D | 3

16.1 $\displaystyle\int_{x_0}^{x} \frac{4t}{2t+1}\,dt = \int_{x_0}^{x}\left(2-\frac{2}{2t+1}\right)dt = \int_{x_0}^{x} 2dt -$

$\displaystyle - \int_{x_0}^{x} \frac{d2t}{2t+1} = 2x - \ln|2x+1| + C$

16.3 $\displaystyle\int_{x_0}^{x} \frac{3t^4}{t^2+t-2}\,dt = 3\int_{x_0}^{x}\left(t^2-t+3-\frac{16}{3(t+2)}+\frac{1}{3(t-1)}\right)dt =$

$$\frac{3t^4}{t^2+t-2} = 3\left(t^2-t+3+\frac{-5t+6}{t^2+t-2}\right)$$

$$\frac{-5t+6}{t^2+t-2} = \frac{-5t+6}{(t-1)(t+2)} = \frac{A}{t+2} + \frac{B}{t-1}$$

$$-5t+6 = A(t-1) + B(t+2) = (A+B)t + (-A+2B)$$

$$\begin{cases} A+B=-5 \\ -A+2B=6 \end{cases} \Longleftrightarrow \begin{cases} B=\frac{1}{3} \\ A=-\frac{16}{3} \end{cases}$$

$$⊜\ 3\left(\frac{x^3}{3}-\frac{x^2}{2}+3x-\frac{16}{3}\ln|x+2|+\frac{1}{3}\ln|x-1|\right)=x^3-\frac{3}{2}x^2+9x-16\ln|x+2|+$$
$$+\ln|x-1|+C$$

16.5 $$\int_{x_0}^{x}\frac{dt}{t^4-1}=\int_{x_0}^{x}\left(\frac{1}{4(t-1)}-\frac{1}{4(t+1)}-\frac{1}{2(t^2+1)}\right)dt\ ⊜$$

$$\frac{1}{t^4-1}=\frac{1}{(t^2-1)(t^2+1)}=\frac{1}{(t-1)(t+1)(t^2+1)}=\frac{A}{t-1}+\frac{B}{t+1}+\frac{Cx+D}{t^2+1}$$

$$1=A(t+1)(t^2+1)+B(t-1)(t^2+1)+(Cx+D)(t-1)(t+1)=$$
$$=(A+B+C)x^3+(A-B+D)x^2+(A+B-C)x+(A-B-D)$$

$$\begin{cases}A+B+C=0\\A-B+D=0\\A+B-C=0\\A-B-D=1\end{cases}\iff\begin{cases}A=\frac{1}{4}\\B=-\frac{1}{4}\\C=0\\D=-\frac{1}{2}\end{cases}$$

$$⊜\ \frac{1}{4}\ln|x-1|-\frac{1}{4}\ln|x+1|-\frac{1}{2}\operatorname{arctg}(t)=\frac{1}{4}\ln|x-1|-\frac{1}{4}\ln|x+1|-\frac{1}{2}\operatorname{arctg}x+C$$

16.9 $$\int_{x_0}^{x}t\cos t\,dt=\left|\begin{matrix}u=t & du=dt\\ dv=\cos t\,dt & v=\sin t\end{matrix}\right|=t\cdot\sin t-\int_{x_0}^{x}\sin t\,dt=x\sin x+\cos x$$

16.11 $$I=\int_{x_0}^{x}e^t\cdot\sin t\,dt=\left|\begin{matrix}u=\sin t, & du=\cos t\,dt\\ dv=e^t dt & v=e^t\end{matrix}\right|=e^t\cdot\sin t-\int_{x_0}^{x}e^t\cdot\cos t\,dt=$$

$$=\left|\begin{matrix}u=\cos t & du=-\sin t\,dt\\ dv=e^t dt & v=e^t\end{matrix}\right|=e^t\cdot\sin t-e^t\cos t+\int_{x_0}^{x}e^t(-\sin t)\,dt=$$
$$=e^x\sin x-e^x\cos x-\int_{x_0}^{x}e^t\cdot\sin t\,dt=e^x(\sin x-\cos x)-I$$

$$I = e^x(\sin x - \cos x) - I$$

$$I = \frac{e^x(\sin x - \cos x)}{2}$$

(16.13) $\int_{x_0}^{x} \arccos t\,dt = \left| \begin{matrix} u = \arccos t & du = -\frac{dt}{\sqrt{1-t^2}} \\ dv = dt & v = t \end{matrix} \right| =$

$= t \cdot \arccos t + \int_{x_0}^{x} \frac{t\,dt}{\sqrt{1-t^2}} = x \cdot \arccos x + \frac{1}{2}\int_{x_0^2}^{x^2} (1-y)^{-\frac{1}{2}} dy =$

$= x\arccos x - \sqrt{1-y} = x\arccos x - \sqrt{1-x^2} + C$

(16.15) $\int_{x_0}^{x} t\,tg^2 t\,dt = \left| \begin{matrix} u = t & du = dt \\ dv = tg^2 t\,dt & v = tg\,t - t \end{matrix} \right| = t(tg\,t - t) -$

$- \int_{x_0}^{x} (tg\,t - t)dt = x\,tg\,x - x^2 - \int_{x_0}^{x} tg\,t\,dt + \int_{x_0}^{x} t\,dt =$

$= x\,tg\,x + \frac{x^2}{2} - x^2 + \ln|\cos x| = x\,tg\,x - \frac{x^2}{2} + \ln|\cos x| + C.$

(16.17) $I = \int_{x_0}^{x} \sin \ln t\,dt = \left| \begin{matrix} u = \sin \ln t & du = \frac{\cos \ln t\,dt}{t} \\ dv = dt & v = t \end{matrix} \right| =$

$= t \cdot \sin \ln t - \int_{x_0}^{x} \cos \ln t\,dt = \left| \begin{matrix} u = \cos \ln t & du = -\frac{\sin \ln t\,dt}{t} \\ dv = dt & v = t \end{matrix} \right| =$

$= x \sin \ln x - t \cos \ln t - \int_{x_0}^{x} \sin \ln t\,dt = x \cdot \sin \ln x - x\cos \ln x - I$

$$I = \frac{x \cdot \sin \ln x - x \cos \ln x}{2}$$