

Q13 21572

$$A = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & -\frac{\sqrt{2}}{2} \\ -\frac{1}{2} & \frac{1}{2} & \frac{\sqrt{2}}{2} \\ \frac{1}{2}\sqrt{2} & -\frac{\sqrt{2}}{2} & 0 \end{pmatrix}$$

$$|A - \lambda E| = \begin{vmatrix} \frac{1}{2} - \lambda & \frac{1}{2} & -\frac{\sqrt{2}}{2} \\ -\frac{1}{2} & \frac{1}{2} - \lambda & \frac{\sqrt{2}}{2} \\ \frac{1}{2}\sqrt{2} & -\frac{\sqrt{2}}{2} & -\lambda \end{vmatrix} = \begin{vmatrix} 1-\lambda & 1-\lambda & 0 \\ \frac{1}{2} & \frac{1}{2}-\lambda & \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & -\lambda \end{vmatrix}$$

$$= \begin{vmatrix} 1-\lambda & 0 & 0 \\ \frac{1}{2} & -\lambda & \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & -\sqrt{2} & -\lambda \end{vmatrix} = (1-\lambda) \begin{vmatrix} -\lambda & \frac{\sqrt{2}}{2} \\ -\sqrt{2} & -\lambda \end{vmatrix}$$

$$= -(1-\lambda)(\lambda^2)$$

$$\lambda_1 = 1$$

$$\lambda_2 = -i$$

$$\lambda_3 = i$$

$$|A - E| = \begin{pmatrix} -\frac{1}{2} & \frac{1}{2} & -\frac{\sqrt{2}}{2} \\ \frac{1}{2} & -\frac{1}{2} & \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & -1 \end{pmatrix} \times (E) = \begin{pmatrix} 1 & -1 & \sqrt{2} \\ 0 & 0 & -2 \\ 0 & 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

x_1	x_2	x_3
1	1	0

$$v_1 = \frac{1}{\sqrt{2}} (1, 1, 0)$$

$$|A + iE| = \begin{vmatrix} \frac{1}{2} + i & \frac{1}{2} & -\frac{\sqrt{2}}{2} \\ \frac{1}{2} & \frac{1}{2} + i & \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & i \end{vmatrix} = \begin{vmatrix} \frac{1+2i}{2} & \frac{1}{2} & -\frac{\sqrt{2}}{2} \\ 0 & \frac{2+6i}{5} & \frac{(1+3-i)\sqrt{2}}{5} \\ 0 & 0 & i \end{vmatrix}$$

$$= \begin{pmatrix} 1+2i & 1 & -\sqrt{2} \\ 0 & 2+6i & (3-i)\sqrt{2} \\ 0 & 0 & i \end{pmatrix}$$

$$\begin{array}{c|c|c} v_1 & v_2 & v_3 \\ \hline -\frac{i\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 1 \end{array}$$

$$v_2 = (-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}, 0)$$

$$v_3 = (0, 0, 1)$$

$$B = Q^{-1} A Q = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}$$

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$$A = \begin{pmatrix} \frac{3}{4} & \frac{1}{4} & -\frac{1}{4}\sqrt{6} \\ \frac{1}{4} & \frac{3}{4} & \frac{1}{4}\sqrt{6} \\ \frac{1}{4}\sqrt{6} & -\frac{1}{4}\sqrt{6} & \frac{1}{2} \end{pmatrix}$$

$$|A - \lambda E| = \begin{vmatrix} \frac{3}{4} - \lambda & \frac{1}{4} & -\frac{\sqrt{6}}{4} \\ \frac{1}{4} & \frac{3}{4} - \lambda & \frac{\sqrt{6}}{4} \\ \frac{\sqrt{6}}{4} & -\frac{\sqrt{6}}{4} & \frac{1}{2} - \lambda \end{vmatrix} = -\lambda^3 + 2\lambda^2 - 2\lambda + 1 =$$

$$= (\lambda - 1) \left(\lambda^2 - \frac{i\sqrt{3} + 1}{2} \right) \left(\lambda - \frac{i\sqrt{3} + 1}{2} \right) = 0$$

$$\lambda_1 = 1$$

$$\lambda_2 = \frac{i\sqrt{3}+1}{2}$$

$$\lambda_3 = \frac{i\sqrt{3}-1}{2}$$

$$|A-E| = \begin{vmatrix} 1-\frac{1}{3} & \frac{1}{3} & -\frac{\sqrt{3}}{3} \\ \frac{1}{3} & 1-\frac{1}{3} & \frac{\sqrt{3}}{3} \\ \frac{\sqrt{3}}{3} & -\frac{\sqrt{3}}{3} & 1-1 \end{vmatrix} = \begin{vmatrix} 1 & -1 & \sqrt{3} \\ 0 & 0 & -1 \end{vmatrix} = \begin{vmatrix} 1 & -1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$\begin{array}{c|c|c} x_1 & x_2 & x_3 \\ \hline 1 & 1 & 0 \end{array}$$

$$C_1 = -i(1, 1, 0)$$

$$|A - \lambda_2 E| = \begin{vmatrix} \frac{2}{3} & -\frac{i\sqrt{3}+1}{2} & \frac{1}{3} & -\frac{\sqrt{3}}{3} \\ \frac{1}{3} & \frac{2}{3} & -\frac{i\sqrt{3}+1}{2} & \frac{\sqrt{3}}{3} \\ \frac{\sqrt{3}}{3} & -\frac{\sqrt{3}}{3} & \frac{1}{2} - \frac{i\sqrt{3}+1}{2} \end{vmatrix} =$$

$$= \begin{pmatrix} \frac{2i\sqrt{3}+1}{3} & \frac{1}{3} & -\frac{\sqrt{3}}{3} \\ 0 & \frac{i\sqrt{3}+1}{13} & \frac{-3i\sqrt{3}-\sqrt{3}}{6} \end{pmatrix}$$

$$\begin{array}{c|c|c} x_1 & x_2 & x_3 \\ \hline -\frac{i\sqrt{3}}{2} & \frac{i\sqrt{3}}{2} & 1 \end{array}$$

$$l_2 = \left(-\frac{i\sqrt{3}}{2}, \frac{i\sqrt{3}}{2}, 0 \right) + (0, 0, 1) \text{ main}$$

$$n = \left(-\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}, 0 \right) \quad m = (0, 0, 1)$$

$$m \perp n \quad |m|, |n| = 1$$

$$z = \frac{1}{2} = \text{main} = x + iy$$

$$x = m = (0, 0, 1), \quad y = \left(-\frac{s_1}{l}, \frac{s_2}{l}, 0\right)$$

$$d = \frac{1}{l}, \quad \beta = -\frac{s_1}{l} \quad \text{given } \lambda$$

$$Q_2 = \begin{pmatrix} \frac{s_1}{l} & \frac{s_2}{l} & 0 \\ \frac{s_1}{l} & -\frac{s_2}{l} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{1}{l} & -\frac{s_1}{l} \\ \frac{s_2}{l} & \frac{1}{l} \end{pmatrix}$$

$$B = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{l} & \frac{s_1}{l} \\ 0 & -\frac{s_1}{l} & \frac{1}{l} \end{pmatrix}$$