Üb 4.2 $\lim\limits_{n\to\infty} \frac{2n^2-1}{5n^2+1}$

$$\frac{2n^2-1}{5n^2+1} = \frac{2-\overbrace{\frac{1}{n^2}}^{\to 0}}{5+\underbrace{\frac{1}{n^2}}_{\to 0}} = \frac{2}{5}$$

4.8 $$\lim_{n\to\infty} \frac{\sin n}{\sqrt{n}} = \underbrace{\sin n}_{\leq 1} \cdot \underbrace{\frac{1}{\sqrt{n}}}_{\to 0} = 0.$$

4.9 $$\lim_{n\to\infty} \frac{\ln(1+n^2)}{2^n} = O(1)\cdot O(1) = O(1) =$$

$$\lim_{n\to\infty} \frac{\ln(1+n^2)}{2^n} = 0$$

$\underline{4.18}$ $x_n = \sum_{k=1}^{n} \frac{2k}{(2k+2)!!}$

$$\sum_{k=1}^{n} \frac{2k}{(2k+2)!!} = \sum_{k=1}^{n} \left(\frac{2k+2}{(2k+2)!!} - \frac{1}{(2k+2)!!} \right) \textcircled{=}$$

$$n!! = \prod_{k=1}^{n} (2k) = n \cdot (n-2) \cdot (n-4) \cdot \ldots \cdot 4 \cdot 2$$

$$\frac{2k+2}{(2k+2)!!} = \cancel{\frac{2k+2}{(2k+2) \cdot 2k!!}} = \frac{1}{2k!!}$$

$$\textcircled{=} \sum_{k=1}^{n} \left(\frac{1}{2k!!} - \frac{1}{(2k+2)!!} \right) = \left(\frac{1}{2!!} - \cancel{\frac{1}{4!!}} \right) + \left(\cancel{\frac{1}{4!!}} - \cancel{\frac{1}{6!!}} \right) + \left(\cancel{\frac{1}{6!!}} - \cancel{\frac{1}{8!!}} \right) -$$

$$\ldots - \frac{1}{2k+2!!} = \frac{1}{2!!} - \underbrace{\frac{1}{(2n+2)!!}}_{\downarrow 0} = \frac{1}{2!!} = \frac{1}{2}$$

$$x_n = \sum_{k=1}^{n} \frac{2k}{(2k+2)!!} = \frac{1}{2}$$

$\underline{4.20.}$ $\lim_{n\to\infty} \left(\frac{1}{5n} \operatorname{arctg} \frac{n^3}{2n+1} + \frac{\sin n - n}{1-4n} \right) =$

$$= \underbrace{\lim_{n\to\infty} \left(\frac{1}{5n} \operatorname{arctg} \frac{3n^2}{2} \right)}_{0} + \underbrace{\lim_{n\to\infty} \left(\frac{\sin n}{1-4n \to 0} \right)}_{0} - \lim_{n\to\infty} \left(\frac{n}{1-4n} \right) \textcircled{=}$$

$$= \lim_{n\to\infty} \left(\frac{n}{1-4n} \right) = \lim_{n\to\infty} \frac{1}{\frac{1}{n} - 4} = -\frac{1}{4}$$

$$0 + 0 + \frac{1}{4} = \frac{1}{4}$$

$$\textcircled{=} \frac{1}{4}$$

$\underline{4.24}$ $\lim_{n\to\infty} \left(\frac{n}{2n^2-1} \cdot \cos \frac{n+1}{2n-1} - \frac{n}{1-2n} \cdot \frac{n(-1)^n}{n^2+1} \right) =$

$$= \lim_{n\to\infty} \left(\frac{n}{2n^2-1} \cdot \cos \frac{n+1}{2n-1} \right) - \lim_{n\to\infty} \left(\frac{n}{1-2n} \cdot \frac{n(-1)^n}{n^2+1} \right) \textcircled{=}$$

$$\lim_{n\to\infty}\left(\frac{n}{2n^2-1}\cdot\cos\frac{n+1}{2n-1}\right)=\frac{1}{2n-\frac{1}{n}}\cdot\cos\frac{n+1}{2n-1}=0$$

$$\lim_{n\to\infty}\left(\frac{n}{1-2n}\cdot\frac{n(-1)^n}{n^2+1}\right)=\frac{1}{\frac{1}{n}-2}\cdot\frac{(-1)^n}{n+\frac{1}{n}}\Rightarrow 0$$

$-\frac{1}{2}$

$(-1)^n\le 1$

$= 0-0=\underline{0}$

4.34. $\lim\limits_{n\to\infty}\sqrt[n]{a}=1\quad(a>0)$

$\sqrt[n]{a}=a^{\frac{1}{n}\to 0}=\underline{1}$.

$\lim\limits_{n\to\infty}\sqrt[n]{a}=1$

Доведено.

Практика ③ б) $\lim\limits_{n\to\infty}\left(\sqrt{n+\sqrt{n+\sqrt{n}}}-\sqrt{n}\right)=\lim\limits_{n\to\infty}\frac{\cancel{n}+\sqrt{n+\sqrt{n}}-\cancel{n}}{\sqrt{n+\sqrt{n+\sqrt{n}}}+\sqrt{n}}$

$=\lim\frac{\sqrt{1+\sqrt{\frac{1}{n}}}}{\sqrt{1+\sqrt{\frac{1}{n}+\frac{1}{n}}}+1}\Rightarrow\frac{1}{2}$

$=\sqrt[3]{n^3+n^2+1}-\sqrt[3]{n^3-n^2+1}=\frac{n^3+n^2+1-n^3+n^2-1}{\ldots}$