

$$\text{P85. } \alpha = \{3; -4\}, \beta = \{1; -2\}, \gamma = \{-1; 2\}$$

$$p = \alpha + \beta + \gamma = \{3; 4\}$$

$$\rho_{31} + \beta = 3 \cdot 1 \\ d = 2 \cdot 3 = 6$$

$$5d = 10 \quad -5d = 15$$

$$d = 2 \quad \beta = -3$$

$$\bar{p}^2 = 2d - 36$$

$$\text{P86. } \bar{\alpha} = \{3; -5; 8\}, \bar{\beta} = \{-1; 2; -4\}$$

$$\alpha + \beta = \{2; -4; 4\} \quad |\alpha + \beta| = \sqrt{4+16+16} = \underline{6}$$

$$\alpha - \beta = \{4; -6; 12\} \quad |\alpha - \beta| = \sqrt{16+36+144} = \sqrt{196} = \underline{14}$$

$$\text{P87. } A(3; -1; 2), B(1; 2; -1), C(-1; 1; -3), D(3; -5; 3)$$

$$\overline{AB} = (-2; 3; -3) \quad \overline{BC} = (-2; -1; -2) \quad \overline{CD} = (4; -6; 6), \overline{DA} = (0; 4; -1)$$

$$\frac{AB}{CD} = -\frac{2}{1} = \frac{3}{-6} = \frac{-3}{6} \quad \text{④}$$

$AB \parallel CD \Rightarrow$ Paralel

$$(?) \quad \alpha = 60^\circ \quad \beta = 120^\circ \quad |\alpha| = 2$$

$$x = |\alpha| \cos \alpha = 2 \cdot 0,5 = 1$$

$$y = |\alpha| \cos \beta = 2 \cdot (-0,5) = -1$$

$$\sqrt{1+1+2^2} = 2$$

$$\frac{r}{z} = 2$$

$$z = \pm \sqrt{2} \quad \alpha = (1; -1; \sqrt{2}) \text{ und } \bar{\alpha} = (1; 1; -\sqrt{2})$$

$$\text{P88. } |\alpha|^2 = \sqrt{144+25+256} = \sqrt{615} = 25 \quad (\alpha = \{12; -5; -16\})$$

$$\cos \varphi_1 = \frac{12}{25} \quad \cos \varphi_2 = -\frac{15}{25} = -\frac{3}{5} \quad \cos \varphi_3 = -\frac{16}{25}$$

$$\text{pgs } |a|=3 \quad |b|=4 \quad \varphi = \frac{2\pi}{3}$$

$$1) ab = 3 \cdot 4 \cdot \cos \frac{2\pi}{3} = 3 \cdot 4 \cdot (-\frac{1}{2}) = -6$$

$$2) a^2 = 9$$

$$3) b^2 = 16$$

$$4) (a+b)^2 = a^2 + 2ab + b^2 = 9 + 2 \cdot (-6) + 16 = 13$$

$$5) (3a+2b)(1+2b) = 3a^2 + 6ab + 4b^2 = 3a^2 + 4ab - 4b^2$$

$$= 3 \cdot 9 + 4 \cdot (-6) - 4 \cdot 16 = 27 + (-24) - 64 = -61$$

$$6) (a-b)^2 = a^2 - 2ab + b^2 = 9 - 2 \cdot (-6) + 9b^2 = 37$$

$$7) (3a+2b)^3 = 9a^3 + 12ab + 4b^3 = 9 \cdot 9 + 12 \cdot (-6) + 4 \cdot 16 = 81 - 72 + 64 = 73$$

282 A(1; -2), B(2; 1), C(3; 2), D(-2; 3), $\vec{p} = \overrightarrow{AD} + \overrightarrow{BD} + \overrightarrow{CD}$

$$\overrightarrow{AB} = (1; 3) \quad \overrightarrow{AC} = (2; 4)$$

$$\overrightarrow{AD} : \begin{cases} 2 + 2\beta = -3 \\ 3\alpha + 4\beta = 5 \end{cases} \quad \alpha = 11 \quad \beta = -7$$

$$\overrightarrow{AD} = 11\overrightarrow{AB} - 2\overrightarrow{AC}$$

$$\overrightarrow{BD} : \begin{cases} \alpha + 2\beta = -1 \\ 3\alpha + 4\beta = 2 \end{cases} \quad \alpha = -8 \quad \beta = 7$$

$$\overrightarrow{BD} = 10\overrightarrow{AB} + 7\overrightarrow{AC}$$

$$\overrightarrow{CD} : \begin{cases} \alpha + 2\beta = -5 \\ 3\alpha + 4\beta = 1 \end{cases} \quad \alpha = 11 \quad \beta = -8$$

$$\overrightarrow{CD} = 11\overrightarrow{AB} - 8\overrightarrow{AC}$$

$$\overrightarrow{p} : \begin{cases} \alpha + 2\beta = -12 \\ 3\alpha + 4\beta = 8 \end{cases} \quad \alpha = 32 \quad \beta = -22$$

$$\overrightarrow{p} = 32\overrightarrow{AB} - 22\overrightarrow{AC}$$

$$4) (\overrightarrow{AB} \cdot \overrightarrow{AC}) \overrightarrow{BC}$$

$$\overrightarrow{AB} \cdot \overrightarrow{AC} = 3 + 8 + 4 \cdot 35$$

$$\overrightarrow{AB} (\overrightarrow{AC} \cdot \overrightarrow{BC})$$

$$\overrightarrow{AC} \cdot \overrightarrow{BC} = -2 \cdot -4 \cdot -20 = -16$$

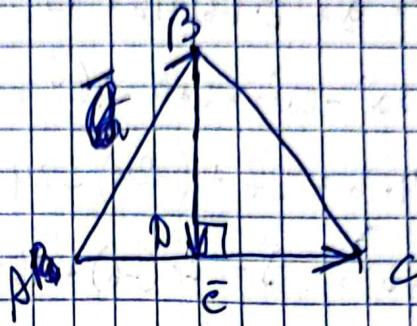
$$35 \cdot \{-2; 2; -10\} =$$

$$= \{-80; 70; -350\}$$

$$-16 \cdot \{3; -4; 12\} = \{-96; 104; -312\}$$

$$307. \quad \overrightarrow{AB} = a$$

$$\overrightarrow{AC} = c$$



$$\overrightarrow{BD} = -\overrightarrow{b} + \overrightarrow{c} \cdot k$$

~~Maximieren~~

$$(\overrightarrow{kc} - \overrightarrow{b}) \cdot \overrightarrow{c} = 0$$

$$k = \frac{\overrightarrow{bc}}{\overrightarrow{c}^2}$$

$$\overrightarrow{BD} = \frac{\overrightarrow{bc}}{\overrightarrow{c}^2} \cdot \overrightarrow{c} = \overrightarrow{b}$$

$$302. |a|=4, |b|=2, |c|=6$$

$$(\hat{a}, \hat{b}) = (\hat{b}, \hat{c}) \neq (\hat{a}, \hat{c}) < 60^\circ \quad p = a+b+c$$

$$p^2 = (a+b+c)^2 = \overline{a}^2 + \overline{b}^2 + \overline{c}^2 + 2ab + 2ac + 2bc = 16 + 4 + 36 +$$

$$+ 2 \cdot 4 \cdot 2 \cdot \frac{1}{2} + 2 \cdot 2 \cdot 6 \cdot \frac{1}{2} + 2 \cdot 4 \cdot 6 \cdot \frac{1}{2} = 56 + 8 + 12 + 24 = 100$$

$$\sqrt{p^2} = 10$$

$$p = 10$$

$$\begin{cases} 2x_1 - y_1 + 3z_1 = -5 \\ x_1 - 3y_1 + 2z_1 = -11/|n| \\ 3x_1 + 2y_1 - 4z_1 = 20 \end{cases}$$

$$\begin{cases} 9x_1 - 3z_1 = 24 & | :3 \\ -6y_1 + 9z_1 = -36 & | :3 \\ 6x_1 + 3y_1 = 21 & | :3 \end{cases}$$

$$x_1 = x = 2$$

$$y = 3$$

$$z = -2$$

$$x = 2i + 3j - 2k$$

84. A(-1; 3; -2), B(2; -1; 5), C(0; 1; -5)

$$\overline{AB} (3; -4; 12) \quad \overline{AC} (1; -2; 2)$$

$$\overline{CB} (2; -2; 10)$$

$$\overline{BC} (-2; 2; -10)$$

$$\overline{BA} (-3; 4; -12)$$

$$\begin{aligned} 1) (2\overline{AB} - \overline{CB})(2\overline{BC} + \overline{BA}) &= 4\overline{AB} \cdot \overline{BC} - 2\overline{BC} \cdot \overline{CB} + 2\overline{AB} \cdot \overline{BA} - \\ &- \overline{CB} \cdot \overline{BA} = 4(-6 - 8 - 120) - 2(-4 - 4 - 100) + 2(-8 - 16 - 144) - \\ &- (-120 - 6 - 8 - 120) = 4 \cdot (-134) + 2 \cdot (-108) + 2 \cdot (-169) - 134 \cdot \\ &\sim -536 + 216 - 338 - 134 = -792 \end{aligned}$$

$$2) \sqrt{\overline{AB}^2} = \sqrt{9 + 16 + 144} = \sqrt{169} = 13$$

$$3) \sqrt{\overline{AC}^2} = \sqrt{1+4+1} = \sqrt{6} = \sqrt{2} \cdot \sqrt{3}$$

$$2) \begin{vmatrix} 4\sin x & 1 \\ 1 & \cos x \end{vmatrix} = 0$$

$$4\sin x \cos x - 1 = 0$$

$$2(2\sin x \cos x) = 1 = 0$$

$$2(\sin 2x + \sin 2x) = 1 = 0$$

$$4\sin 2x = 1 = 0$$

$$\sin 2x = \frac{1}{4}$$

~~2 sin 2x = 1/4~~

$$2x = \arcsin\left(\frac{1}{4}\right) + \pi n, n \in \mathbb{Z}$$

$$x = \frac{\arcsin\left(\frac{1}{4}\right)}{2} + \frac{\pi}{2}n, n \in \mathbb{Z}$$

83B A(-2; 3; -4), B(3; 2; 5), C(1; -1; 2), D(3; 2; -4).

np_{CD} AB - ?

$$\overline{AB}(5; -1; 9) \quad \overline{CD}(2; 3; -6)$$

$$\overline{AB} \cdot \overline{CD} \cdot \sin \angle ABD = 10 - 3 = 5 = -42$$

$$|\overline{CD}| = \sqrt{4+9+36} = \sqrt{49} = 7$$

$$\text{np}_{\overline{CD}} \overline{AB} = \frac{\overline{AB} \cdot \overline{CD}}{|\overline{CD}|} = -\frac{42}{7}$$

83B a = 2i - j + 3k =

$$b = i - 3j + 2k$$

$$c = 3i + 2j - 4k.$$

$$x \{ x \in \mathbb{N}, z^2 \}$$

$$8) \begin{vmatrix} \cos 8x & -\sin 5x \\ \sin 8x & \cos 5x \end{vmatrix} = 0$$

$$\cos 8x \cos 5x + \sin 5x \sin 8x = 0$$

$$\cos(8x - 5x) = 0$$

$$\cos 3x = 0$$

$$3x = \frac{\pi}{2} + \pi n, n \in \mathbb{Z}$$

$$x = \frac{\pi}{6} + \frac{\pi}{3}n, n \in \mathbb{Z}.$$

12.2.

$$\left| \begin{array}{ccc|c} 1 & 2 & 0 \\ 0 & 1 & 3 \\ 5 & 0 & -1 \end{array} \right| \quad 1 \cdot 1 \cdot (-1) + 2 \cdot 3 \cdot 5 + 0 \cdot 0 \cdot 0 = 0 - 0 + 30 = 30$$

$$= -1 + 30 = 29$$

12.5

$$1) \left| \begin{array}{cc|c} 2 & x-4 \\ 1 & 4 \end{array} \right| = 0$$

$$2 \cdot 4 - (x-4) \cdot 1 = 0$$

$$8 - x + 4 = 0$$

$$\underline{12 = x}$$

$$2) \left| \begin{array}{cc|c} 1 & 4 \\ 3x & x+2x \end{array} \right| = 0$$

$$x+2x - 12x = 0$$

$$11x = 0$$

$$\underline{x = 0}$$

$$3) \left| \begin{array}{cc|c} x & x+1 \\ -4 & x+2 \end{array} \right| = 0$$

$$x^2 + x - 4x - 4 = 0$$

$$x^2 - 3x - 4 = 0$$

$$x_1 = 4 \quad x_2 = -1$$

$$4) \left| \begin{array}{cc|c} 3x & 1 \\ x & 2x-3 \end{array} \right| = \frac{3}{2}$$

$$6x^2 - 9x + x = \frac{3}{2}$$

$$6x^2 - 8x - \frac{3}{2} = 0 \quad | \cdot 2$$

$$\cancel{12x^2 - 16x - 3 = 0}$$

$$x_1 = \frac{9}{6} \quad x_2 = -\frac{1}{6}$$

$$5) \left| \begin{array}{cc|c} x+1 & -5 \\ 1 & x-1 \end{array} \right| = 0$$

$$x^2 + x - x - 1 + 5 = 0$$

$$x^2 + 4 = 0$$

$$\cancel{x^2 = 0}$$

$$6) \left| \begin{array}{cc|c} x^2-4 & -1 \\ x+4 & x+2 \end{array} \right| = 0$$

$$x^3 - 4x + x^2 - 8 + x + 4 = 0$$

$$x^3 + x^2 - 3x - 12 = 0$$

$$\cancel{x = 0}$$

$$\text{121. } \begin{array}{|ccc|} \hline & -2 & 1 & 3 \\ \hline & 2 & 0 & -2 \\ \hline \end{array} \mid = 3 \cdot 1 \cdot (-2) + (-2) \cdot 0 \cdot 1 +$$

D/3

$$\text{123S. 1) } \begin{array}{|ccc|} \hline & 3 & -2 & 1 \\ \hline & 1 & x & -2 \\ \hline & 1 & -1 & 2 & -1 \\ \hline \end{array} \mid < 1$$

$$3 \cdot x \cdot (-1) + (-2) \cdot (-2) \cdot (-1) + 1 \cdot 1 \cdot (-1) - 1 \cdot x \cdot (-1) + (-2) \cdot 1 \cdot (-1) - 3 \cdot (-2) \cdot 2 \leq 1 \text{ false}$$

$$-3x - 4 - 1 + x - 2 + 12 < 1$$

$$-2x < 1 + 4 + 1 + 2 - 12$$

$$-2x < -4$$

$$2x > 4$$

$$\underline{x > 2}$$

$$\text{2) } \begin{array}{|ccc|} \hline & 2 & x+1 & -1 \\ \hline & 1 & 1 & -2 \\ \hline & 5 & -3 & x \\ \hline \end{array} \mid \geq 0$$

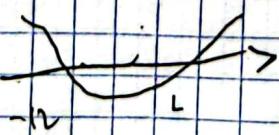
$$2 \cdot 1 \cdot x + (x+2) \cdot (-2) \cdot 5 + (-1) \cdot 1 \cdot (-3) - (-1) \cdot 1 \cdot 5 - (x+2) \cdot 1 \cdot x - 2 \cdot (-2) \cdot (-1)$$

$$-2x - 10x - 20 + 3 + 5 - x^2 - 2x - 12 \geq 0$$

$$-x^2 - 14x - 24 \geq 0$$

$$x^2 + 14x + 24 \leq 0$$

$$x_1 = -12 \quad x_2 = -2$$



$$\times g(-12, 2)$$