$\lim\limits_{x\to 0} \frac{\ln(1+\dots)}{x\dots} \cdot \frac{\dots}{x\dots}$

Домашня робота 1

11.10. $f(x)=\operatorname{arctg}\frac{1+x}{1-x} = \operatorname{arctg}(t)$

$f'(x) = f'(t)\cdot(t)' = \frac{1}{\frac{(x+1)^2}{(1-x)^2}+1}\cdot\left(\frac{x+1}{(1-x)}\right)' =$

$= \frac{1}{\frac{(x+1)^2}{(1-x)^2}+1}\cdot\frac{(x+1)'(1-x)-(1-x)'(x+1)}{(1-x)^2} = \frac{1}{\frac{(x+1)^2}{(1-x)^2}+1}\cdot$

$\cdot\frac{(1-x)\cdot(x+1)}{(1-x)^2} = \frac{2}{(1-x)^2\left(\frac{(x+1)^2}{(1-x)^2}+1\right)}$

11.8 $f(x) = 10^{\frac{x}{\log_x 3}} = a^{t(x)}$

$a^{t(x)'} = a^{t(x)}\cdot\ln a\cdot t'(x) = 10^{\frac{x}{\log_x 3}}\cdot\ln 10\cdot\left(\frac{x}{\log_x 3}\right)' =$

$\Downarrow$

$\frac{x\ln x}{\ln 3}$

$= \ln 10\cdot 10^{\frac{x\ln x}{\ln 3}}\cdot\frac{1}{\ln 3}\cdot(x\cdot\ln x)' =$

$= \frac{\ln 10\cdot 10^{\frac{x\ln x}{3}}}{\ln 3}\cdot\left((x)'\cdot\ln x+(\ln x)'\cdot x\right) =$

$\Downarrow$

$1\cdot\ln x+\frac{1}{\cancel{x}}\cdot\cancel{x}$

$$= \frac{\ln 10 \cdot 10^{\frac{x \ln x}{\ln x}} (\ln x + 1)}{\ln x}$$

11.13 $f(x) = 2^{\operatorname{tg} \frac{1}{x}}$; $x_0 = \frac{1}{\pi}$

$$f'(x) = 2^{\operatorname{tg}\frac{1}{x}} \ln 2 \cdot \left(\operatorname{tg}\frac{1}{x}\right)' = 2^{\operatorname{tg}\frac{1}{x}} \cdot \ln 2 \cdot \frac{-\frac{1}{x^2}}{\cos^2 \frac{1}{x}}$$

$$f'(x_0) = 2^{\operatorname{tg}\pi} \cdot \ln 2 \cdot \frac{-\pi^2}{\cos^2 \frac{1}{\pi}} = -\pi^2 \ln 2.$$

11.14. $f(x) = \operatorname{sgn} x$; $x_0 = 0$

$\lim\limits_{x \to 0} \operatorname{sgn}(x) = \varnothing$; не диференційовна

11.15. $f(x) = \begin{cases} x, & x \le 0 \\ \sin x, & x > 0 \end{cases}$

1) $x_0 = -\pi$

$$f'_л = \lim_{\Delta x \to 0} \frac{f(x_0) - f(x_0 - \Delta x)}{\Delta x} = \frac{-\pi + \pi + \Delta x}{\Delta x} = 1$$

$$\parallel$$

$$f'_п = \lim_{\Delta x \to 0} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x} = \frac{-\pi + \Delta x - \pi}{\Delta x} = 1$$

$f(x)$ - диференційована в $x_0 = \pi$.

2) $x_0 = 0$; $f'_л = \lim\limits_{\Delta x \to 0} \frac{f(x_0) - f(x_0 - \Delta x)}{\Delta x} = \frac{0 - 0 + \Delta x}{\Delta x} = 1$

$$\parallel$$

$$f'_п = \lim_{x \to 0} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x} = \frac{\sin \Delta x}{\Delta x} = 1$$

$f(x)$ - диференційована в $x_0 = 0$.

11.13. $f(x) = x \cdot |x|$, $x_0 = 0$

$$f(x) = \begin{cases} x^2; & x \geq 0 \\ -x^2 & x < 0 \end{cases}$$

$$f'_{л} = \frac{f(x_0) - f(x_0 - \Delta x)}{\Delta x} = \frac{\Delta x^2}{\Delta x} =$$

$$= \Delta x = 0$$

$$f'_{п} = \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x} = \frac{\Delta x^2}{\Delta x} = \Delta x \to 0$$

$f(x)$ – диференційовна