Bài

61. $x_n = 1 + \frac{n}{n+1}\sin\frac{n\pi}{2}$

$x_{4k-3} = 1 + \frac{4k-3}{4k-2}\sin\frac{\pi}{2}$

$x_{4k-2} = 1 + \frac{4k-2}{4k-1}\cdot 0 = 1$

$x_{4k-1} = 1 + \frac{4k-1}{4k}\cdot(-1) = 1 - \frac{4k-1}{4k}$

$x_{4k} = 1 + \frac{4k}{4k+1}\cdot 0 = 1$

$\lim\limits_{n\to\infty} \frac{4k-3}{4k-2} = 1 + \frac{\cancel{4k}\left(1-\frac{3}{4k}\right)}{\cancel{4k}\left(1-\frac{2}{4k}\right)} = 1 + 1 = 2$

$\lim_{k\to\infty} 1 = 1$

$\lim_{k\to\infty} 1 + \frac{4k-1}{4k} = 1 - \frac{4k(1-\frac{1}{4k})}{4k} = 1-1 = 0$

$\lim_{k\to\infty} 1 = 1$

	lim	inf	sup
$n=4k$	1	1	1
$n=4k-3$	0	0	$\frac{1}{4}$
$n=4k-2$	1	1	1
$n=4k-3$	2	$\frac{3}{2}$	2

$\inf = \underline{\lim} = 0 \qquad \sup = \overline{\lim} = 2.$

<u>5.16</u> $x_n = \frac{1}{n^3}\sum_{k=1}^{n} k^2$

За т. Штольца

$x_n = \frac{1}{n^3}\sum_{k=1}^{n} k^2 = \frac{z_n}{y_n}$, $z_n \to \sum_{k=1}^{n} k^2$, $y_n \to n^3$

$\lim_{n\to\infty} \frac{z_{n+1}-z_n}{y_{n+1}-y_n} = \lim_{n\to\infty} \frac{(n+1)^2}{(n+1)^3 - n^3} = \lim_{n\to\infty} \frac{n^2+2n+1}{n^3+1n+3n^2+3n-n^3} =$

$= \lim_{n\to\infty} \frac{n^2+2n+1}{3n^2+3n+1} = \lim_{n\to\infty} \frac{1+\frac{2}{n}+\frac{1}{n^2}}{3+\frac{3}{n}+\frac{1}{n^2}} = \frac{1}{3}$

8.13 $x_n = \sum_{k=1}^{n} \frac{1}{2n+2k-1}$, $\frac{1}{2n+1} + \frac{1}{2n+3} + \dots$

$= \underbrace{\frac{1}{2n+1} + \frac{1}{2n+3} + \dots + \frac{1}{4n-1}}_{S_{4n}}$ $+ \underbrace{1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{2n}} + \underbrace{\frac{1}{2n+1} \dots + \frac{1}{4n}} =$

$- \underbrace{\left(1 + \frac{1}{2} + \dots + \frac{1}{2n}\right)}_{\ln 2n + c} - \underbrace{\left(\frac{1}{2n+2} + \dots + \frac{1}{4n}\right)}_{?} =$

$= \ln 4n + \cancel{c} - \ln 2n - \cancel{c} - \left(\frac{1}{2(n+1)} + \dots + \frac{1}{4n}\right) = \ln 2 - \frac{1}{2} \ln \frac{m}{n}$

$\lim \ln 2 - \frac{1}{2} \ln 2 = \sqrt{2}$

8.7 $x_{n+1} = x_n^2 - 2x_n + 2$, $n \geq 1$, $x_1 \in (1,2)$

Нехай $\lim_{n \to \infty} x_n = a$

$a = a^2 - 2a + 2$

$a^2 - 3a + 2 = 0$

$n = 1$ $x \in (1,2)$ $x_1 < 2$

$x_{n+1} = x_n^2 - 2x_n + 2 < 2^2 - 2 \cdot 2 + 2 = \underline{2}$

$x_{n+1} - x_n = x_n^2 - 2x_n + 2 - x_n = x_n^2 - 3x_n + 2 = (x_n - 1)(x_n - 2) \leq 0$

$\forall n$

$x_{n+1} \leq x_n \Rightarrow$ незростаюча

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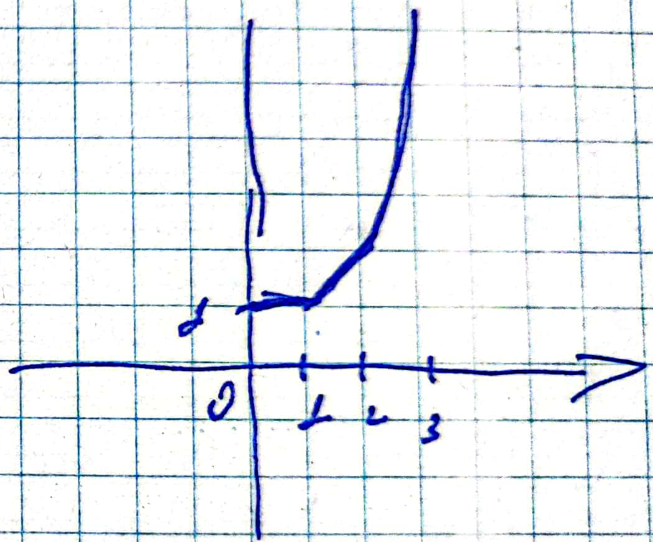
6.13 $f(x) = \lim_{n\to\infty} \sqrt[n]{1+x^n+\left(\frac{x^2}{2}\right)^n}, \ x \geq 0$

$f(x) = \lim_{n\to\infty} \sqrt[n]{x^{2n}\left(\frac{1}{x^{2n}}+\frac{1}{x^n}+\frac{1}{2^n}\right)} =$

$= \begin{cases} 1, & 0 \leq x \leq 1 \\ x, & 1 < x \leq 2 \\ \frac{x^2}{2} & x \geq 2 \end{cases}$



BTB