Ob

307

$$\begin{vmatrix} x_1 & a_1b_2 & a_1b_3 & \dots & a_1b_n \\ a_2b_1 & x_2 & a_2b_3 & \dots & a_2b_n \\ a_3b_1 & a_3b_2 & x_3 & \dots & a_3b_n \\ \dots & \dots & \dots & \dots & \dots \\ a_nb_1 & a_nb_2 & a_nb_3 & \dots & x_n \end{vmatrix} =$$

$$= \begin{vmatrix} x_1 - a_1b_1 & a_1b_2 & \dots & a_1b_n \\ a_2b_1 - a_1b_1 & x_2 & \dots & a_2b_n \\ a_3b_1 - a_1b_1 & a_3b_2 & \dots & a_3b_n \\ \dots & \dots & \dots & \dots \\ a_nb_1 - a_1b_1 & a_nb_2 & \dots & x_n \end{vmatrix} + \begin{vmatrix} a_1b_1 & a_1b_2 & \dots & a_1b_n \\ a_2b_1 & x_2 & \dots & a_2b_n \\ a_3b_1 & a_3b_2 & \dots & a_3b_n \\ \dots & \dots & \dots & \dots \\ a_nb_1 & a_nb_2 & \dots & x_n \end{vmatrix} =$$

$$= (x_1 - a_1b_1)(x_2 - a_2b_2) \dots (x_n - a_nb_n) \cdot \left(1 + \frac{a_1b_1}{x_1} + \frac{a_2b_2}{x_2} + \dots + \frac{a_nb_n}{x_n}\right).$$

331

$$\begin{vmatrix} (x+a_1)^n & (x+a_1)^{n-1} & \dots & x+a_1 & 1 \\ (x+a_2)^n & (x+a_2)^{n-1} & \dots & x+a_2 & 1 \\ \dots & \dots & \dots & \dots & \dots \\ (x+a_{n+1})^n & (x+a_{n+1})^{n-1} & \dots & (x+a_{n+1}) & 1 \end{vmatrix} = \prod_{1 \le i < j \le n} (a_i - a_j)$$

332

$$\begin{vmatrix} 1 & \sin\varphi_1 & \sin^2\varphi_1 & \dots & \sin^{n-1}\varphi_1 \\ 1 & \sin\varphi_2 & \sin^2\varphi_2 & \dots & \sin^{n-1}\varphi_2 \\ \dots & \dots & \dots & \dots & \dots \\ 1 & \sin\varphi_n & \sin^2\varphi_n & \dots & \sin^{n-1}\varphi_n \end{vmatrix} = \prod_{1 \le j < i \le n} (\sin\varphi_i - \sin\varphi_j) =$$

$$= 2^{\frac{n(n-1)}{2}} \prod_{1 \le j < i \le n} \left(\cos\frac{\varphi_i - \varphi_j}{2} \sin\frac{\varphi_i + \varphi_j}{2}\right)$$

556

$$\begin{cases} 2x_1 + 5x_2 + 4x_3 + x_4 = 20 \\ x_1 + 3x_2 + 2x_3 + x_4 = 11 \\ 2x_1 + 10x_2 + 9x_3 + 7x_4 = 40 \\ 3x_1 + 8x_2 + 9x_3 + 2x_4 = 37 \end{cases}$$

$\Delta \neq 0 \Rightarrow x_i = \frac{\Delta_i}{\Delta}$

$$\Delta = \begin{vmatrix} 2 & 5 & 4 & 1 \\ 1 & 3 & 2 & 1 \\ 2 & 10 & 9 & 7 \\ 3 & 8 & 9 & 2 \end{vmatrix} \overset{-2\text{I}}{=} \begin{vmatrix} 2 & 5 & 0 & 1 \\ 1 & 3 & 0 & 1 \\ 2 & 10 & 5 & 7 \\ 3 & 8 & 3 & 2 \end{vmatrix} \overset{-2\text{II}}{=}$$

$=$

$$2\begin{vmatrix} 0 & -1 & 0 & -1 \\ 1 & 3 & 0 & 1 \\ 2 & 10 & 5 & 7 \\ 3 & 8 & 3 & 2 \end{vmatrix} = 2(-1)^3(-1)\begin{vmatrix} 1 & 0 & 1 \\ 2 & 5 & 7 \\ 3 & 3 & 2 \end{vmatrix} + (-1)^5\cdot(-1)\begin{vmatrix} 1 & 3 & 0 \\ 2 & 10 & 5 \\ 3 & 8 & 3 \end{vmatrix} =$$

$$= 2(10+6-15-25+30+48-8-40) = -3$$

$$\Delta_1 = \begin{vmatrix} 20 & 5 & 4 & 1 \\ 11 & 3 & 2 & 1 \\ 40 & 10 & 9 & 7 \\ 29 & 8 & 3 & 2 \end{vmatrix} = \begin{vmatrix} 0 & 1 & 3 & 1 \\ 11 & 1 & 1 & 1 \\ 0 & 1 & 2 & 7 \\ 5 & -1 & 7 & 2 \end{vmatrix} =$$

-4II -3III $-\text{IV}$ $+5\text{II}$

$$= (-1)^3(-3)\begin{vmatrix} 1 & 3 & 1 \\ 1 & 2 & 7 \\ 4 & 12 & 7 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 1 \\ 1 & -1 & 7 \\ 4 & 0 & 7 \end{vmatrix} = -7+4 = -3$$

-3I

$$\Delta_2 = \begin{vmatrix} 2 & 20 & 4 & 1 \\ 1 & 11 & 2 & 1 \\ 2 & 40 & 9 & 7 \\ 3 & 37 & 9 & 2 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 4 & 1 \\ 0 & 1 & 2 & 1 \\ -5 & -5 & 9 & 7 \\ 1 & -8 & 9 & 2 \end{vmatrix} =$$

$-\text{II}$ -6III $-\text{III}$

$$= -\begin{vmatrix} 1 & 0 & 3 & 1 \\ 0 & 1 & 1 & 1 \\ -5 & -5 & 2 & 7 \\ 1 & -8 & 7 & 2 \end{vmatrix} = 2\begin{vmatrix} 1 & 0 & 3 & 1 \\ -1 & 1 & 1 & 1 \\ 6 & -5 & 2 & 2 \\ 9 & -8 & 7 & 2 \end{vmatrix} + \text{I} = 2$$

$-\text{II}$

$$= \begin{vmatrix} 1 & 0 & 3 & 1 \\ 0 & 1 & 4 & 2 \\ 0 & -5 & 2 & 7 \\ 9 & -8 & 7 & 2 \end{vmatrix} = \begin{vmatrix} 1 & 4 & 2 \\ -5 & 2 & 7 \\ -8 & 7 & 2 \end{vmatrix} - 9 \begin{vmatrix} 0 & 3 & 1 \\ 1 & 4 & 2 \\ -5 & 2 & 7 \end{vmatrix} =$$

$$= 4 - 70 - 224 + 32 + 40 - 48 - 9(2 - 30 + 20 - 21) = -288 + 252 = -6$$

$$\Delta_3 = \begin{vmatrix} 2 & 5 & 20 & 1 \\ 1 & 3 & 11 & 1 \\ 2 & 10 & 40 & 7 \\ 3 & 8 & 37 & 2 \end{vmatrix} \begin{matrix} \\ \\ \\ -2\text{I} - 4\text{II} - \text{I} \end{matrix} = \begin{vmatrix} 2 & 1 & 0 & -1 \\ 1 & 1 & -1 & 0 \\ 2 & 6 & 0 & 5 \\ 3 & 2 & 5 & -1 \end{vmatrix} \begin{matrix} \\ \\ \\ +5\text{II} \end{matrix} =$$

$$= \begin{vmatrix} 2 & 1 & 0 & -1 \\ 1 & 1 & -1 & 0 \\ 2 & 6 & 0 & 5 \\ 8 & 7 & 0 & -1 \end{vmatrix} = \begin{vmatrix} 2 & 1 & -1 \\ 2 & 6 & 5 \\ 8 & 7 & -1 \end{vmatrix} =$$

$$= \begin{vmatrix} 0 & 2 & -1 \\ 1 & 1 & 5 \\ 0 & -6 & -1 \end{vmatrix} = -8 + 2 = -6$$

$$\Delta_4 = \begin{vmatrix} 2 & 5 & 4 & 20 \\ 1 & 3 & 2 & 11 \\ 2 & 10 & 9 & 40 \\ 3 & 8 & 9 & 37 \end{vmatrix} \begin{matrix} \\ \\ \\ -2\text{I} \quad -\text{II} \quad -5\text{II} \end{matrix} = \begin{vmatrix} 2 & 1 & -1 & 0 \\ 1 & 1 & -1 & 1 \\ 2 & 6 & -1 & -5 \\ 3 & 2 & 1 & -3 \end{vmatrix} \begin{matrix} -\text{II} \\ \\ \\ \\ \end{matrix} =$$

$$= 2\begin{vmatrix} 1 & 0 & 0 & -2 \\ 1 & 1 & -1 & 1 \\ 2 & 6 & -1 & -5 \\ 3 & 2 & 1 & -8 \end{vmatrix} = 2\cdot\begin{vmatrix} 1 & 0 & 0 & -2 \\ 0 & 0 & -1 & 0 \\ 1 & 5 & -1 & -6 \\ 4 & 3 & -1 & -7 \end{vmatrix} =$$

$$= 2\begin{vmatrix} 1 & 0 & -1 \\ 1 & 5 & 6 \\ 4 & 3 & -1 \end{vmatrix} = -35 - 3 + 20 + 18 = 0$$

$x_1 = 1 \qquad x_2 = 2 \qquad x_3 = 2 \qquad x_4 = 0$

566

$$\left(\begin{array}{cccc|c} 4 & -3 & 3 & 5 & 7 \\ 1 & -2 & -2 & -3 & 3 \\ 3 & -1 & 2 & 1 & -1 \\ 2 & 3 & 2 & -3 & -7 \end{array}\right) = 2\left(\begin{array}{cccc|c} 1 & -2 & -2 & -3 & 3 \\ 2 & 3 & 2 & -3 & -7 \\ 3 & -1 & 2 & 1 & -1 \\ 4 & -3 & 3 & 5 & 7 \end{array}\right) \begin{matrix} \\ -2\text{I} \\ -3\text{I} \\ -4\text{I} \end{matrix} =$$

$$= 2\left(\begin{array}{cccc|c} 1 & -2 & -2 & -3 & 3 \\ 0 & 7 & 6 & -2 & -13 \\ 0 & 5 & 8 & 10 & -10 \\ 0 & 5 & 11 & 17 & -5 \end{array}\right) \begin{matrix} \\ -\text{IV} \\ -\text{II} \\ \\ \end{matrix} = 2\left(\begin{array}{cccc|c} 1 & -2 & -2 & -3 & 3 \\ 0 & 2 & -5 & -15 & -8 \\ 0 & -1 & 2 & 12 & 3 \\ 0 & 5 & 11 & 17 & -5 \end{array}\right) \begin{matrix} \\ \\ \\ -\text{II}+\text{IV} \end{matrix} =$$

$$= 2\left(\begin{array}{cccc|c} 1 & -2 & -2 & -3 & 3 \\ 0 & 1 & 18 & 44 & 6 \\ 0 & -2 & 2 & 12 & 3 \\ 0 & 2 & -5 & -19 & -8 \end{array}\right) \begin{matrix} \\ \\ +2\text{II} \\ -2\text{II} \end{matrix} = 2\left(\begin{array}{cccc|c} 1 & -2 & -2 & -3 & 3 \\ 0 & 1 & 18 & 44 & 6 \\ 0 & 0 & 38 & 100 & 15 \\ 0 & 0 & -41 & -115 & -20 \end{array}\right) \begin{matrix} \\ \\ \\ +\frac{41}{38}\text{III} \end{matrix} =$$

$$= 2\left(\begin{array}{cccc|c} 1 & -2 & -2 & -3 & 3 \\ 0 & 1 & 18 & 44 & 6 \\ 0 & 0 & 38 & 100 & 15 \\ 0 & 0 & 0 & -20+\frac{41\cdot 15}{38} & \end{array}\right)$$

$\left(-13+\frac{41-105}{33}\right)x_4 = -104+\frac{15\cdot 42}{33} = \left(-\frac{5}{1}\right)x_4$

$26x_4 = 104\left(-\frac{5}{2}\right) = 15$

$x_4 = \left(\frac{15}{2}\right)$

$x_2 + 26\cdot\frac{15}{6} = 43 = \left(-\frac{5}{2}\right)$ (26)

$x_1 - 2x_2 - 2x_3 - 3x_4 = 3$

$x_1 = \left(\frac{15}{2}\right)$

520
$$\begin{pmatrix} 1 & 1 & -6 & -4 & 6 \\ 3 & -1 & -6 & -4 & 2 \\ 2 & 3 & 9 & 2 & 6 \\ 3 & 2 & 3 & 8 & -2 \end{pmatrix} \begin{matrix} \\ -3\text{I} \\ -2\text{I} \\ -3\text{I} \end{matrix} = \begin{pmatrix} 1 & 1 & -6 & -4 & 6 \\ 0 & -4 & 12 & 8 & -16 \\ 0 & 1 & 21 & 10 & -6 \\ 0 & -1 & 21 & 20 & -25 \end{pmatrix} =$$

$$= \begin{pmatrix} 1 & 1 & -6 & -4 & 6 \\ 0 & 1 & 21 & 10 & -6 \\ 0 & -1 & 42 & 20 & -15 \\ 0 & -4 & 12 & 8 & -16 \end{pmatrix} \begin{matrix} \\ \\ +\text{II} \\ +4\text{II} \end{matrix} = \begin{pmatrix} 1 & 1 & -6 & -4 & 6 \\ 0 & 1 & 21 & 10 & -6 \\ 0 & 0 & 42 & 30 & -21 \\ 0 & 0 & 96 & 48 & -40 \end{pmatrix} -\text{III} =$$

$$= \begin{pmatrix} 1 & 1 & -6 & -4 & 6 \\ 0 & 1 & 21 & 10 & -6 \\ 0 & 0 & 12 & -12 & 22 \\ 0 & 0 & 42 & 30 & -31 \end{pmatrix} -3\text{III} = \begin{pmatrix} 1 & 1 & -6 & -4 & 6 \\ 0 & 1 & 21 & 10 & -6 \\ 0 & 0 & 12 & -12 & 22 \\ 0 & 0 & 6 & 66 & -97 \end{pmatrix} \frac{:}{2\text{II}}$$

$$= \begin{pmatrix} 1 & 1 & -6 & -4 & 6 \\ 0 & 1 & 21 & 10 & -6 \\ 0 & 0 & 6 & 66 & -97 \\ 0 & 0 & 0 & -143 & 143 \end{pmatrix}$$

$-143x_4 = 143 \Rightarrow x_4 = \left(\frac{-3}{1}\right)$

$6x_3 + 66\left(\frac{-3}{1}\right) = -97 \Rightarrow x_3 = \left(\frac{1}{3}\right)$

$x_2 + 7 + 15 = 6 \Rightarrow x_2 = (2)$

$x_1 + 2 - 2 + 6 = 6 \Rightarrow x_1 = (10)$