Д/З (172) $\int\limits_{x_0}^{x} \frac{dt}{t(\sqrt{t}+\sqrt[5]{t^2})} = \int\limits_{x_0}^{x} \frac{dt}{t^{\frac{3}{2}}+t^{\frac{7}{5}}} = \left| \begin{array}{l} y = t^{\frac{1}{10}} \quad dy = \frac{dt}{10t^{\frac{9}{10}}} \\ dt = 10t^{\frac{9}{10}}dy \\ dt = 10y^9 dy \end{array} \right| =$

$$= \int\limits_{\sqrt[10]{x_0}}^{\sqrt[10]{x}} \frac{10y^9 dy}{y^{15}+y^{14}} = 10\int\limits_{\sqrt[10]{x_0}}^{\sqrt[10]{x}} \frac{dy}{y^6+y^5}$$

$$\frac{1}{y^5(y+1)} = \frac{A}{y^5} + \frac{B}{y^4} + \frac{C}{y^3} + \frac{D}{y^2} + \frac{E}{y} + \frac{F}{y+1} \;=$$

$$1 = Ay + A + By^2 + By + Cy^3 + Cy^2 + Dy^4 + Dy^3 + Ey^5 + Ey^4 + Fy^5$$

$$\begin{array}{ll} y^5: E+F=0 & \\ y^4: D+E=0 & \\ y^3: C+D=0 & \\ y^2: B+C=0 & \\ y^1: A+B=0 & \\ y^0: A=1 & \end{array} \qquad \begin{cases} A=1 \\ B=-1 \\ C=1 \\ D=-1 \\ E=1 \\ F=-1 \end{cases}$$

$$= \frac{1}{y^5} - \frac{1}{y^4} + \frac{1}{y^3} - \frac{1}{y^2} + \frac{1}{y} - \frac{1}{y+1}$$

$$10\int\limits_{\sqrt[10]{x_0}}^{\sqrt[10]{x}} \frac{dy}{y^5} - 10\int\limits_{\sqrt[10]{x_0}}^{\sqrt[10]{x}} \frac{dy}{y^4} + 10\int\limits_{\sqrt[10]{x_0}}^{\sqrt[10]{x}} \frac{dy}{y^3} - 10\int\limits_{\sqrt[10]{x_0}}^{\sqrt[10]{x}} \frac{dy}{y^2} + 10\int\limits_{\sqrt[10]{x_0}}^{\sqrt[10]{x}} \frac{dy}{y} - 10\int\limits_{\sqrt[10]{x_0}}^{\sqrt[10]{x}} \frac{dy}{y+1} =$$

$$= \frac{10y^{-4}}{-4}\Big|_{\sqrt[10]{x_0}}^{\sqrt[10]{x}} - \frac{10y^{-3}}{-3}\Big|_{\sqrt[10]{x_0}}^{\sqrt[10]{x}} + \frac{10y^{-2}}{-2}\Big|_{\sqrt[10]{x_0}}^{\sqrt[10]{x}} - \frac{10y^{-1}}{-1}\Big|_{\sqrt[10]{x_0}}^{\sqrt[10]{x}} + 10\ln|y|\Big|_{\sqrt[10]{x_0}}^{\sqrt[10]{x}} -$$

$$- 10\ln|y+1|\Big|_{\sqrt[10]{x_0}}^{\sqrt[10]{x}} = -\frac{5}{2\sqrt[5]{x^2}} + \frac{10}{3\sqrt[10]{x^3}} - \frac{5}{\sqrt[5]{x}} + \frac{10}{\sqrt[10]{x}} + \ln x - 10\ln|\sqrt[10]{x}+1|$$

$$= \ln\frac{x}{(\sqrt[10]{x}+1)^{10}} - \frac{5}{2\sqrt[5]{x^2}} + \frac{10}{3\sqrt[10]{x^3}} - \frac{5}{\sqrt[5]{x}} + \frac{10}{\sqrt[10]{x}} =$$

17.8 $$\int_{x_0}^{x} \frac{3t^3}{\sqrt{t^2+4t+5}}\,dt = (At^2+Bt+C)\sqrt{t^2+4t+5} + \lambda \int_{x_0}^{x} \frac{dt}{\sqrt{t^2+4t+5}}$$

$$\frac{3t^2}{\sqrt{t^2+4t+5}} = (2At+B)\sqrt{t^2+4t+5} + (At^2+Bt+C)\frac{t+2}{\sqrt{t^2+4t+5}} + \lambda\frac{1}{\sqrt{t^2+4t+5}}$$

$$3t^3 = (2At+B)\cdot(t^2+4t+5) + (At^2+Bt+C)(t+2) + \lambda =$$
$$= 2At^3+8At^2+10At+Bt^2+4Bt+5B+At^3+Bt^2+Ct+2At^2+2Bt+2C+\lambda$$

t^3: $3A=3$ $\qquad A=1$

t^2: $10A+2B=0$ $\qquad B=-5$

t^1: $10A+6B+C=0$ $\qquad C=20$

t^0: $5B+2C+\lambda=0$ $\qquad \lambda=-15$

$$\int_{x_0}^{x} \frac{3t^3\,dt}{\sqrt{t^2+4t+5}} = (t^2-5t+20)\sqrt{t^2+4t+5} - 15\int_{x_0}^{x}\frac{dt}{\sqrt{(t+2)^2+1}} =$$

$$= (x^2-5x+20)\sqrt{x^2+4x+5} - 15\ln\left|x+2+\sqrt{(x+2)^2+1}\right|$$

17.9 $$\int_{x_0}^{x}\frac{dt}{t\sqrt{t^2+4t-4}} = \left| \begin{array}{l} y=\frac{1}{t} \\ dy=-\frac{1}{t^2}dt \end{array}\right| = \int_{x_0}^{x}\frac{dt}{t\cdot|t|\cdot\sqrt{1-\frac{4}{t}-\frac{4}{t^2}}} =$$

$$= \operatorname{sgn} t \cdot \int_{x_0}^{x}\frac{dt}{t^2\sqrt{1-\frac{4}{t}-\frac{4}{t^2}}} = -\operatorname{sgn} y \int_{\frac{1}{x_0}}^{\frac{1}{x}}\frac{dy}{\sqrt{1-4y-4y^2}} = -\operatorname{sgn} y\int_{\frac{1}{x_0}}^{\frac{1}{x}}\frac{dy}{\sqrt{-(2y-1)^2+2}} =$$

$$= -\frac{1}{2}\operatorname{sgn} y\int_{\frac{1}{x_0}}^{\frac{1}{x}}\frac{d(2y-1)}{\sqrt{-(2y-1)^2+2}} = -\frac{1}{2}\operatorname{sgn} y\cdot \arcsin\frac{2y-1}{\sqrt{2}} = -\frac{1}{2}\operatorname{sgn} x\cdot\arcsin\frac{\frac{2}{t}-1}{\sqrt{2}}$$

(12.13) $\int_{x_0}^{x} \frac{dt}{1+\sqrt{1-2t-t^2}}$. $c>0 \to$ II підст. Ейлера

$\sqrt{1-2t-t^2} = yt - 1$

$1-2t-t^2 = y^2t^2 - 2yt + 1$

$t = \frac{2y-2}{y^2+1}$; $dt = \frac{-2y^2+4y+2}{(y^2+1)^2}$

$\int \frac{-2y^2+4y+2}{(y^2+1)^2 \cdot \frac{2y^2-2y}{y^2+1}} dy = \int \frac{2(-y^2+2y+1)}{2(y^2+1)(y-1)y} dy$

$\frac{-y^2+2y+1}{(y^2+1)(y-1)y} = \frac{Ay+B}{y^2+1} + \frac{C}{y} + \frac{D}{y-1}$

$-y^2+2y+1 = (Ay+B)(y^2-y) + C(y^2+1)(y-1) + D(y^2+1)y =$

$= Ay^3 - Ay^2 + By^2 - By + Cy^3 - Cy^2 + Cy - C + Dy^3 + Dy$

$y^3: A+C+D=0$ $\quad \begin{cases} B+D=-1 \\ -B+D=3 \end{cases} \Rightarrow \begin{matrix} D=1 \\ B=-2 \end{matrix}$

$y^2: -A+B-C=-1$ $\quad A=0$

$y^1: -B+D+C=2$ $\quad C=-1$

$y^0: -C=1$

$\int \frac{(-y^2+2y+1)}{(y^2+1)y(y-1)} = \int \frac{-2}{y^2+1} dy - \int \frac{1}{y} dy + \int \frac{1}{y-1} dy =$

$= -2\operatorname{arctg} y - \ln|y| + \ln|y-1| = -2\operatorname{arctg} y + \ln\left|\frac{y}{y-1}\right|$

(12.14) $\int_{x_0}^{x} t^{-1}\left(1+t^{\frac{1}{3}}\right)^{-3} dt$

$p=-3$ $\quad \frac{(-1+1)}{\frac{1}{3}} \cdot 3 = 0$ $\quad 0-3=-3$

Викор. I підстановку

$\int_{x_0}^{x} t^{-1}\left(1+t^{\frac{1}{3}}\right)^{-3} dt = \left| \begin{matrix} t = y^3 \\ dt = 3y^2 dy \end{matrix} \right| = \int_{\sqrt[3]{x_0}}^{\sqrt[3]{x}} y^{-3}\left(1+(y^3)^{\frac{1}{3}}\right)^{-3} dy = \int_{\sqrt[3]{x_0}}^{\sqrt[3]{x}} \frac{3y^2 dy}{y^3(1+y)^3}$

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$$\frac{3y^2}{y^3(1+y)^3} = \frac{A}{y} + \frac{B}{y^2} + \frac{C}{y^3} + \frac{D}{y+1} + \frac{E}{(y+1)^2} + \frac{F}{(y+1)^3}$$

$$3y^2 = Ay^2(1+y)^3 + By(1+y)^3 + C(1+y)^3 + Dy^3(y+1)^2 + Ey^3(y+1) + F(y^3)$$

$$3y^2 = Ay^5 + 3Ay^4 + 3Ay^3 + Ay^2 + By^4 + 3By^3 + 3By^2 + By + Cy^3 + 3Cy^2 + 3Cy + C + Dy^5 + 2Dy^4 + Dy^3 + Ey^4 + Ey^3 + Fy^3$$

$y^5: A + D = 0$

$y^4: 3A + B + 2D + E = 0$

$y^3: 3A + 3B + C + D + E + F = 0 \qquad E = -3 \qquad F = -3$

$y^2: A + 3B + 3C = 3 \qquad A = 3$

$y^1: 3B + 3C = 0 \qquad B = 0$

$y^0: C = 0$

$$\int_{\sqrt[3]{x_0}}^{\sqrt[3]{x}} \left(\frac{3}{y} - \frac{3}{y+1} - \frac{3}{(y+1)^2} - \frac{3}{(y+1)^3}\right) dy = \Big(3\ln|y| - 3\ln|y+1| + 3\frac{1}{y+1} + \frac{3}{2}\cdot\frac{1}{(y+1)^2}\Big)\Bigg|_{\sqrt[3]{x_0}}^{\sqrt[3]{x}} = 3\ln\left|\frac{\sqrt[3]{x}}{\sqrt[3]{x}+1}\right| + \frac{6\sqrt[3]{x}+9}{2(\sqrt[3]{x}+1)^2}$$

(13.2) $$\int_{x_0}^{x} \sin^2 t \cdot \cos^3 t \, dt = \frac{1}{2}\int_{x_0}^{x} \sin t\,(1-\sin^2 t)^{\frac{3-1}{2}} \cdot 2\sin t\cos t \, dt =$$

$$= \left| \begin{matrix} y = \sin^2 t \\ dy = 2\sin t\cos t\,dt \end{matrix} \right| = \frac{1}{2}\int_{\sin^2 x_0}^{\sin^2 x} y^{\frac{1}{2}}(1-y)\,dy = \frac{1}{2}\int_{\sin^2 x_0}^{\sin^2 x} y^{\frac{1}{2}} - \frac{1}{2}\int_{\sin^2 x_0}^{\sin^2 x} y^{\frac{3}{2}} =$$

$$= \left(\frac{1}{2}\cdot\frac{2y^{\frac{3}{2}}}{3} - \frac{1}{2}\cdot\frac{2y^{\frac{5}{2}}}{5}\right)\Bigg|_{x_0}^{x} = \frac{1}{3}\sin^3 x - \frac{1}{5}\sin^5 t$$

(13.6) $$\int_{x_0}^{x} \frac{dt\cdot\cos t}{\cos t\cdot\cos t} = \int_{x_0}^{x} \frac{\cos t\,dt}{1-\sin^2 t} = \left| \begin{matrix} y = \sin t \\ dy = \cos t\,dt \end{matrix} \right| =$$

$$= \int_{x_0}^{x} \frac{dy}{1-y^2} = -\frac{1}{2}\ln\left|\frac{y-1}{y+1}\right| = -\frac{1}{2}\ln\left|\frac{\sin x-1}{\sin x+1}\right|.$$

(18.6) $$\int_{x_0}^{x} \frac{dt}{5-\sin t} = \left| \begin{matrix} y = tg\frac{t}{2} \\ dt = \frac{2dy}{1+y^2} \end{matrix} \right| = \int_{x_0}^{x} \frac{2dy}{(1+y^2)(5-\frac{6y}{1+y^2})} =$$

$$= \int_{x_0}^{x} \frac{2dy}{5+5y^2-6y} = \int_{x_0}^{x} \frac{2dy}{5(y^2-\frac{6}{5}y+\frac{2}{25}+\frac{16}{25})} = \int_{x_0}^{x} \frac{2dy}{5((y-\frac{3}{5})^2+\frac{16}{25})} =$$

$$= \frac{2}{5}\cdot\frac{5}{4}\, \operatorname{arctg}\frac{(y-\frac{3}{5})\cdot 5}{4} = \frac{1}{2}\operatorname{arctg}\left(\frac{5\,tg\frac{x}{2}-3}{4}\right)$$