Д/з (19.18) $a_n = \sum_{k=1}^{n} \frac{(4k-3)^3}{k^4} = \frac{1}{n}\sum_{k=1}^{n}\left(\frac{64}{\frac{k}{n}} - \frac{144}{\frac{k^2}{n}} + \frac{108}{\frac{k^3}{n}} - \frac{27}{\frac{k^4}{n}}\right) =$

$= \lim_{n\to\infty} a_n = \int_0^1 \frac{64}{x}\,dx - \int_0^1 \frac{144}{x^2}\,dx + \int_0^1 \frac{108}{x^3}\,dx - \int_0^1 \frac{27}{x^4}\,dx =$

$= 144 - 54 + 9 = 99.$

(19.23) $\lim_{n\to\infty} a_n = \lim\left(\prod_{k=1}^{2^n-1}\left(1+\frac{k}{2^n}\right)\right)^{\frac{1}{2^n}} = e^{\lim_{n\to\infty}\frac{1}{2^n}\ln\prod_{k=1}^{2^n-1}\left(1+\frac{k}{2^n}\right)} \,(=)$

$\lim_{n\to\infty}\frac{1}{2^n}\cdot\ln\prod_{k=1}^{2^n-1}\left(1+\frac{k}{2^n}\right) = \lim_{n\to\infty}\frac{1}{2^n}\cdot\sum_{k=1}^{2^n-1}\ln\left(1+\frac{k}{2^n}\right) =$

$= \int_0^1 \ln(1+x)\,dx = \left|\begin{matrix} u = \ln(1+x) & du = \frac{dx}{1+x} \\ dv = dx & v = x \end{matrix}\right| =$

$= \ln(1+x)x\Big|_0^1 - \int_0^1 \frac{x\,dx}{1+x} = \ln 2 - \int_0^1 \frac{x+1-1}{1+x}\,dx =$

$= \ln 2 - x\Big|_0^1 + \ln|x+1|\Big|_0^1 = \ln 4 - 1 = \ln\frac{4}{e}$

$(=)\ e^{\ln\frac{4}{e}} = \frac{4}{e}$

19.27 $f(x) = \left[\frac{1}{5x}\right]$, $[a,b] = \left[\frac{1}{3}, 1\right]$

Це функція обмежена на $[a,b]$ і має скінченну кількість точок розриву ⇒ інтегровна.

(20.1) $\int_0^{\frac{\pi}{2}} f(\sin x)\,dx = \int_0^{\frac{\pi}{2}} f(\cos x)\,dx$, $f \in C([0,1])$

$\int_0^{\frac{\pi}{2}} f(\cos x)\,dx = \int_0^{\frac{\pi}{2}} f\left(\sin\left(\frac{\pi}{2} - x\right)\right)dx = \left|\begin{matrix} \frac{\pi}{2} - x = t \\ dx = -dt \end{matrix}\right| =$

$$= -\int_{\frac{\pi}{2}}^{0} f(\sin t)\,dt = \int_{0}^{\frac{\pi}{2}} f(\sin t)\,dt$$

20.3 $$\int_{0}^{1} \frac{\operatorname{arctg} x}{x}\,dx = \frac{1}{2}\int_{0}^{\frac{\pi}{2}} \frac{t\,dt}{\sin t}$$

$$\int_{0}^{1} \frac{\operatorname{arctg} x}{x}\,dx = \left| \begin{array}{ll} y = \operatorname{arctg} x & \operatorname{tg} y = x \\ dx = \frac{1}{\cos^2 y}\,dy & \end{array} \right| = \int_{0}^{\frac{\pi}{4}} \frac{y\,dy}{\operatorname{tg} y \cdot \cos^2 y} =$$

$$= \int_{0}^{\frac{\pi}{4}} \frac{2y\,dy}{\sin 2y} = \left| \begin{array}{ll} 2y = t & 2dy = dt \\ dy = \frac{dt}{2} & \end{array} \right| = \frac{1}{2}\int_{0}^{\frac{\pi}{2}} \frac{t\,dt}{\sin t}$$

20.7 $$\int_{0}^{100\pi} \sqrt{1-\cos 2x}\,dx = \int_{0}^{100\pi} \sqrt{2}\,|\sin x|\,dx = \sqrt{2}\cdot 100 \int_{0}^{\pi} \sin x\,dx =$$

$$= \sqrt{2}\cdot 100 \cdot 2 = 200\sqrt{2}$$

20.14 $$\int_{0}^{1} \sqrt{x^2+1}\,dx, \quad x = \frac{1}{\cos t}$$

$\frac{1}{\cos t} \neq 0$, тому виконати заміну не можливо.