Д/З

20.12 $\int_1^2 \frac{dx}{x+x^3} = \int_1^2 \frac{dx}{x(1+x^2)} = \int_1^2 \frac{A}{x}dx + \int_1^2 \frac{Bx+C}{1+x^2} = \ln x\Big|_1^2 -$

$-\frac{1}{2}\ln(1+x^2)\Big|_1^2 = \frac{3}{2}\ln 2 - \ln\sqrt{5} = \frac{1}{2}\ln\frac{8}{5}$

$A(1+x^2)+Bx^2+Cx=1$

$A=1$

$B=-1$

20.28 $\lim\limits_{x\to\infty}\frac{\left(\int_0^x e^{t^2}dt\right)^2}{\int_0^x e^{2t^2}dt} = \lim\limits_{x\to\infty}\frac{\frac{d}{dx}\left(\int_0^x e^{t^2}dt\right)^2}{\frac{d}{dx}\left(\int_0^x e^{2t^2}dt\right)} =$

$= \lim\limits_{x\to\infty}\frac{2\int_0^x e^{t^2}dt\cdot e^{x^2}}{e^{2x^2}} = \lim\limits_{x\to\infty}\frac{2e^{x^2}}{e^{x^2}\cdot 2x} = \lim\limits_{x\to\infty}\frac{1}{x}=0$

$I_1 = \int_0^{\pi} x\sin x\,dx \qquad I_2 = \int_{\pi}^{2\pi} x\sin x\,dx$

$x-\pi=t$
$x=\pi+t$ $\quad \int_0^{\pi}(\pi+t)\sin(\pi+t)\,dt = -\int_0^{\pi}(\pi+t)\sin t\,dt =$

$= -\int_0^{\pi}\pi\sin t\,dt - \int_0^{\pi}t\sin t\,dt$

$I_2 = -I_1 - \pi\int_0^{\pi}\sin t\,dt = -I_1 - 2\pi$

$I_1 > I_2$

20.34 $I = \int_0^2 e^{x^2-x}dx$

$e^{x^2-x} \in C[0,2]$

$I = \int_0^2 e^{x^2-x}dx = e^{\varepsilon^2-\varepsilon}\cdot\int_0^2 dx = e^{\varepsilon^2-\varepsilon}x\Big|_0^2 = 2e^{\varepsilon^2-\varepsilon}$

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$x \Big|_1^2 -$

$$2e^{\frac{1}{4}} = \inf_{x\in[0;1]} e^{x^2-x} \le e^{\varepsilon^2-\varepsilon} \le \sup_{x\in[0;1]} e^{x^2-x} = 2e^2$$

$$2e^{\frac{1}{4}} \le I \le 2e^2$$

20.37 $$\int_0^1 \frac{x^{19}dx}{\sqrt[3]{1+x^6}}$$

$f_1 = \frac{x^5}{\sqrt[3]{1+x^5}}$ неспадна

$f_2 = x^{14}$ неспадна

1) $$I = \int_0^1 \frac{x^{19}dx}{\sqrt[3]{1+x^6}} = f_1(1)\int_\varepsilon^1 f_2(x)\,dx = \frac{1}{\sqrt[3]{2}}\int_\varepsilon^1 x^{14}dx =$$

$$= \frac{1}{\sqrt[3]{2}} \cdot \frac{x^{15}}{15}\Big|_\varepsilon^1 = \frac{1}{\sqrt[3]{2}}\left(\frac{1}{15} - \frac{\varepsilon^{15}}{15}\right)$$

$\varepsilon \in [0;1]$

$$0 \le I \le \frac{1}{15\sqrt[3]{2}}$$

2) $$I = \int_0^1 \frac{x^{19}dx}{\sqrt[3]{1+x^6}} = f_2(1)\int_\varepsilon^1 f_1(x)\,dx = 1\int_\varepsilon^1 \frac{x^5}{\sqrt[3]{1+x}}\,dx =$$

$$= \left| \begin{array}{l} u = 1+x^6 \\ du = 6x^5dx \\ dx = \frac{du}{6x^5} \end{array} \right| = \int_\varepsilon^1 \frac{x^5}{\sqrt[3]{u}} \cdot \frac{du}{6x^5} = \frac{1}{6}\int_\varepsilon^1 u^{-\frac{1}{3}}du = \frac{1}{6}\cdot\frac{3\sqrt[3]{u^2}}{2}\Big|_\varepsilon^1 =$$

$$= \frac{\sqrt[3]{u^2}}{4}\Big|_\varepsilon^1 = \frac{\sqrt[3]{(x^6+1)^2}}{4}\Big|_\varepsilon^1 = \frac{\sqrt[3]{4}}{4} - \frac{\sqrt[3]{(\varepsilon^6+1)^2}}{4}$$

$\varepsilon \in [0,1]$, $\quad 0 \le I \le \frac{\sqrt[3]{4}-1}{4}$

$$\frac{1}{15\sqrt[3]{2}} < \frac{\sqrt[3]{4}-1}{4}.$$