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(15.1) $$\int \frac{x^5+2x^3-4x^2-x+11}{x^2}\,dx = \int x^3+2x-4-\frac{1}{x}+\frac{11}{x^2}\,dx =$$

$$= \frac{x^4}{4}+x^2-4x-\ln|x|-\frac{11}{x}+C$$

(15.2) $$\int \frac{(1-x)^2}{x\sqrt{x}}\,dx = \int x^{-\frac{3}{2}}\,dx - 2\int \frac{dx}{\sqrt{x}} + \int \sqrt{x}\,dx = x =$$

$$= -\frac{2}{\sqrt{x}} - 4\sqrt{x} + \frac{2x^{\frac{3}{2}}}{3} + C = \frac{2(x^2-6x-3)}{3\sqrt{x}} + C$$

(15.3) $$\int \frac{(1+x)^2}{x(1+x^2)}\,dx = \int \frac{1+2x+x^2}{x(1+x^2)}\,dx = \int\left(\frac{1+x^2}{x(1+x^2)}+\frac{2x}{x(1+x^2)}\right)dx =$$

$$= \int \frac{dx}{x} + 2\int \frac{dx}{1+x^2} = \ln|x| + \operatorname{arctg} x + C$$

(15.4) $$\int 2\cos^2\frac{x}{2}\,dx = \int (1+\cos x)\,dx = x+\sin x + C$$

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15.5 $\int (\cos 2x \sin x - \sin 2x \cos x)\,dx = \int \sin(x-2x)\,dx = \int -\sin x\,dx = \cos x + C$

15.6 $\int tg^2 x\,dx = \int \frac{\sin^2 x}{\cos^2 x}\,dx = \int \frac{1-\cos^2 x}{\cos x} = \int \frac{1}{\cos^2 x} - \int dx =$

$= tg\,x - x + C$

15.7 $\int sh\,x \cdot ch\,x\,dx = \frac{1}{2}\int sh\,2x\,d2x = \frac{1}{4} ch\,2x$

15.8 $\int \frac{3^{x+1} + e^{3x} - e^{x-1}}{e^x}\,dx = \int 3\left(\frac{3}{e}\right)^x + e^{2x} - \frac{1}{e} \cdot dx =$

$= \frac{3\left(\frac{3}{e}\right)^x}{\ln\frac{1}{e}} + \frac{1}{2}e^{2x} - \frac{1}{e}x + C.$

15.9 $\int \frac{dx}{(2x-3)^5} = \left| \begin{matrix} 2x-3=t \\ 2dx=dt \end{matrix} \right| = \frac{1}{2}\int \frac{dt}{t^5} = \frac{1}{2}\left(-\frac{1}{4}\right)\frac{1}{t^4} + C =$

$= -\frac{1}{8} \cdot \frac{1}{(2x-3)^4} + C$

15.10 $\int \sqrt[3]{(5-8x)^4}\,dx = \left| \begin{matrix} 5-8x=t \\ -8dx=dt \end{matrix} \right| = -\frac{1}{8}\int t^{\frac{4}{3}}\,dt =$

$= \underbrace{-\frac{1}{8} \cdot \frac{3t^{\frac{7}{3}}}{7}}_{56} = -\frac{3(5-8x)^{\frac{7}{3}}}{56} + C.$

(15.11) $\int e^{-3x+1}dx = \left| \begin{matrix} -3x+1 = t \\ -3dx = dt \end{matrix} \right| = -\frac{1}{3}\int e^t dt = -\frac{1}{3}e^{-3x+1} + C$

(15.12) $\int \sin(2x-3)dx = \left| \begin{matrix} 2x-3 = t \\ 2dx = dt \end{matrix} \right| = \frac{1}{2}\int \sin t\, dt = -\frac{1}{2}\cos t =$

$= -\frac{\cos(2x-3)}{2} + C$

(15.13) $\int \frac{dx}{2x^2+9} = \frac{1}{9}\int \frac{dx}{1+\frac{2}{9}x^2} = \left| \begin{matrix} t = \sqrt{\frac{2}{9}}x \\ \sqrt{\frac{2}{9}}dx = dt \end{matrix} \right| =$

$= \frac{\sqrt{9}}{9\sqrt{2}}\int \frac{dt}{1+t^2} = \frac{1}{\sqrt{18}}\operatorname{arctg}\sqrt{\frac{2}{9}}x + C$

(15.14) $\int \frac{dx}{\sqrt{4-9x^2}} = \left| \begin{matrix} \frac{3}{2}x = t \\ \frac{3}{2}dx = dt \end{matrix} \right| = \frac{1}{3}\int \frac{2}{\sqrt{4-4t^2}}dt =$

$= \frac{1}{3}\int \frac{1}{\sqrt{1-t^2}}dt = \frac{1}{3}\arcsin t = \frac{\arcsin\frac{3x}{2}}{3}$

(15.15) $\int_{x_0}^{x} \frac{t^3}{t^2-4}dt = \left| \begin{matrix} y = t^2-4 \\ dy = 2tdt \end{matrix} \right| = \frac{1}{2}\int \frac{y+4}{y}dy =$

$= \frac{1}{2}\int \left(\frac{4}{y}+1\right)dy = \frac{1}{2}\left(4\int\frac{1}{y}dy + \int dy\right) = \frac{1}{2}\cdot 4\cdot \ln y + \frac{y}{2} =$

$= 2\ln y + \frac{y}{2} = 2\ln(t^2-4) + \frac{t^2-4}{2} + C.$

$+C$

15.16 $\int_{x_0}^{x} \frac{t}{\sqrt{b^2t^2+a^2}}\,dt = \left| \begin{array}{l} y=b^2t^2+a^2 \\ dy=2b^2t\cdot dt \end{array}\right| =$

$= \frac{1}{2b^2}\int\frac{1}{\sqrt{y}}\,dy = \frac{1}{\cancel{2}b^2}\cdot\cancel{2}\sqrt{y} = \frac{\sqrt{y}}{b^2} = \frac{\sqrt{b^2t^2+a^2}}{b^2}+C.$

15.17 $\int_{x_0}^{x} t^4\sin(t^5+3)\,dt = \left| \begin{array}{l} y=t^5+3 \\ dy=5t^4dt \end{array}\right| = \frac{1}{5}\int \sin y\,dy =$

$= \frac{1}{5} -\cos y = -\frac{\cos(t^5+3)}{5}+C$

15.18 $\int_{x_0}^{x} t\sqrt{b^2t^2+a^2}\,dt = \left| \begin{array}{l} y=b^2t^2+a^2 \\ dy=2b^2t\,dt \end{array}\right| = \frac{1}{2b^2}\int\sqrt{y}\,dy =$

$= \frac{1}{\cancel{2}b^2}\cdot\frac{\cancel{2}y^{\frac{3}{2}}}{3} = \frac{\sqrt{(b^2t^2+a^2)^3}}{3b^2}+C$

15.19 $\int_{x_0}^{x} \frac{1}{t^3}\cdot e^{-\frac{1}{t^2}}\,dt = \left| \begin{array}{l} y=-\frac{1}{t^2} \\ dy=\frac{2}{t^3}dt \end{array}\right| =$

$= \frac{1}{2}\int_{x_0}^{x} e^y\,dy = \frac{1}{2}e^y = \frac{e^{-\frac{1}{t^2}}}{2}+C$

(15.20) $\int_{x_0}^{x} e^{\sin t} \cdot \cos t \, dt = \left| \begin{matrix} y = \sin t \\ dy = \cos t \, dt \end{matrix} \right| =$

$= \int e^{y} dy = e^{y} = e^{\sin t} + C$

(15.21) $\int_{x_0}^{x} \frac{dt}{t \ln t} = \left| \begin{matrix} y = \ln t \\ t\,dy = dt \end{matrix} \right| = \int_{x_0}^{x} \frac{1}{y} \, dy = \ln y =$

$= \ln(\ln t) + C.$

(15.22) $\int_{x_0}^{x} \frac{dt}{t \ln t \ln\ln t} = \left| \begin{matrix} y = \ln \ln t \\ dy = \frac{1}{t \ln t} dt \end{matrix} \right| = \int_{x_0}^{x} \frac{1}{y} dy =$

$= \ln y = \ln \ln \ln t + C.$