ДЗ 11

1) $f(x,\theta) = \begin{cases} \frac{1}{\theta^{v}\Gamma(v)} x^{v-1} e^{-\frac{x}{\theta}}, & x>0 \\ 0, & x\leq 0 \end{cases}$

~ гамма розподіл, v - відомий

$\hat{\theta} = \frac{1}{nv}\sum_{i=1}^{n}\xi_i$

$L(\vec{x};\theta) = \prod_{k=1}^{n} f(\xi_k,\theta) = \prod_{k=1}^{n} \frac{1}{\theta^{v}\Gamma(v)}\xi_k^{v-1} e^{-\frac{\xi_k}{\theta}}$

$\ln L = \sum_{k=1}^{n} \ln\left(\frac{1}{\theta^{v}\Gamma(v)}(\xi_k)^{v-1} e^{-\frac{\xi_k}{\theta}}\right) = \sum_{k=1}^{n} -\ln\theta\cdot v -$

$-\ln(\Gamma(v)) + (v-1)\ln\xi_k - \frac{\xi_k}{\theta} = \frac{\partial}{\partial\theta}\ln L = \sum_{k=1}^{n} -\frac{v}{\theta} + \frac{\xi_k}{\theta^2} =$

$= \sum_{k=1}^{n}\frac{\xi_k}{\theta} - \frac{v}{\theta}$

$\hat{\sum_{k=1}^{n}}\frac{\xi_k}{\theta^2} = \frac{nv}{\theta} = \sum_{k=1}^{n}\frac{nv\xi_k}{nv\theta^2} - \frac{nv\,\theta}{\theta^2} = \frac{(\hat{\theta}-\theta)nv}{\theta^2}$ - ефективна

2) $P(k, p) = (1-p)^k p, \quad k = 0, 1 \ldots$

Мат. спод. в. геом. розподілу $= \frac{1}{p}$

$$\frac{1}{p} = \frac{\sum_{i=1}^{n} \xi_i}{n}$$

$$\frac{1}{p} = \frac{1}{\xi} = \frac{1}{M_1}$$

3) $M\xi = \theta = M_1 = \frac{\sum_{k=1}^{n} \xi_k}{n}$

$$D\xi = \frac{2\sigma\sqrt{3}}{12} = \sigma^2$$

$$\sigma^2 + \theta^2 = \hat{M}_2 = \frac{\sum_{k=1}^{n} \xi_k^2}{n}$$

$$\begin{cases} \hat{\theta} = \sum_{k=1}^{n} \xi_k \\ \sigma' = \sqrt{\frac{\sum_{k=1}^{n} \xi_k^2}{n} - \left(\frac{\sum_{k=1}^{n} \xi_k}{n}\right)} \end{cases}$$

$$M\hat{\theta} = \frac{\sum_{k=1}^{n} M(\xi_k)}{n} = \frac{n\theta}{n} = \theta \rightarrow \text{незсунута}$$

$$D\hat{\theta} = \frac{1}{n^2} \sum_{k=1}^{n} D(\xi_k) = \frac{n\sigma}{n^2} \rightarrow \text{конзистентна}$$

$$\sqrt{\frac{\sum_{k=1}^{n} M(\xi_k^2)}{n}} = \sqrt{\frac{n(\theta - \sigma\sqrt{3})^2 + \theta - n^3 + (\theta + \sigma\sqrt{3})^2}{3n} - \theta^2} =$$

$$= \sqrt{\sigma^2 + \theta^2 - \theta^2} = \sqrt{\sigma^2}$$

~~[illegible]~~ 5) $P(k;\lambda) = \frac{\lambda^k}{k!} e^{-\lambda}, \ k = 0,1..$

$$L = \prod_{k=1}^{n} = \frac{\lambda^{\xi_k}}{\xi_k} e^{-\lambda}$$

$$\ln L = \sum_{k=1}^{n} \xi_k \cdot \ln\lambda - \ln(\xi_k) - \lambda$$

$$U = \frac{\partial \ln L}{\partial x} = \sum_{k=1}^{n} \frac{\xi_k}{\lambda} - 1$$

$$\lambda = \frac{\sum \xi_k}{n} = \overline{\xi_k}$$

$$M\lambda = \sum_{k=1}^{n} \frac{M\xi_k}{n} = \frac{\sum_{k=1}^{n} \lambda}{n} = \lambda$$ - незсунута

$$D\lambda = \frac{1}{n^2} \sum_{k=1}^{n} D(\xi_k) = \frac{n\lambda}{n^2} \to 0$$ конзистентна

$$\frac{\partial V}{\partial \lambda} = \sum_{k=1}^{n} - \frac{\xi_k}{\lambda^2}$$

$$I(\lambda) = -M\frac{\partial V}{\partial \lambda} = \sum_{k=1}^{n} M\left(\frac{\xi_k}{\lambda^2}\right) = \sum_{k=1}^{n} \frac{\lambda}{\lambda^2} = \frac{n}{\lambda}$$

$$D_\lambda = \frac{\lambda}{n} = \frac{1}{\frac{n}{\lambda}} = \frac{1}{I(\lambda)}$$ - ефективна