

(B.1) $f(x) = \sqrt[3]{1+x}, n=5$

$$f(x) = (1+x)^{\frac{1}{3}} = 1 + \frac{1}{3}x + \frac{\frac{1}{3}(\frac{1}{3}-1)}{2!}x^2 + \frac{\frac{1}{3}(-\frac{2}{3})(-\frac{2}{3}-1)}{3!}x^3 + \frac{\frac{1}{3}(-\frac{2}{3})(\frac{5}{3})(-\frac{5}{3}-1)}{4!}x^4 + \frac{\frac{1}{3}(\frac{2}{3}) \cdot \frac{5}{3} \cdot (-\frac{8}{3}) \cdot (-\frac{8}{3}-1)}{5!}x^5 + \dots$$

$$+ O(x^5) = 1 + \frac{1}{3}x - \frac{1}{9}x^2 + \frac{5}{81}x^3 - \frac{10}{243}x^4 + \frac{25}{729}x^5 + O(x^5)$$

(B.3) $n=5$ $f(x) = \frac{1}{1+x} = (1+x)^{-1} = 1 - x + \frac{-1(-2)}{2!}x^2 + \frac{2(-3)}{6!}x^3 +$

$$+ \frac{2(-3)(-4)}{4!}x^4 + \frac{2 \cdot 12 \cdot (-5)}{5!}x^5 + O(x^5) = 1 - x + x^2 - x^3 + x^4 - x^5 + O(x^5)$$

(B.5) $n=5$ $f(x) = \cosh(x) = \frac{e^x + e^{-x}}{2} = \frac{1+x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} - \frac{x^5}{5!}}{2}$

$$= \frac{2 + 2\frac{x^2}{2!} + 2\frac{x^4}{4!}}{2} = 1 + \frac{x^2}{2} + \frac{x^4}{24} + O(x^5)$$

$$(13.6) \quad f(x) = \sinh(x) = \frac{e^x - e^{-x}}{2} = \frac{1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} - 1 + x - \frac{x^2}{2} + \frac{x^3}{3!} - \frac{x^4}{4!} + \frac{x^5}{5!}}{2}$$

$$= \frac{2x + 2\frac{x^3}{3!} + 2\frac{x^5}{5!}}{2} = x + \frac{x^3}{3!} + \frac{x^5}{5!} + o(x^5)$$

$$(13.9) \quad f(x) = e^{2x-x^2} = 1 + (2x-x^2) + \frac{(2x-x^2)^2}{2!} + \frac{(2x-x^2)^3}{3!} + \frac{(2x-x^2)^4}{4!} + \frac{(2x-x^2)^5}{5!} + o(x^5)$$

$$= 1 + 2x - x^2 + \frac{4x^2 - 4x^3 + x^4}{2} + \frac{8x^3 - 12x^4 + 6x^5}{6} + \frac{16x^4 - 32x^5}{24} + \frac{32x^5}{120} + o(x^5)$$

$$= 1 + 2x - x^2 + 2x^2 - 2x^3 + \frac{x}{2} + 2x^3 - 2x^4 + \frac{4}{3}x^3 + \frac{2}{3}x^4 - \frac{4}{3}x^5 + \frac{4}{15}x^5 + o(x^5) = 1 + 2x + x^2 - \frac{2}{3}x^3 - \frac{5x^4}{6} - \frac{x^5}{15} + o(x^5)$$

$$(13.10) \quad f(x) = \sin(\sin x)$$

$$g(x) = \sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} + o(x^5)$$

$$\sin(g(x)) = \left(x - \frac{x^3}{3!} + \frac{x^5}{5!}\right) - \frac{\left(x - \frac{x^3}{3!} + \frac{x^5}{5!}\right)^3}{3!} + \frac{\left(x - \frac{x^3}{3!} + \frac{x^5}{5!}\right)^5}{5!} + o(x^5)$$

$$= x - \frac{x^3}{6} + \frac{x^5}{120} - \frac{x^3 - \frac{1}{2}x^5 + o(x^5)}{6} + \frac{x^5 + o(x^5)}{120} = x - \frac{x^3}{6} + \frac{x^5}{120} - \frac{x^3}{6} + \frac{x^5}{120} + \frac{x^5}{120} + o(x^5) = x - \frac{x^3}{3} + \frac{x^5}{10} + o(x^5)$$

$$(13.13) \quad \lim_{x \rightarrow 0} \frac{e^x \sin x - x(1+x)}{3x^3} = \lim_{x \rightarrow 0} \frac{(1+x+\frac{x^2}{2})(x-\frac{x^3}{6}) - x - x^2}{3x^3}$$

$$= \lim_{x \rightarrow 0} \frac{x + x^2 + \frac{x^3}{6} - \frac{x^3}{6} - x - x^2}{3x^3} = \frac{\frac{x^3}{6}}{3x^3} = \frac{1}{18}$$