Д/З 2

№4. $\sum_{k=1}^{n} \frac{1}{k^2} \le 2 - \frac{1}{n}$

При $n=1$ $\quad \frac{1}{1} \le 2-1$ ⊕

Для $n+1$: $\sum_{k=1}^{n+1} \frac{1}{k^2} \le 2 - \frac{1}{n+1} + \frac{1}{(n+1)^2}$

$$\sum_{k=1}^{n} \frac{1}{k^2} + \frac{1}{(k+1)^2} \le 2 - \frac{1}{n+1} + \frac{1}{(n+1)^2} = \frac{2(n+1)^2 - n - 1 + 1}{(n+1)^2} =$$

$$= \frac{2n^2+4n+1-n}{(n+1)^2} = \frac{2n^2+3n+1}{(n+1)^2}$$

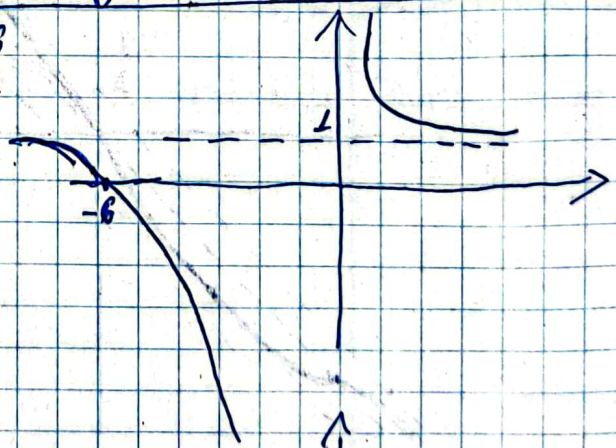
$$\frac{2n^2+3n+1}{(n+1)^2} - \frac{(2n-1)(n+1)^2+n}{n(n+1)^2} = \frac{2(n+1)^2-n}{(n+1)^2} - \frac{(2n-1)(n+1)^2+n}{n(n+1)^2} =$$

$$= \frac{2n(n+1)^2 - n^2 - (2n-1)(n+1)^2 + n}{n(n+1)^2} = \frac{(n+1)^2(2n-(2n-1)) - n}{n(n+1)^2} =$$

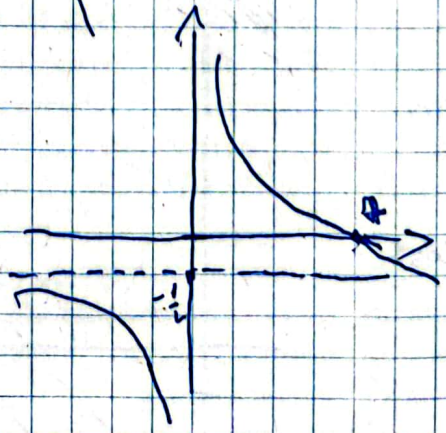
$$= \frac{(n+1)^2 - n(n-1)}{n(n+1)^2} = \frac{n^2+2n+1-n^2}{n(n+1)^2} = \frac{(n+1)}{n(n+1)} = \frac{1}{n(n+1)} \ge 0$$

Що і треба було довести

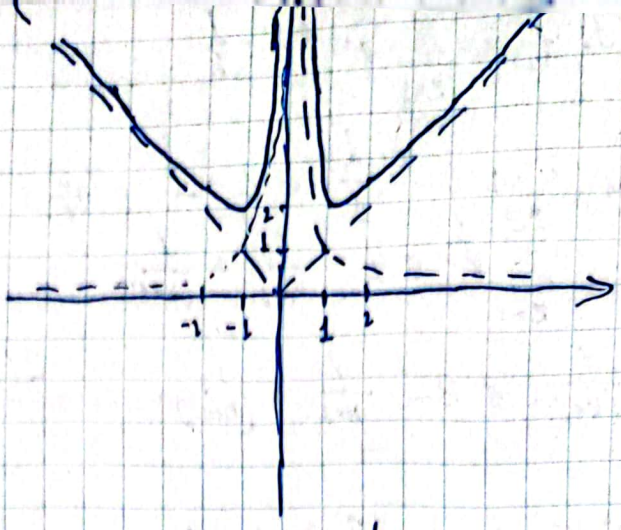
№2.1 $f(x) = \frac{6+x}{x} = 1 + \frac{6}{x}$



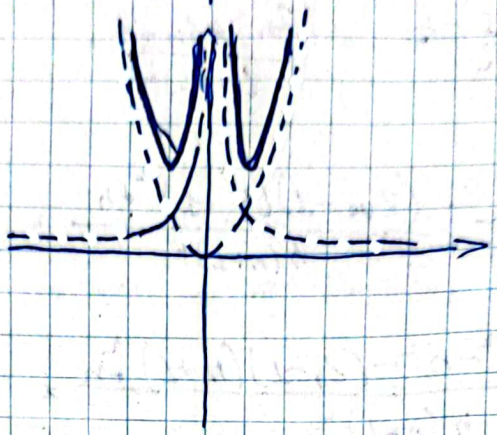
№2 $y = \frac{7-x}{2x} = -\frac{1}{2} + \frac{7}{2x}$



23 $f(x) = |x| + \frac{1}{|x|}$

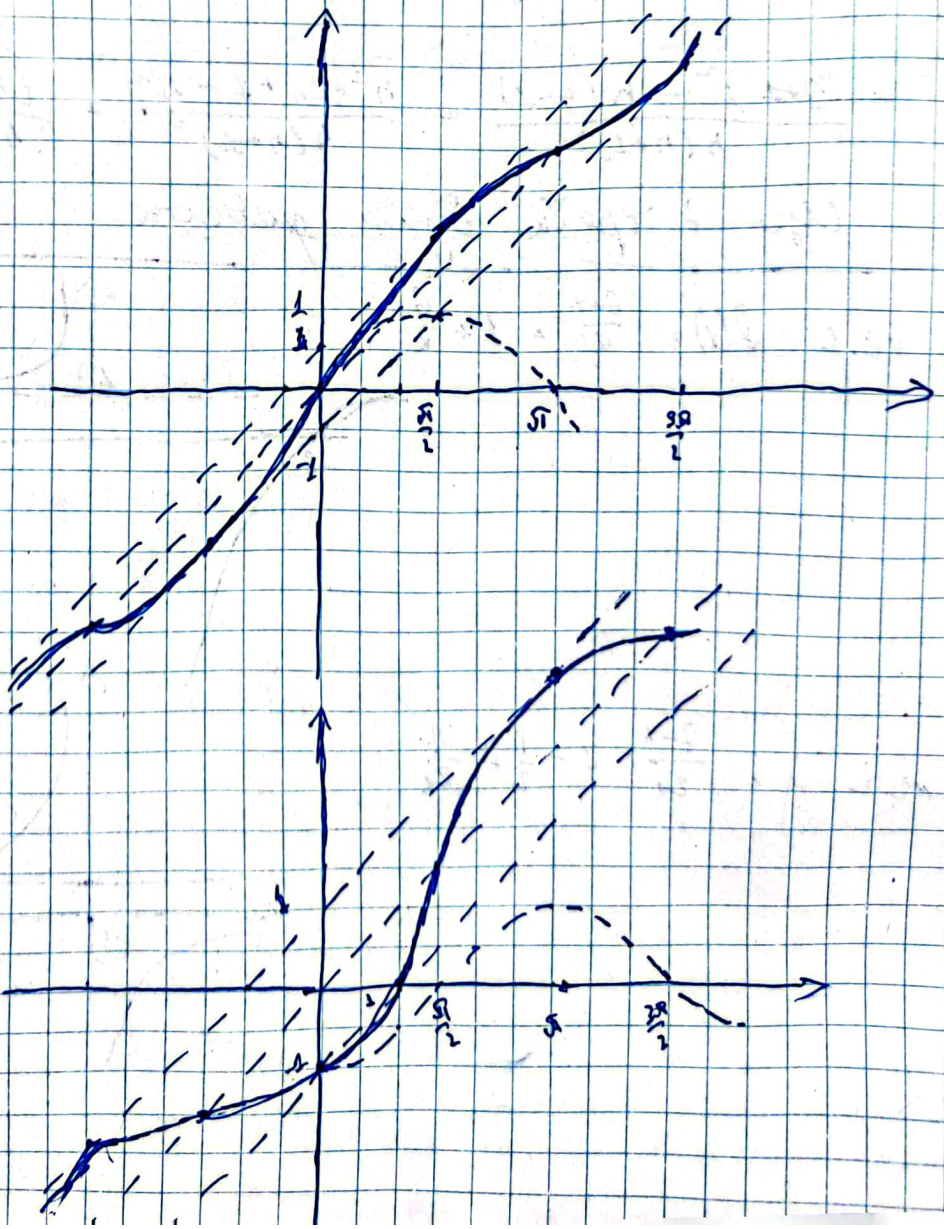


24. $f(x) = x^2 + \frac{1}{|x|}$



25. $y = x + \sin x$

$x - 1 < y < x + 1$

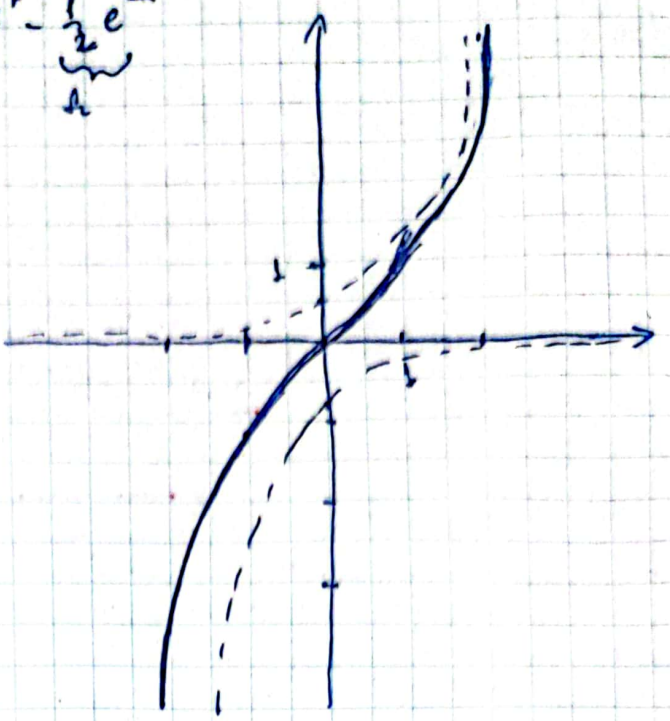


26. $f(x) = x - \cos x$

2.2 $f(x) = \operatorname{sh} x = \frac{e^x - e^{-x}}{2} = \underbrace{\frac{1}{2}e^x}_{f_1} - \underbrace{\frac{1}{2}e^{-x}}_{f_2}$

x	-2	-1	0	1	2
$f_1(x)$	0,07	0,19	0,5	1,35	3,65
$f_2(x)$	-3,65	-1,35	-0,5	-0,19	-0,07

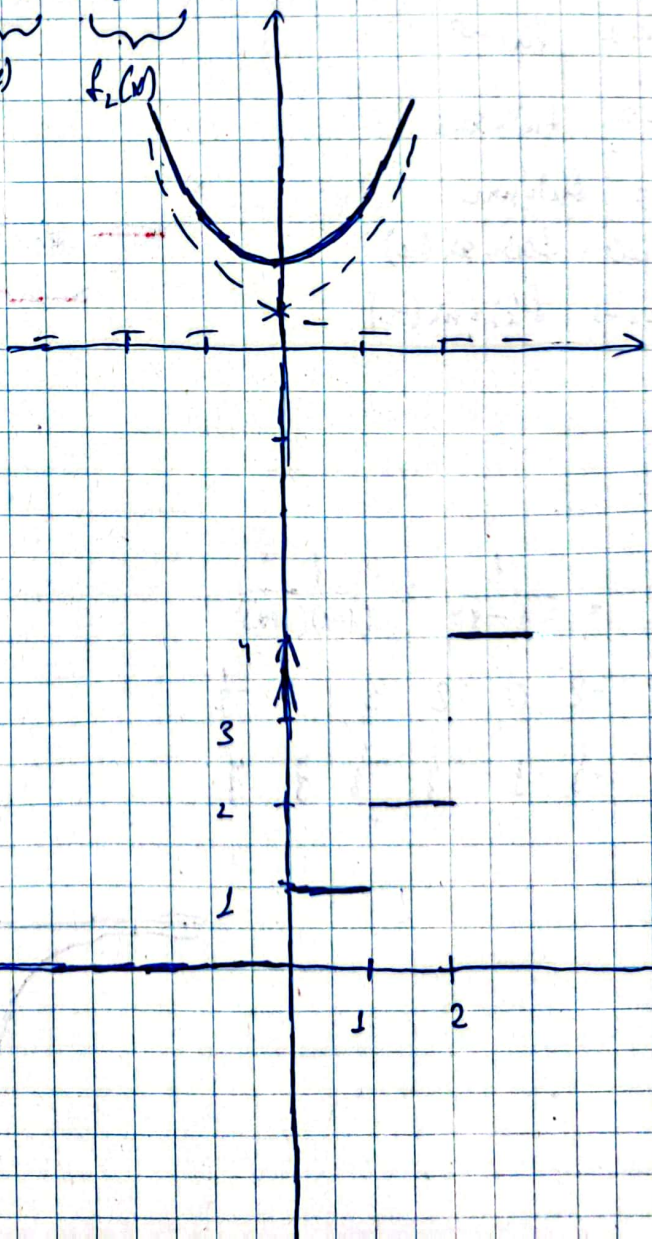
-3,58 -1,16 0 1,54 3,72



2.8 $f(x) = \operatorname{ch} x = \frac{e^x + e^{-x}}{2} = \underbrace{\frac{1}{2}e^x}_{f_1(x)} + \underbrace{\frac{1}{2}e^{-x}}_{f_2(x)}$

x	-2	-1	0	1	2
$f_1(x)$	0,07	0,19	0,5	1,35	3,65
$f_2(x)$	3,65	1,35	0,5	0,19	0,07

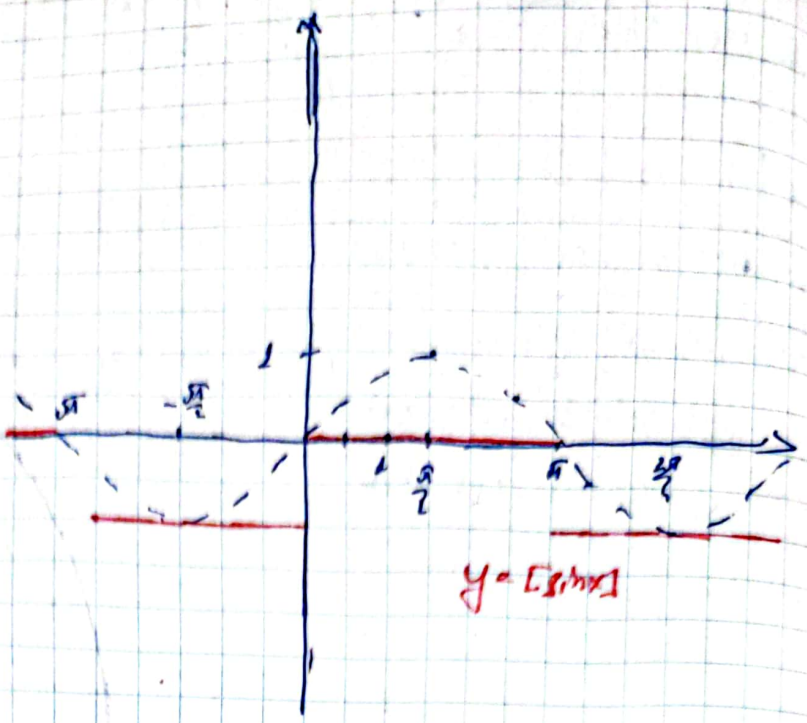
3,72 1,54 1 1,54 3,72



2.14 $f(x) = [2^x]$

x	-2	-1	0	1	2
f(x)	0	0	1	2	4

218 $f(x) = [\sin x]$



212 $f(x) = \sin[x]$

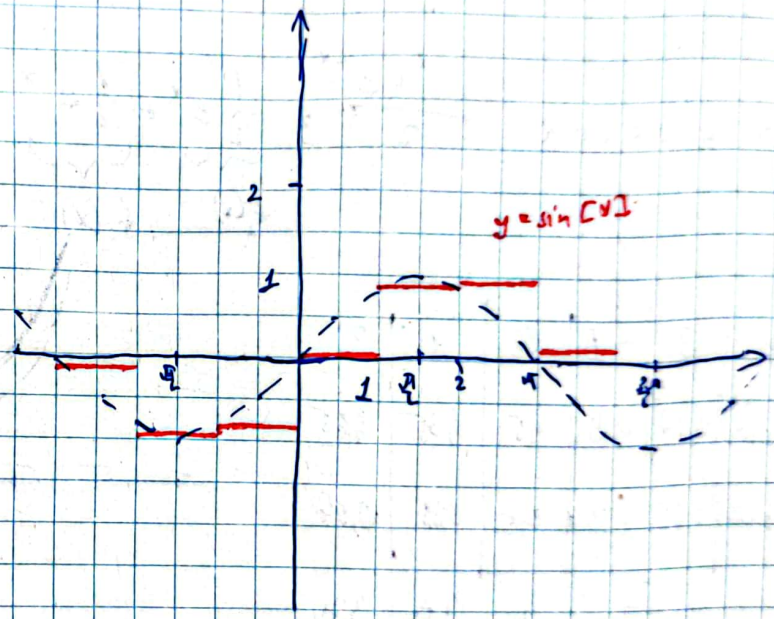
$0 \le x < 1$ $f(x) = 0$

$1 \le x < 2$ $f(x) = \sin 1$

$2 \le x < 3$ $f(x) = \sin 2$

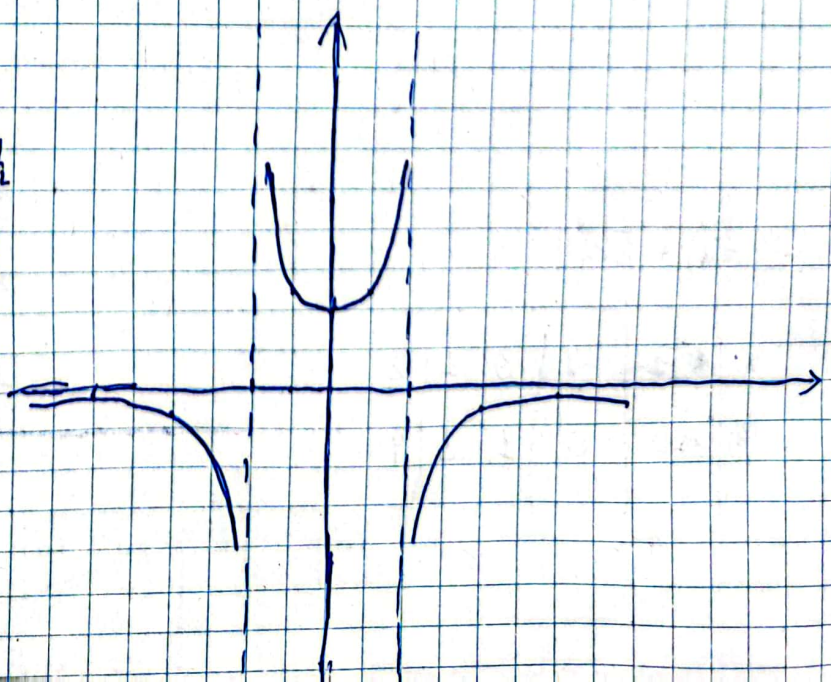
$-1 \le x < 0$ $f(x) = \sin(-1)$

$-2 \le x < -1$ $f(x) = \sin(-2)$



220 $f(x) = \frac{1}{1-x^2} = \frac{1}{(1-x)(1+x)}$

x	-2	-2	0	2	3	$\frac{1}{2}$	$-\frac{1}{2}$
y	$-\frac{1}{8}$	$-\frac{1}{3}$	1	$-\frac{1}{3}$	$-\frac{1}{8}$	$\frac{4}{3}$	$\frac{4}{3}$



220 $f(x) = e^{\sin x}$

x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	$-\frac{3\pi}{2}$	$-\pi$	$-\frac{\pi}{2}$
y	1	$2,4$	1	$0,10$	$2,7$	1	$0,37$