## Exercises on univariate time series

1. Simulate $n = 50$ values from an $MA(2)$ process with $\theta_1 = 0.1$, $\theta_2 = -0.6$, $\mu = 100$, and $\sigma_\epsilon^2 = 400$.
   (a) Plot the time series.
   (b) Commpute the the theoretical *ACF*
   (c) Plot the estimated *ACF* and *PACF*.
   (d) Using the first 45 observations, estimate the model.
   (e) Evaluate the model fitting.
   (f) Predict the last 5 observations and evaluate the predictive performance of the model.
2. Repeat the previous exercise with $n = 500$, estimating the model with the first 495 observations and predicting the last 5 ones.
3. Simulate $n = 50$ values from an $AR(2)$ process with $\phi_1 = 1.5$, $\theta_2 = -0.75$, $\mu = 100$ and $\sigma_\epsilon^2 = 400$.
   (a) Plot the time series.
   (b) Compute the the theoretical *ACF*
   (c) Plot the estimated *ACF* and *PACF*.
   (d) Evaluate the model fitting.
   (e) Using the first 45 observations, estimate the model.
   (f) Predict the last 5 observations and evaluate the predictive performance of the model.
4. Repeat the previous exercise with $n = 500$, estimating the model with the first 495 observations and predicting the last 5 ones.
5. Generate a sequence of 500 observations from $\epsilon_t \sim iidN(0, 0.1)$
   (a) Set $Y_0 = 0$ and simulate 500 observations from
   $$Y_t = Y_{t-1} + \epsilon_t, \qquad t = 1, \ldots, 500$$
   where the $\epsilon_t$'s are the ones you previously generated.
   (b) Plot $\epsilon_t$ and $\nabla^2 Y_t$. What can you notice by comparing the two plots?
   (c) Set $W_0 = 0$ and
   $$W_t = Y_1 + \ldots + Y_t, \qquad t = 1, \ldots, 500.$$
   Compare the plots of the time series $\epsilon_t$, $Y_t$ and $W_t$.
   (d) Write a script producing the simulations and the plots you previously obtained through the iterative commands.
6. Simulate 500 observations form an $ARMA(1, 1)$ process with $\phi = 0.7$, $\theta = -0, 5$, $\mu = 20$ and $\sigma_\epsilon^2 = 10$
   (a) Plot the time series.

(b) Compute the the theoretical *ACF*

(c) Plot the estimated *ACF* and *PACF*.

(d) Using the first 490 observations, estimate the model.

(e) Evaluate the model fitting.

(f) Predict the last 10 observations and evaluate the predictive performance of the model.

7. Load the data in the csv file `Tbrate.csv`. The multivariate data set contains three quarterly time series, from the first quarter 1959 to the fourth quarter 1996.

   (a) Plot the the three time series in the data set as well as their estimated *ACF* and *PACF*.

   (b) Describe the signs of nonstationarity in the time series and *ACF* plots.

   (c) Use the *ADF* test to decide which of the series are nonstationary. Do the tests agree with the conclusions based on the time series and estimated *ACF* plots?

   (d) Using the `auto.arima` of the package `forecast`, select the best *ARIMA* model for the 91-day treasury bill rate in terms of the *AIC* criterion.

   (e) Change the criterion to *BIC*. Does the best fitting model then change?

   (f) Run the diagnostics on the models you have previously fitted to the 91-day treasury bill rate.

   (g) Do you think that there is residual autocorrelation? If so, describe this autocorrelation and suggest a more appropriate model for the T-bill series.

   (h) Plot the 0.95 prediction interval produced by the best fitting model.

8. Load the data set `IncomeUK` from the package `Ecdat`.

   (a) Describe the behavior of consumption. What types of differencing, seasonal, nonseasonal, or both, would you recommend? Do you recommend fitting a seasonal *ARIMA* model to the data with or without a log transformation? Consider also using *ACF* plots to help answer these questions.

   (b) Find an *ARIMA* model that provides a good fit to `log(consumption)`. What model order did you select? (Give the orders of the nonseasonal and seasonal components.)

   (c) Run the diagnostics in order to evaluate the model fitting. Is the fitted model satisfactory?

   (d) Apply `auto.arima` to `log(consumption)` using *BIC*. What model is selected? Compare the goodness of fit of this model with the one of the model you estimated previously.