Dz

837. $\begin{pmatrix} 3 & 4 \\ 5 & 7 \end{pmatrix} = A$

$$A^{-1} = \frac{1}{A}\begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}^T = \begin{pmatrix} 7 & -5 \\ -4 & 3 \end{pmatrix}^T = \begin{pmatrix} 7 & -4 \\ -5 & 3 \end{pmatrix}$$

842 $$\left(\begin{array}{ccc|ccc} 2 & 7 & 3 & 1 & 0 & 0 \\ 3 & 9 & 4 & 0 & 1 & 0 \\ 1 & 5 & 3 & 0 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 1 & 6 & 1 & 0 & -1 \\ 0 & 12 & 4 & 0 & 1 & -3 \\ 1 & -5 & -1 & 0 & 0 & 1 \end{array}\right) \sim$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 1 & 6 & 1 & 0 & -1 \\ 0 & 12 & 4 & 0 & 1 & -3 \\ 0 & -6 & -7 & -1 & 0 & 2 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 1 & 6 & 1 & 0 & -1 \\ 0 & 0 & -10 & -2 & 1 & 1 \\ 0 & -6 & -7 & -1 & 0 & 2 \end{array}\right) \sim$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & -5 & -1 & 0 & 0 & 1 \\ 0 & -6 & -7 & -1 & 0 & 2 \\ 0 & 0 & 10 & -2 & 1 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & -5 & -1 & 0 & 0 & 1 \\ -1 & 1 & -6 & -1 & 0 & 1 \\ 0 & 0 & -10 & -2 & 1 & 1 \end{array}\right) \sim$$

$$\sim \left(\begin{array}{ccc|ccc} -1 & -1 & -6 & -1 & 0 & 1 \\ 0 & -6 & -7 & -1 & 0 & 2 \\ 0 & 0 & -10 & -2 & 1 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 1 & 6 & 1 & 0 & -1 \\ 0 & 6 & 7 & 1 & 0 & -2 \\ 0 & 0 & 10 & 2 & -1 & -1 \end{array}\right) \sim$$

$1 \quad -1 \quad -1$

$$\sim \begin{pmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$A^{-1} = \frac{1}{7}$$

$$\sim\left(\begin{array}{ccc|ccc} 1 & 0 & \frac{4}{6} & \frac{5}{6} & 0 & -\frac{2}{3} \\ 0 & 1 & \frac{2}{3} & \frac{1}{6} & 0 & -\frac{1}{3} \\ 0 & 0 & 1 & \frac{2}{10} & -\frac{1}{10} & -\frac{1}{20} \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -\frac{2}{15} & 0 & -\frac{11}{60} \\ 0 & 1 & 0 & -\frac{1}{15} & 0 & \frac{5}{6} \\ 0 & 0 & 1 & 0{,}2 & -0{,}1 & -0{,}4 \end{array}\right)$$

A^{-1}

<u>зад4</u>

$$\left(\begin{array}{cccc|cccc} 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & -1 & -1 & 0 & 1 & 0 & 0 \\ 1 & -1 & 1 & -1 & 0 & 0 & 1 & 0 \\ 1 & -1 & -1 & 1 & 0 & 0 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{cccc|cccc} 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & -2 & -2 & 1 & 0 & -1 & 0 \\ 0 & -2 & 0 & -2 & 1 & 0 & 0 & -1 \\ 0 & -2 & -2 & 0 & 1 & -1 & 0 & 0 \end{array}\right) \sim$$

$$\sim\left(\begin{array}{cccc|cccc} 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & \frac{1}{2} & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 2 & -2 & 0 & 0 & 1 & -1 \\ 0 & 0 & 2 & 2 & 1 & -1 & 0 & 0 \end{array}\right) \sim \left(\begin{array}{cccc|cccc} 1 & 0 & 1 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 1 & 0 & 1 & \frac{1}{2} & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & \frac{1}{2} & -\frac{1}{2} \\ 0 & 0 & 0 & 4 & 1 & -1 & -1 & 1 \end{array}\right) \sim$$

$$\sim\left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 1 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 1 & 0 & 1 & \frac{1}{2} & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & \frac{1}{2} & -\frac{1}{2} \\ 0 & 0 & 0 & 1 & \frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} & \frac{1}{4} \end{array}\right) \sim$$

$$A^{-1} = \frac{1}{4}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}$$

861 $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \cdot X = \begin{pmatrix} 3 & 5 \\ 5 & 9 \end{pmatrix} =$

$$\left(\begin{array}{cc|cc} 1 & 2 & 3 & 5 \\ 3 & 4 & 5 & 9 \end{array}\right) = \left(\begin{array}{cc|cc} 1 & 2 & 3 & 5 \\ 0 & -2 & -4 & -6 \end{array}\right) = \left(\begin{array}{cc|cc} 1 & 0 & 1 & -1 \\ 0 & 1 & 2 & 3 \end{array}\right)$$

$$X = \begin{pmatrix} -1 & -1 \\ 2 & 3 \end{pmatrix}$$

865 $X \cdot \begin{pmatrix} 5 & 3 & 1 \\ 1 & -3 & -2 \\ -5 & 2 & 1 \end{pmatrix} = \begin{pmatrix} -8 & 3 & 0 \\ -8 & 9 & 0 \\ -2 & 15 & 0 \end{pmatrix}$

$$\left(\begin{array}{ccc} 5 & 3 & 1 \\ 1 & -3 & -2 \\ -5 & 2 & 1 \\ \hline -8 & 3 & 0 \\ -5 & 9 & 0 \\ -2 & 15 & 0 \end{array}\right) = \left(\begin{array}{ccc} 8 & 2 & 1 \\ -2 & -2 & -2 \\ -3 & 1 & 1 \\ \hline -5 & 3 & 0 \\ 4 & 9 & 0 \\ 13 & 15 & 0 \end{array}\right) = \left(\begin{array}{ccc} 2 & -6 & 1 \\ 0 & 1 & -2 \\ -4 & 4 & 1 \\ \hline -5 & 8 & 0 \\ 4 & 5 & 0 \\ 13 & 2 & 0 \end{array}\right) =$$

$$= \left(\begin{array}{ccc} 7 & 1 & 1 \\ 0 & 1 & -2 \\ -4 & 0 & 1 \\ \hline -5 & 3 & 0 \\ 4 & 9 & 0 \\ 13 & 15 & 0 \end{array}\right) = \left(\begin{array}{ccc} 7 & 1 & 9 \\ 0 & 1 & 0 \\ -4 & 0 & 1 \\ \hline -5 & 3 & 6 \\ 4 & 9 & 18 \\ 13 & 15 & 30 \end{array}\right) = \left(\begin{array}{ccc} 19 & 1 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ \hline 14 & 3 & 6 \\ 26 & 9 & 18 \\ 33 & 15 & 30 \end{array}\right) =$$

$$= \begin{pmatrix} 1 & 1 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ \hline 1 & 3 & 6 \\ 4 & 9 & 13 \\ 7 & 15 & 30 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ \hline 1 & 2 & 6 \\ 4 & 5 & 13 \\ 7 & 8 & 30 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ \hline 1 & 2 & 3 \\ 4 & 5 & 1 \\ 7 & 8 & 9 \end{pmatrix}$$

$$\underline{866} \quad \begin{pmatrix} 2 & -3 & 1 \\ 4 & -5 & 2 \\ 5 & -7 & 3 \end{pmatrix} \cdot X \cdot \begin{pmatrix} 9 & 7 & 6 \\ 1 & 1 & 2 \\ 1 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 0 & -2 \\ 18 & 12 & 9 \\ 23 & 15 & 11 \end{pmatrix}$$

$$X = A^{-1} \cdot C \cdot B^{-1}$$

$$(A|C) \sim (E|A^{-1}C)$$

$$\left(\frac{B}{A^{-1}C}\right) \sim \left(\frac{E}{A^{-1}CB^{-1}}\right)$$

$$\left(\begin{array}{ccc|ccc} 2 & -3 & 1 & 2 & 0 & -2 \\ 4 & -5 & 2 & 18 & 12 & 9 \\ 5 & -7 & 3 & 23 & 15 & 11 \end{array}\right) \begin{matrix} \\ -2I \\ -II \end{matrix} = \left(\begin{array}{ccc|ccc} 2 & -3 & 1 & 2 & 0 & -2 \\ 0 & 1 & 0 & 14 & 12 & 13 \\ 1 & -2 & 1 & 5 & 3 & 2 \end{array}\right) -III + II =$$

$$= \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 11 & 9 & 9 \\ 0 & 1 & 0 & 14 & 12 & 13 \\ 1 & -2 & 1 & 5 & 3 & 2 \end{array}\right) = \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 11 & 9 & 9 \\ 0 & 1 & 0 & 14 & 12 & 13 \\ 0 & 0 & 1 & 22 & 18 & 19 \end{array}\right)$$

$$\begin{pmatrix} 9 & 7 & 6 \\ 1 & 1 & 2 \\ 1 & 1 & 1 \\ \hline 11 & 9 & 9 \\ 14 & 12 & 13 \\ 22 & 18 & 19 \end{pmatrix} = \begin{pmatrix} 2 & 1 & 6 \\ 0 & -1 & 2 \\ 0 & 0 & 1 \\ \hline 2 & 0 & 9 \\ 2 & -1 & 13 \\ 4 & -1 & 19 \end{pmatrix} = \begin{pmatrix} 1 & -1 & 6 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \\ \hline 1 & 0 & 9 \\ 1 & 1 & 13 \\ 2 & 1 & 19 \end{pmatrix}$$

$A^{-1}C$

$$= \left(\begin{array}{ccc} 1 & 0 & 6 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \\ \hline 1 & 1 & 9 \\ 1 & 2 & 13 \\ 2 & 3 & 19 \\ & -17 & -29 \end{array}\right) = \left(\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ \hline 1 & 1 & 1 \\ 1 & 2 & 3 \\ 2 & 3 & 1 \end{array}\right)$$

$$" X$$

<u>3c)</u> $X \cdot \begin{pmatrix} 3 & 6 \\ 4 & 8 \end{pmatrix} = \begin{pmatrix} 2 & 4 \\ 9 & 18 \end{pmatrix}$

$$\left(\begin{array}{cc} 3 & 6 \\ 4 & 8 \\ \hline 2 & 4 \\ 9 & 18 \end{array}\right) = \left(\begin{array}{cc} 0 & 6 \\ 0 & 8 \\ \hline 0 & 4 \\ 0 & 18 \end{array}\right) = \left(\begin{array}{cc} 0 & 3 \\ 0 & 4 \\ \hline 0 & 2 \\ 0 & 9 \end{array}\right)$$

$$X \begin{pmatrix} 0 & 3 \\ 0 & 4 \end{pmatrix} = \begin{pmatrix} 0 & 2 \\ 0 & 9 \end{pmatrix}$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 0 & 3 \\ 0 & 4 \end{pmatrix} = \begin{pmatrix} 0 & 2 \\ 0 & 9 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 3a+4b \\ 0 & 3c+4d \end{pmatrix} = \begin{pmatrix} 0 & 2 \\ 0 & 9 \end{pmatrix} \qquad \begin{cases} 3a+4b = 2 \\ 3c+4d = 9 \end{cases}$$

$$a = \frac{2-4b}{3} \qquad c = \frac{9-4d}{3} \qquad b \in \mathbb{R} \quad d \in \mathbb{R}$$

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$$X = \begin{pmatrix} \frac{2-4b}{3} & b \\ \frac{2-4d}{3} & d \end{pmatrix}$$

468 $\begin{vmatrix} a & b & c & d \\ -b & a & d & -c \\ -c & -d & a & b \\ -d & c & -b & a \end{vmatrix} = \Delta = \det A$

$$AA^T = \begin{pmatrix} a & b & c & d \\ -b & a & d & -c \\ -c & -d & a & b \\ -d & c & -b & a \end{pmatrix} \begin{pmatrix} a & -b & -c & -d \\ b & a & -d & c \\ c & d & a & -b \\ d & -c & b & a \end{pmatrix}$$

$$= \begin{pmatrix} a^2+b^2+c^2+d^2 & 0 & 0 & 0 \\ 0 & a^2+b^2+c^2+d^2 & 0 & 0 \\ 0 & 0 & a^2+b^2+c^2+d^2 & 0 \\ 0 & 0 & 0 & a^2+b^2+c^2+d^2 \end{pmatrix}$$

$$\Delta^2 = (\det A)^2 = \det A \cdot \det A^T = (a^2+b^2+c^2+d^2)^4$$

$$\Delta = \pm (a^2+b^2+c^2+d^2)$$

469 $(x-1-i)(x-1+i)(x+1+i)(x+1-i)$

$(x^2 - x + \cancel{ix} - x + 1 \cancel{-i} \cancel{-ix} \cancel{+i} - i^2)(x^2 + x \cancel{-xi} + x + 1 \cancel{-i} + \cancel{ix} \cancel{+i} - i^2) =$

$= (x^2 - 2x + 2)(x^2 + 2x + 2) =$

104 $\dfrac{1+i\,tg\,\alpha}{1-i\,tg\,\alpha} = \dfrac{(1+i\,tg\,\alpha)(1+tg\,\alpha)}{(1-tg\,\alpha)(1+tg\,\alpha)} = \dfrac{\ldots}{1+tg^2\alpha}$

$= \dfrac{1-tg^2\alpha}{1+tg^2\alpha} + i\,\dfrac{2tg\,\alpha}{1+tg^2\alpha} = \cos 2\alpha + i\sin 2\alpha$

108 b) $\begin{cases}(2+i)x+(2-i)y=6\\(3+2i)x+(3-2i)y=8\end{cases}$

$x = \dfrac{6+(i-2)y}{2+i} \cdot (2-i) = \dfrac{12-6i+(2i+1-4+2i)y}{4+1} =$

$= \dfrac{12-6i+(4i-3)y}{5}$

$3+2i \cdot \dfrac{12-6i+(4i-3)y}{5} + (3-2i)y = 8$

$\dfrac{(36-18i+24i-12i)+(12i-9-8-6i)y}{5} + (3-2i)y = 8$

$\dfrac{(24+6i)+(-17+6i)y}{5} + (3-2i)y = 8$

$\dfrac{6i(25-16)y}{5} + (3-2i)y = 8$

$\dfrac{6i\cdot 9y}{5}\,(3+2i) + (3-2i)y = 8\,(3+2i)\cdot 5$

$\dfrac{(3+2i)(6i\cdot 9y)+(3-2i)(3+2i)\cdot 5}{5\cdot(3+2i)} = 8$

$\cancel{18+27\ldots} \cancel{\ldots 27\cdot 8y^2}$

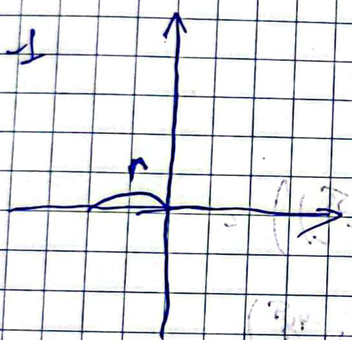
$\cancel{162iy\ldots - 108y + 9\ldots}$

109 a) $\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)^2 = \frac{1}{4} - 2\cdot\frac{1}{2}\cdot\frac{i\sqrt{3}}{2} + \frac{3i^2}{4} =$

$= \frac{1}{4} - \frac{3}{4} - \frac{i\sqrt{3}}{2}$

$= -\frac{1}{2} - \left(\frac{i\sqrt{3}}{\sqrt{2}}\right)^2 = -\frac{1}{2} - \sqrt{-\frac{3}{2}}$

110 b) -1



$z = r\cos\alpha + i\sin\alpha$

$z = a + ib$

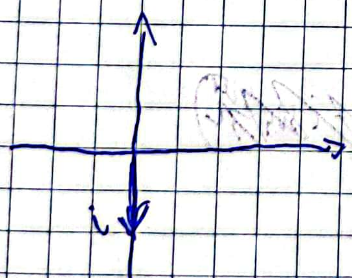
$r = \sqrt{a^2+b^2}$

$r\cos\alpha = a$

$r\sin\alpha = b$

$1 = 1(\cos\pi + i\sin\pi)$

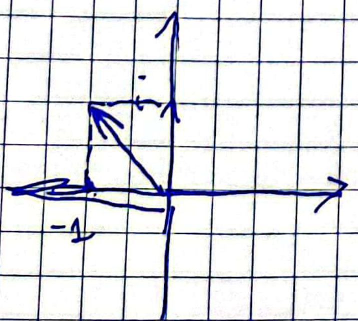
d) $-i$



$r = 1$

$-i = \cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}$

e) $-1+i$

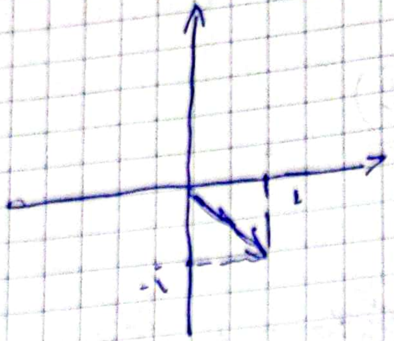


$r = \sqrt{1+1} = \sqrt{2}$

$\alpha = \frac{3\pi}{4}$

$-1+i = \sqrt{2}\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}$

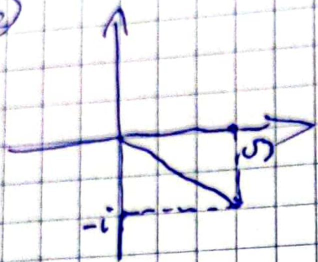
h) $1-i$



$r=\sqrt{2}$ $\alpha=\frac{7\pi}{4}$

$1-i=\sqrt{2}\left(\cos\frac{3\pi}{4}+i\sin\frac{7\pi}{4}\right)$

j) $\sqrt{3}-i$ ②

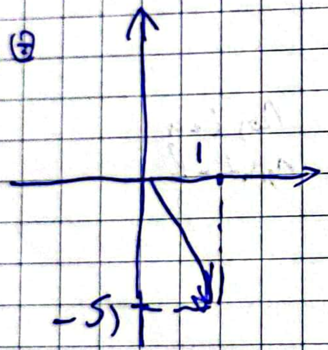


$r=2$

② $2\left(\frac{\sqrt{3}}{2}+i\left(-\frac{1}{2}\right)\right)=$

$=2\left(\cos\frac{\pi}{6}+i\sin\frac{2\pi}{3}\right)$

$1-i\sqrt{3}$ ②



$r=2$

② $2\left(\frac{1}{2}+i\sin\left(-\frac{\sqrt{3}}{2}\right)\right)=$

$=2\left(\cos\frac{\pi}{3}+i\sin\frac{4\pi}{3}\right)$

B6 c) $\frac{(1-i\sqrt{3})(\cos\varphi+i\sin\varphi)}{2(1-i)(\cos\varphi-i\sin\varphi)}=\cancel{2\left(\cos\frac{5\pi}{3}+i\sin\frac{7\pi}{2}\right)}$

$=\frac{\cancel{2}\left(\cos\left(\frac{5\pi}{3}+\varphi\right)+i\left(\sin\frac{5\pi}{3}\varphi\right)\right)}{\cancel{2}\sqrt{2}\left(\cos\left(\frac{7\pi}{4}+\varphi\right)+i\left(\sin\frac{7\pi}{4}-\varphi\right)\right)}$

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