2 = 100.63 \text{ nF}$,
 $L = 22.65 \text{ mH}$

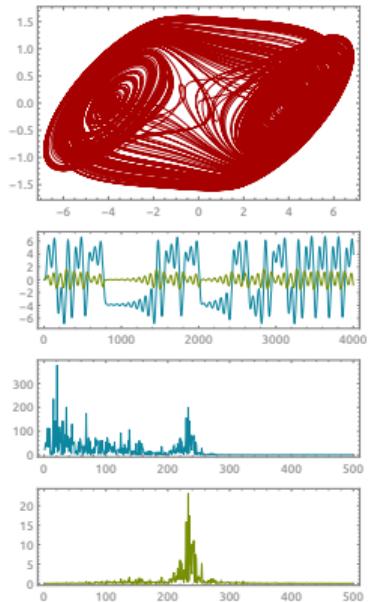
```
(* Characteristic function *)
g[u_] := $Gb * u + ($Gb - $Ga) / 2 *
(Abs[u - $E1] - Abs[u + $E1]);

(* Equations and initial conditions *)
eqs = {$C1 * u1'[t] == $G * (u2[t] - u1[t]) - g[u1[t]],
        $C2 * u2'[t] == $G * (u1[t] - u2[t]) + iL[t],
        $L * iL'[t] == -u2[t]};
init = {u1[0] == $u10, u2[0] == $u20, iL[0] == $iL0};

(* Solving... *)
sol = NDSolve[{eqs, init}, {u1, u2, iL}, {t, 0, $time},
    MaxSteps -> Infinity, WorkingPrecision -> $precision];
```

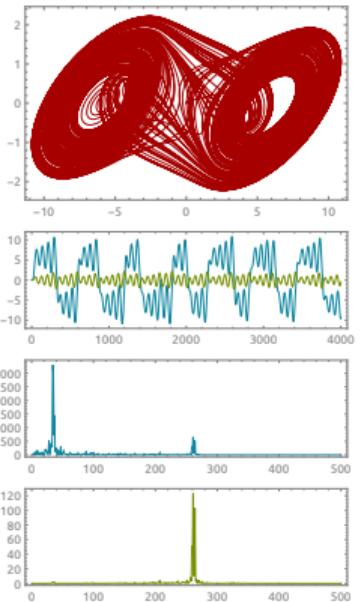
图 11 数值计算部分代码

模拟结果 (I)



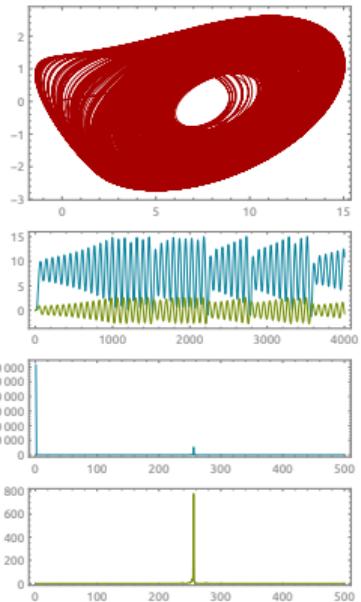
(a) 双吸引子 (I)

$$R = 1800 \Omega$$



(b) 双吸引子 (II)

$$R = 2000 \Omega$$

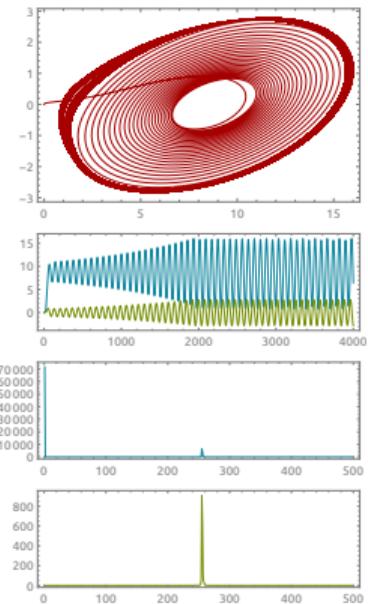


(c) 单吸引子

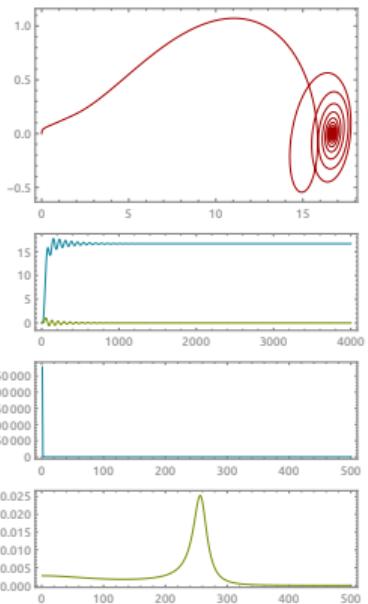
$$R = 2127 \Omega$$



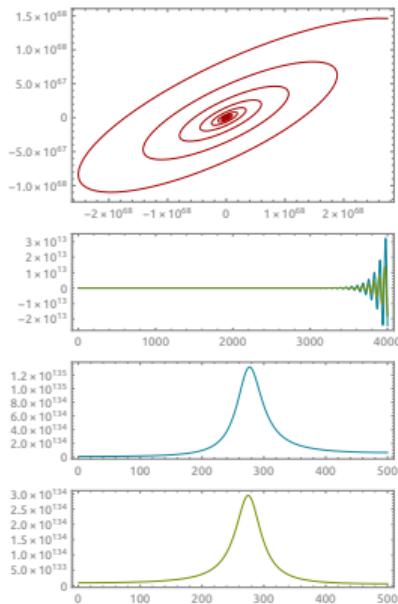
模拟结果 (II)



(d) 极限环

 $R = 2150 \Omega$ 

(e) 不动点

 $R = 2320 \Omega$ 

(f) 相图发散

 $R = 1700 \Omega$ 

Show time!

图 12 双吸引子的形成



保密通信 I——模拟电路



原理

► 加密:

$$U_0 + u_A = \tilde{U}$$



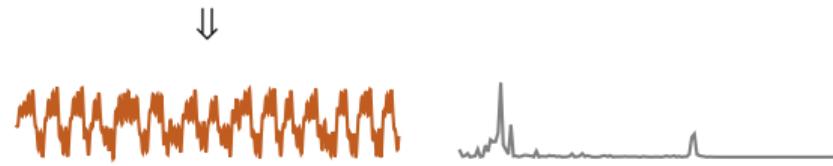
► 解密:

$$\tilde{U} - u_B = U_0$$



► 两混沌信号同步，即

$$u_A = u_B$$



► 保密性依据:

混沌信号之不可预测

(a) 加密示意图

(b) Fourier 频谱

混沌信号同步 (I)

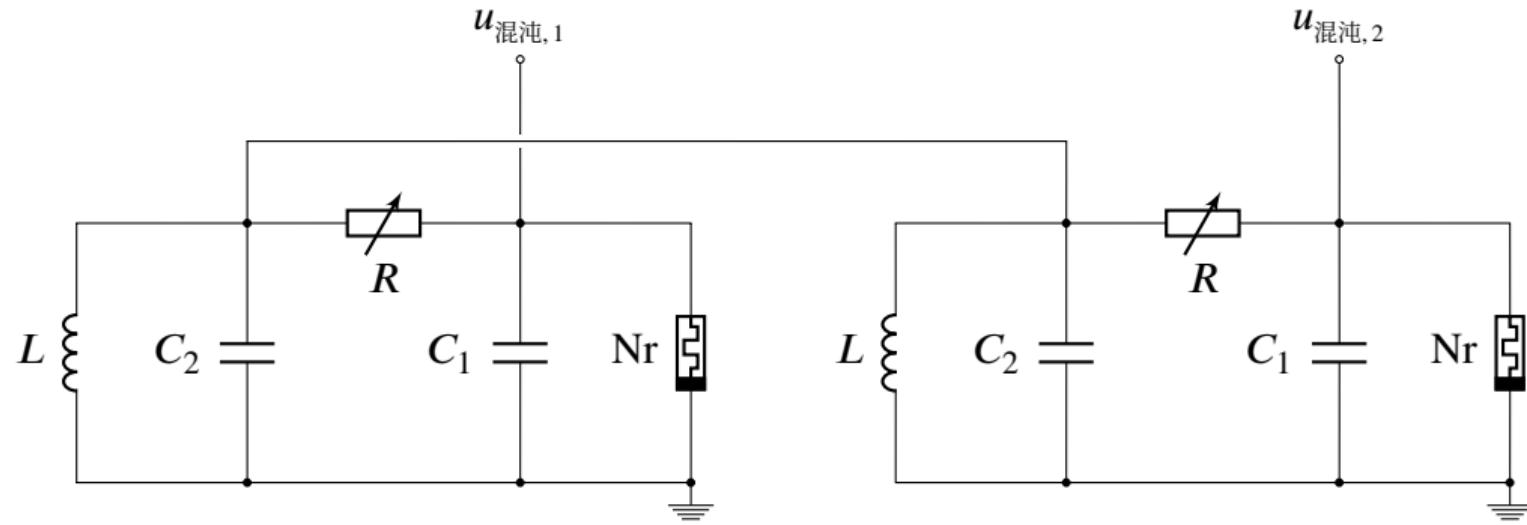
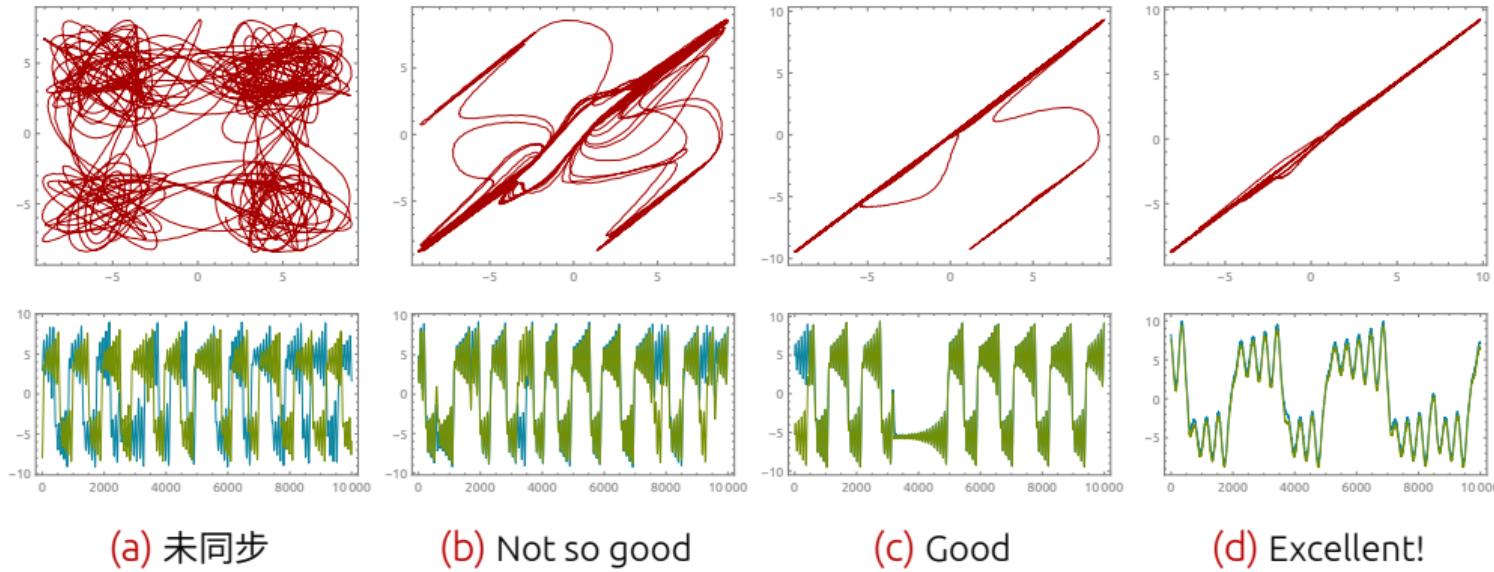


图 13 混沌保密通信电路图



混沌信号同步 (II)



(a) 未同步

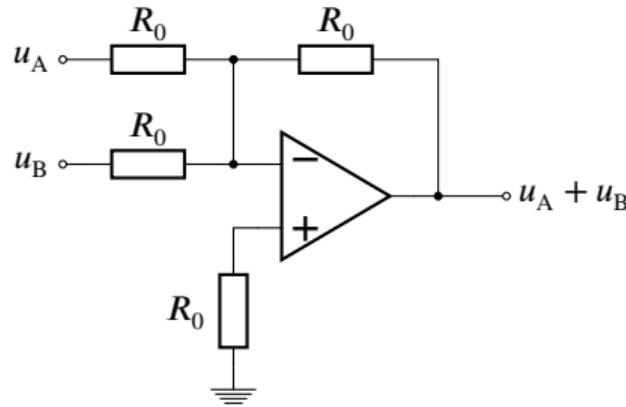
(b) Not so good

(c) Good

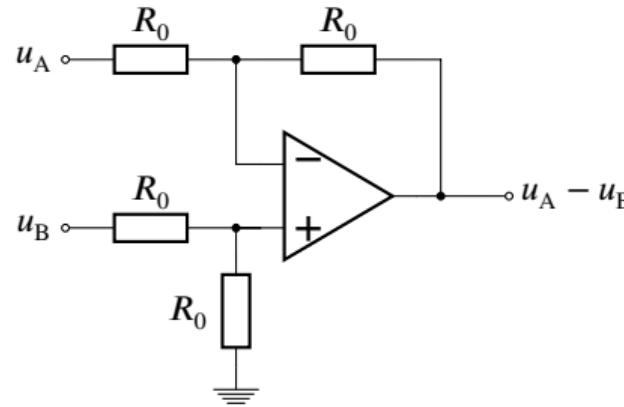
(d) Excellent!



加减法器



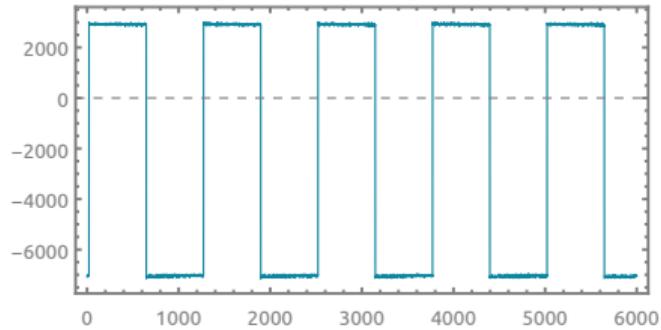
(a) 加法器



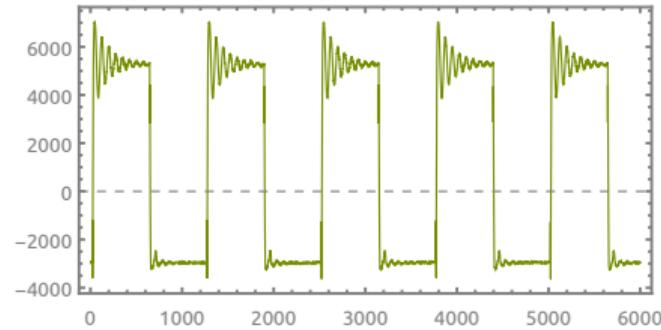
(b) 减法器

- ▶ 运算放大器需要加上 $\pm 15\text{ V}$ 的电压源
- ▶ 所有电阻 R_0 均为 $1\text{ k}\Omega$

然而……



(a) 原始方波信号



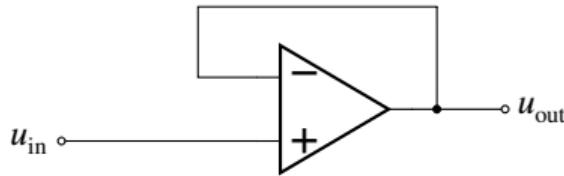
(b) 方波 + 混沌信号

- ▶ 阻抗过小，相当于直接接地



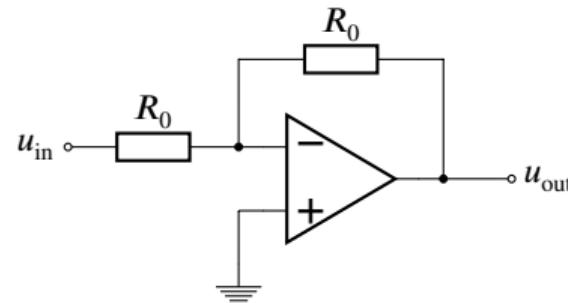
More than 加减法器

- ▶ 方波 + 方波 \Rightarrow 正常
- ▶ 方波 + 混沌 \Rightarrow 混沌消失
- ▶ 使用射极跟随器（共集极放大器）



(a) 射极跟随器

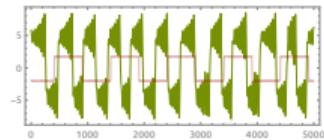
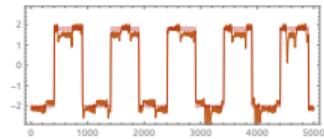
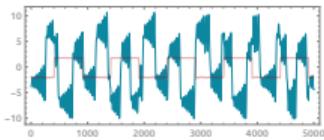
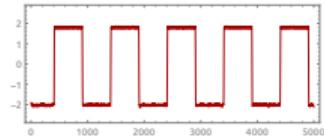
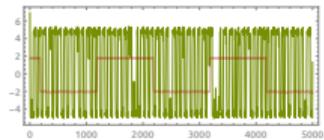
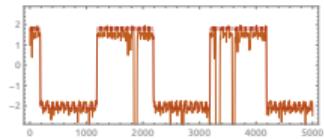
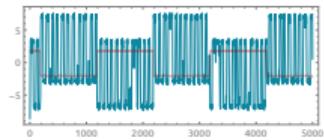
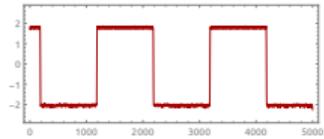
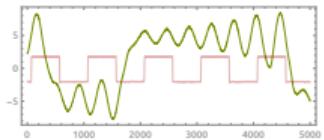
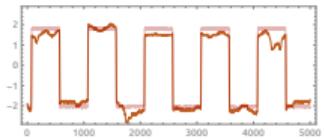
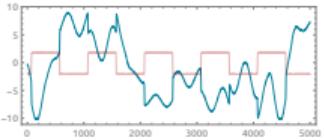
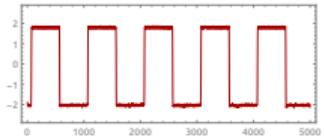
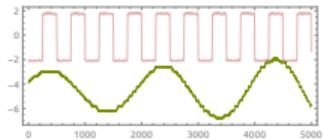
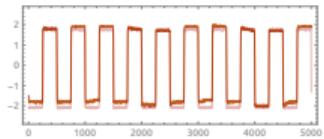
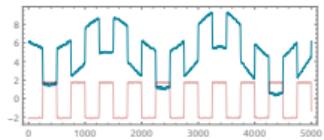
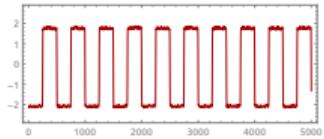
- ▶ 减法器 = 反相减法器
- ▶ 再来一个反相器



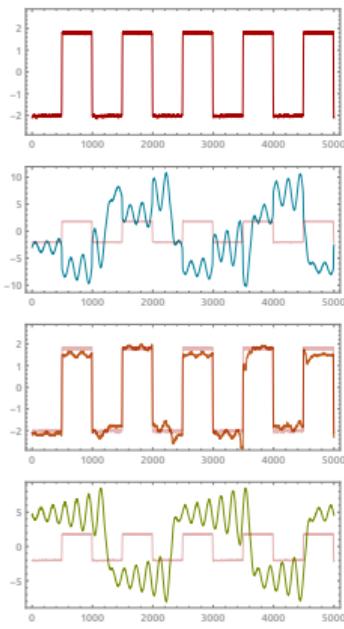
(b) 反相器



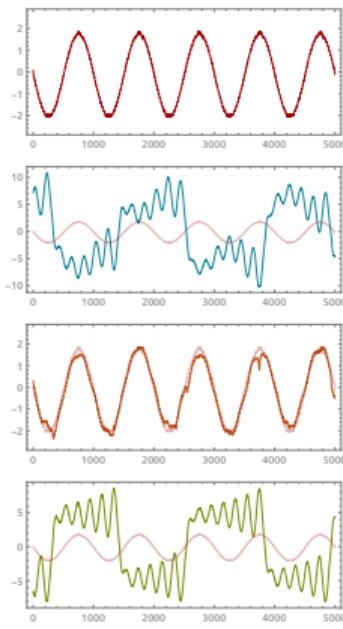
加密 / 解密结果 (I)

(a) $f = 10 \text{ Hz}$, h.scale = 0.05(b) $f = 100 \text{ Hz}$, h.scale = 0.01(c) $f = 1 \text{ kHz}$, h.scale = 0.001(d) $f = 10 \text{ kHz}$, h.scale = 0.0002

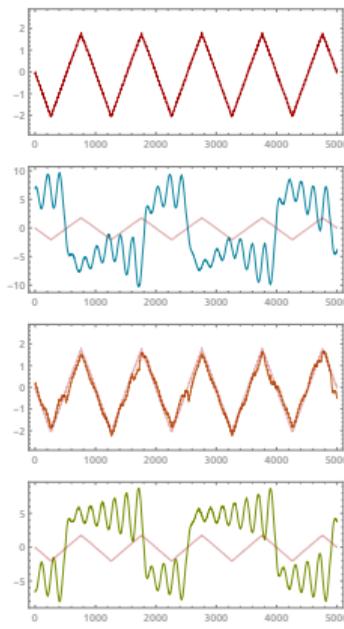
加密 / 解密结果 (II)



(a) 方波



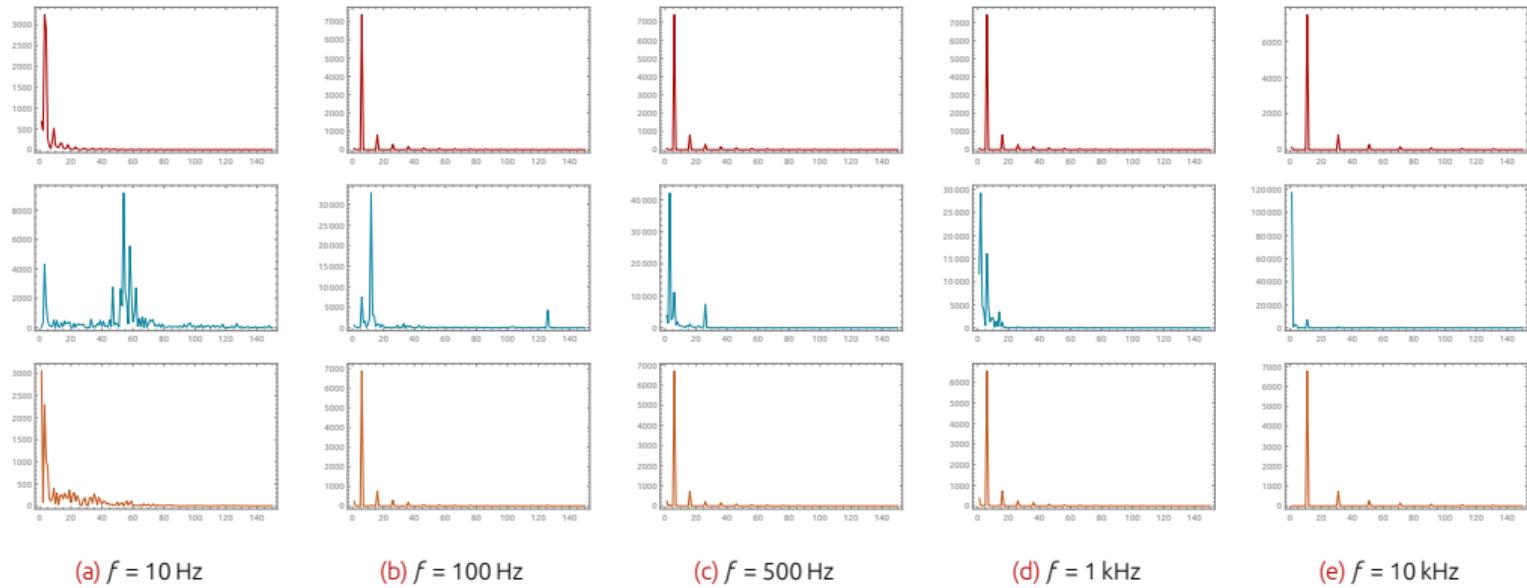
(b) 正弦波



(c) 三角波

$f = 500 \text{ Hz}$, $\text{h.scale} = 0.002$. 自上而下依次为原始信号、加密信号、解密信号与混沌信号. 以上数据经过 Gauss 平滑处理 ($r = 5$).

频谱分析



以上均采用方波信号，颜色与之前对应。此处数据未经过 Gauss 平滑处理。



保密通信 II——数字电路



原理

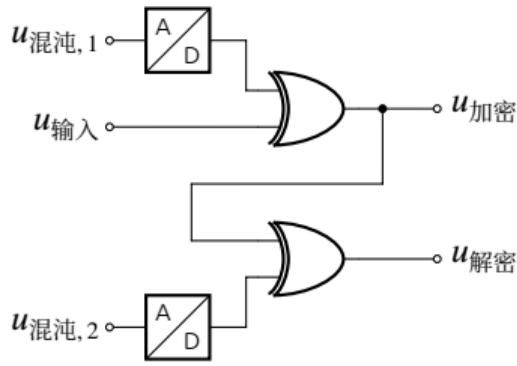


图 14 加 / 解密逻辑电路

输入	混沌 1	加密	混沌 2	解密
0	0	0	0	0
0	1	1	1	0
1	0	1	0	1
1	1	0	1	1

表 1 异或运算 (xor) 真值表



模数转换器 (ADC)

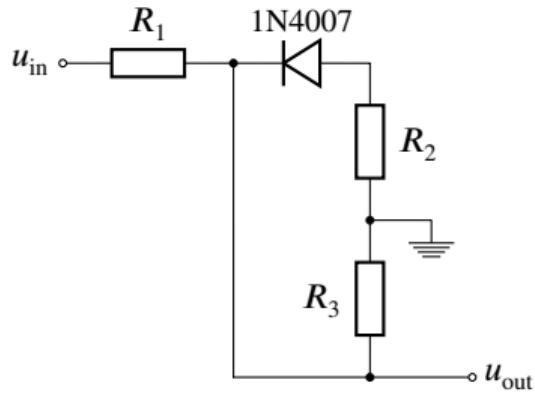


图 15 模数转换器 (伪)

$$R_1 = 100 \text{ k}\Omega, \quad R_2 = 10 \text{ k}\Omega, \quad R_3 = 30 \text{ k}\Omega$$

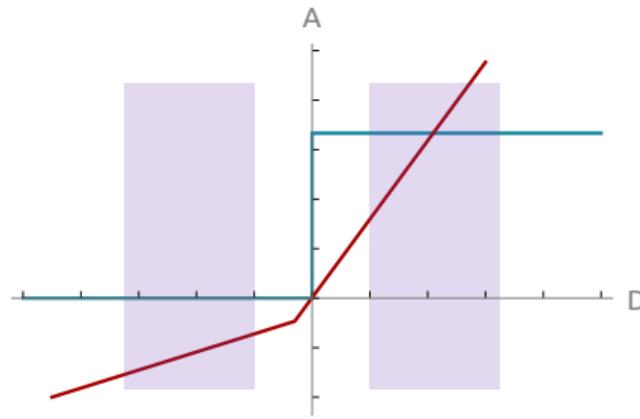
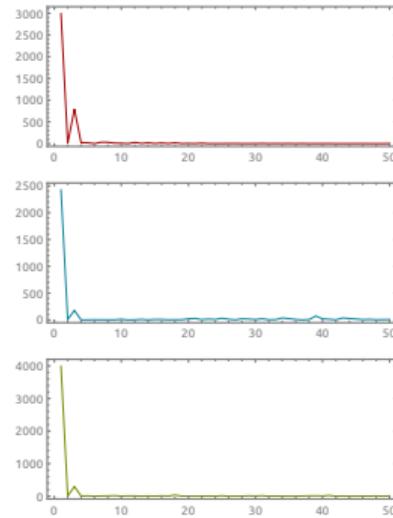
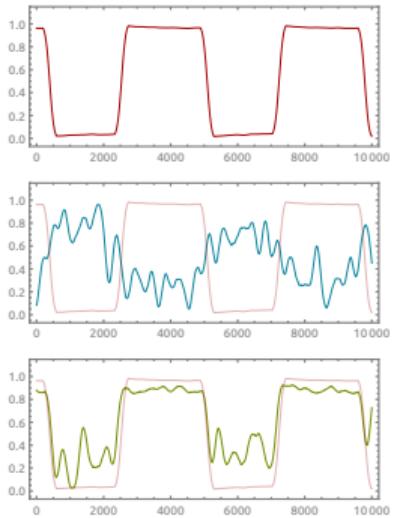
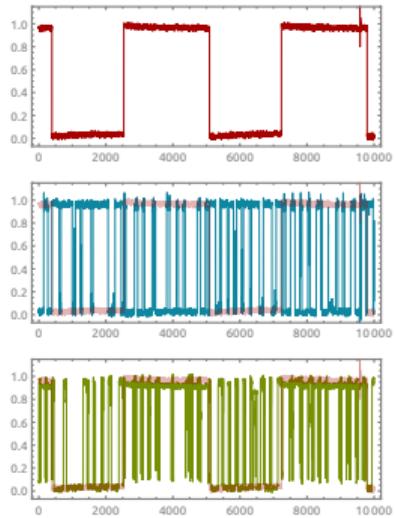


图 16 输入输出关系

蓝线：理想 ADC；红线：我们的装置；阴影：大部分时间的电压值



加密 / 解密结果



自上而下依次为原始信号、加密信号与解密信号。数据经过了归一化处理。



互信息 (I)

假设在某种编码下， x 有 u 的概率为 0，有 $1 - u$ 的概率为 1。当 $x = 0$ 时， y 有 a 的概率为 1；当 $x = 1$ 时， y 有 b 的概率为 0。则 $p(x)$ 、 $p(y)$ 、 $p(x, y)$ 取值如下：

随机变量 X 和 Y 的互信息定义为：

$$I(X; Y) = \sum_{y \in Y} \sum_{x \in X} p(x, y) \log \frac{p(x, y)}{p(x)p(y)}$$

- ▶ $p(x, y)$: 联合概率分布
- ▶ $p(x)$ 、 $p(y)$: 边缘概率分布

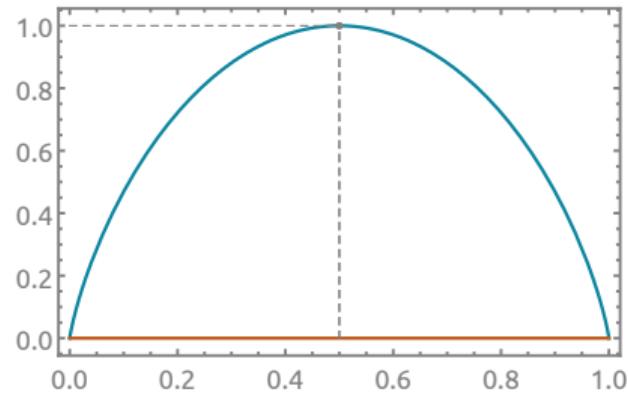
	0	1
$p(x)$	u	$1 - u$
$p(y)$	$u(1 - a) + (1 - u)b$	$ua + (1 - u)(1 - b)$

	0, 0	0, 1	1, 0	1, 1
$p(x, y)$	$u(1 - a)$	ua	$(1 - u)b$	$(1 - u)(1 - b)$

表 2 边缘概率分布 & 联合概率分布



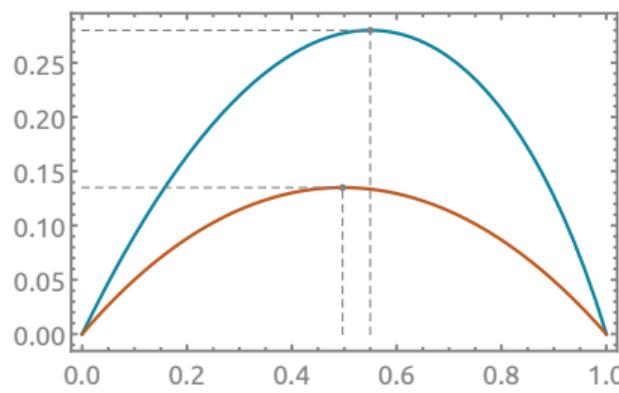
互信息 (II)



(a) 理论

解密前: $a = 0.5, b = 0.5;$

解密后: $a = 0, b = 0.$



(b) 实际

解密前: $a = 0.7322, b = 0.6750;$

解密后: $a = 0.0534, b = 0.3979.$

解密后的极值 $u_{\max} = 0.5498$. 解密前后的信道容量分别为 0.1335 和 0.2799.



总结



Conclusion

- ▶ 观察了 Chua 电路在不同条件下的相图，并与数值模拟相对照；
- ▶ 利用多种方法测量了非线性负阻的 $I-U$ 特性；
- ▶ 分别给出了混沌保密通信的模拟电路和数字电路实现方案.



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Questions?



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Note: Some Chinese characters may not display normally when copying.

See <https://github.com/CTeX-org/ctex-kit/issues/286>.

