Д/З

№3338

$f(x,y) = \frac{x}{y}$, $y \neq 0$, $P_0(1,1)$, $n=2$

$\frac{x}{y} = \frac{1+(x-1)}{1+(y-1)} = \frac{1+u}{1+v}$

$1+u = 1+u+\frac{1(1-1)}{2!}u^2 = 1+(x-1)+0 = 1+(x-1)$

$1+v = 1-v+\frac{(-1)(-1-1)}{2!}v^2 = 1-(y-1)+(y-1)^2$

$(1+(x-1))(1-(y-1)+(y-1)^2) = 1-(y-1)+(y-1)^2+(x-1) -$

$-(x-1)(y-1)+o(|h|^2)$

№3.42

$f(x,y) = \sin x \sin y$, $P_0\left(\frac{\pi}{2}, \frac{\pi}{3}\right)$, $n=2$

$f(x,y) = \sin\frac{\pi}{2}\sin\frac{\pi}{3} + \frac{1}{1!}\left(\cos\frac{\pi}{2}\sin\frac{\pi}{3}(\ldots) + \cos y \sin x\right) +$

$+\frac{1}{2!}\left(-\sin x \sin y\,(x-\frac{\pi}{2})^2 - \sin y \sin x\,(y-\frac{\pi}{3})^2 + 2\cos x \cos y\right) =$

$= \frac{\sqrt{3}}{2} + \frac{1}{2}(y - \frac{\pi}{3}) + \frac{\sqrt{3}}{4}(x - \frac{\pi}{2})^2 - \frac{\sqrt{3}}{4}(y - \frac{\pi}{3})^2 + o(|h|^2)$

$h = (x - x_0, y - y_0)$

№ 24.6

$f(x,y) = xy + \frac{50}{x} + \frac{20}{y}$

$df = dxy + dyx - \frac{50}{x^2} - \frac{20}{y^2}$

$f'_x = y - \frac{50}{x^2} = 0 \Rightarrow y = \frac{50}{x^2} \Rightarrow y = 2$

$f'_y = x - \frac{20}{y^2} = 0 \Rightarrow x - \frac{20x^4}{2500} = 0 \Rightarrow x = 5$

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$f''_{xx} = \frac{100}{x^3}$

$f''_{yy} = \frac{40}{y^3}$

$f''_{xy} = 1$

$\delta(5,2) = 3 > 0$

$\delta(x,y) = \begin{vmatrix} \frac{100}{x^3} & 1 \\ 1 & \frac{40}{y^3} \end{vmatrix} = \frac{4000}{1000} - 1 = 999 > 0$

$f_{min} = f(5,2) = 30$

№ 24.12 $f(x,y,z) = xy^2z^3(a - x - 2y - 3z)$, $a > 0$; $x, y, z > 0$

$df = (y^2z^3dx + 2xyz^3dy + 3y^2x\,dz) + (a - x - 2y - 3z) + xy^2z^3 \cdot$

$\cdot (-dx - 2dy - 3dz)$

$\begin{cases} f'_x = ay^2z^3 - 2xy^2z^3 - 3y^2z^4 \\ f'_y = 2axyz^3 - 2x^2yz^3 - 6xy^2z^3 - 6xyz^4 \\ f'_z = 3az^2y^2x - 3z^2y^2x^2 - 6z^2y^3x \ldots \end{cases} \Leftrightarrow$

$$\Leftrightarrow \begin{cases} y^2z^3(a-2x-2y-3z)=0 \\ 2xyz^3(a-x-3y-3z)=0 \\ 3z^2y^2x(a-x-2y-4z)=0 \end{cases} \Rightarrow \begin{cases} 3a=4x+7y+10z \\ a=x+3y+3z \\ a=x+2y+4z \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} x=\frac{a}{7} \\ y=\frac{a}{7} \\ z=\frac{a}{7} \end{cases} \Rightarrow M_1=\left(\frac{a}{7}, \frac{a}{7}, \frac{a}{7}\right)$$

$$\begin{cases} f''_{xx}=-2y^2z^3 \\ f''_{yy}=-6xyz^3 \\ f''_{zz}=-12xy^2z^2 \end{cases} \quad \begin{cases} f''_{xy}=-2y^2z^3 \\ f''_{yz}=-6xyz^3 \\ f''_{xz}=-3y^2z^3 \end{cases}$$

$$\Delta_1=-\frac{2a^5}{7^5}<0$$

$$\Delta_2=\begin{pmatrix} -\frac{2a^5}{7^5} & -\frac{a^5}{7^5} \\ -\frac{a^5}{7^5} & -6\frac{a^5}{7^5} \end{pmatrix}=\frac{11a^{10}}{7^{10}}>0$$

$$\Delta_3<0$$

$M_2=(x,0,0)$

$M_3=(0,y,0)$

$M_4=(0,0,z)$

$\Rightarrow$ loc max $=f\left(\frac{a}{7}, \frac{a}{7}, \frac{a}{7}\right)=\left(\frac{a}{7}\right)^7=$

№24.25 $f(x,y)=x+2y$, $x^2+y^2=5$

$\Phi(x,y,\lambda)=x+2y+\lambda(x^2+y^2-5)$

$$\begin{cases} 1+\lambda x=0 \\ 2+\lambda y=0 \\ x^2+y^2=5 \end{cases} \Rightarrow \begin{cases} x=-\frac{1}{\lambda} \\ y=-\frac{2}{\lambda} \\ \frac{1}{\lambda^2}+\frac{4}{\lambda^2}=5 \end{cases} \Rightarrow \overset{P_0}{\begin{cases} x=-1 \\ y=-2 \\ \lambda=1 \end{cases}} \text{ або } \overset{P_1}{\begin{cases} x=1 \\ y=2 \\ \lambda=1 \end{cases}}$$

Другий диференціал ф-ї Лагранжа

$d^2L = 2\lambda dx^2 + 2x\,dx\,d\lambda + 2\lambda dy^2 + 2y\,dy\,d\lambda$

$d^2L(P_0) > 0 \Rightarrow$ loc min $f(1,-2) = -5$

$d^2L(P_1) < 0 \Rightarrow$ loc max $f(1,2) = 5$

№ 4.37

$f(x,y) = x^2 - xy + y^2$, $D = \{(x,y) \in R^2 \mid |x| + |y| \le 1\}$

$$\begin{cases} f'_x = 2x - y = 0 \\ f'_y = 2y - x = 0 \end{cases} \Rightarrow \begin{cases} y = 2x \\ x = 0 \end{cases} \Rightarrow \begin{cases} y = 0 \\ x = 0 \end{cases}$$

№ 3.32 $\quad \frac{x}{z} = \ln\frac{z}{y} + 1$

$\frac{x}{z} + \ln y = \ln z + 1$

$\ln z = \frac{x}{z} + \ln y - 1$

$\frac{dz}{|z|} = \frac{dx\,z - dz\,x}{z^2} + \frac{dy}{|y|}$

$dz = \frac{dx\,z - dz\,x}{z} + \frac{z\,dy}{y}$

$dz\left(1 + \frac{x}{z}\right) = \frac{z\,dy}{y} + dx$

$dz = \left(\frac{z\,dy + y\,dx}{y}\right)\left(\frac{z}{x+z}\right)$

$z'_x = \frac{z}{x+z}$

$z'_y = \frac{z^2}{y(x+z)}$

$z''_{xx} = \frac{d_z(x+z) - z(d_x + d_z)}{(x+z)^2} = -\frac{z}{(x+z)^2}$

$$z''_{yy} = \frac{2z\,dz\,(y(x+z)) - z^2\left(dy\,(x+z) + y\,(\overset{\to 0}{dx} + dz)\right)^{\to 0}}{y^2(x+z)^2} =$$

$$= -\frac{z^2(x+z)}{y^2(x+z)^2} = -\frac{z^2}{y^2(x+z)}$$

$$z''_{xy} = \frac{2z\,\overset{0}{dz}\,(y\,(x+z)) - z^2\left(\overset{0}{dy}\,(x+z) + y\,(dx + \overset{0}{dz})\right)}{y^2(x+z)^2} =$$

$$= -\frac{z^2 y}{y^2(x+z)^2} = -\frac{z^2}{y(x+z)^2}$$