12.7

$$f(x) = \frac{ax+b}{cx+d} = \frac{a}{c} + \frac{bc-ad}{c(cx+d)}$$

$$f^{(n)}(x) = \sum_{k=0}^{n} C_k^n \left(\frac{a}{c}\right)^{n-k} \cdot \left(-\frac{d}{c(cx+d)}\right)^{k} \cdot \frac{(n-k)!}{(cx+d)^{n-k+1}}$$

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(12.8) $f(x) = \frac{x^2}{1-x}$

$$f'(x) = \frac{2x(1-x) - x^2(-1)}{(1-x^2)} = \frac{2x - x^2}{1-x^2} = \frac{x(2-x)}{(1-x)^2}$$

$$f''(x) = \frac{(2-2x)(1-x)^2 - (2x-x^2)\cdot 2(1-x)(-1)}{(1-x)^4} =$$

$$= \frac{\cancel{(1-x)}((1-x)(2-2x)\ (2x-x^2) - 2(1-x)(-1))}{(1-x)^3} =$$

$$= \frac{2-2x-2x+2x^2+4x-2x^2}{(1-x)^3} = \frac{2}{1-x^3}$$

$$f'''(x) \ \frac{-2\cdot(-3)\ (1-x)^2}{(1-x)^6} = \frac{6}{(1-x)^4}$$

$$f^{(n)}(x) = \frac{n!}{(1-x)^{n+1}}, \quad n \geq 2$$

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(12.13) $f(x) = \operatorname{arctg} x$

$$f'(x) = \frac{1}{1+x^2}$$

$$f'' = \frac{-2x}{(1+x^2)^2}$$

$$f^{(3)}(x) = \frac{-2(1+x^2)^2 + 2x \cdot 2 \cdot 2x(1+x^2)}{(1+x^2)^4} = \frac{(-2-2x^2+8x^2)\cancel{(1+x^2)}}{(1+x^2)^4} =$$

$$= \frac{6x^2-2}{(1+x^2)^3}$$

$$f^{(4)}(x) = \frac{12x(1+x^2)^3 - (6x^2-2)\cdot 3 \cdot 2x(1+x^2)^2}{1+x^2} =$$

$$= \frac{\cancel{(1+x^2)^2}(12x + \overset{-24x^3}{\cancel{\ldots}} + 12x)}{(1+x^3)^{\cancel{6}\,4}} = \frac{-24x^3+24x}{(1+x^2)^4}$$

$$f^{(5)}(x) = \frac{(-24\cdot 3x^2+24)(1+x^2)^4 - (-24x^3+24x)\cdot 4\cdot 2x(1+x^2)^3}{(1+x^2)^8} =$$

$$= \frac{\cancel{(1+x^2)^3}(-72x^2 - 72x^4 + 24 + 24x^2 + 192x^4 - 192x^2)}{(1+x^2)^{\cancel{8}\,5}} =$$

$$= \frac{-240x^2+120x^4+24}{(1+x^2)^5}$$

$$f'(0) = \frac{1}{1} = 1$$

$$f''(0) = 0$$

$$f^{(3)}(0) = -2$$

$$f^{(4)}(0) = 0$$

$$f^{(5)}(0) = 24$$

$$f^{(n)}(0) = \begin{cases} 0, & n = 2k \\ (n-1)!, & n = 2k-1. \end{cases}$$

12.10 $f(x) = \frac{1}{\sqrt{1-2x}}$

$$f'(x) = -\cancel{\frac{1}{2}}(1-2x)^{-\frac{3}{2}} \cdot \cancel{(-2)} = \frac{1}{\sqrt{(1-2x)^3}}$$

$$f''(x) = -\frac{3}{2}(1-2x)^{-\frac{5}{2}}(-2) = \frac{3}{\sqrt{(1-2x)^5}}$$

$$f'''(x) = -\frac{5}{2}(1-2x)^{-\frac{7}{2}}(-2) = \frac{5}{\sqrt{(1-2x)^7}}$$

$$f^{(n)}(x) = \frac{2n}{\sqrt{(1-2x)^{2n+1}}}$$

12.11 $f(x) = \sin^2 x$

$$f'(x) = \sin 2x$$

$$f''(x) = 2\cos 2x$$

$$f^{(3)}(x) = 2(-\sin 2x) \cdot 2 = -4\sin 2x$$

$$f^{(4)}(x) = -4\cos 2x \cdot 2 = -8\cos 2x$$

$$f^{(5)}(x) = -8 \cdot (-\sin 2x) \cdot 2 = 2 = 16\sin 2x$$

$$f^{(n)}(x) = \begin{cases} -2^{n-1}\cos 2x, & n = 4k \\ 2^{n-1}\sin 2x, & n = 4k+1 \\ 2^{n-1}\cos 2x, & n = 4k+2 \\ -2^{n-1}\sin 2x, & n = 4k+3 \end{cases}$$

(12.14) $f(x) = (\arcsin x)^2$

$$f'(x) = \frac{2\arcsin x}{\sqrt{1-x^2}}$$

$$f''(x) = \left(\frac{2}{\sqrt{1-x^2}}\right)^3 - \frac{2x \cdot \arcsin x}{\sqrt{(1-x^2)^3}}$$

$$f'''(x) = \frac{4x}{(1-x^2)^2} + \frac{2\arcsin x}{(\sqrt{1-x^2})^3} + \frac{2x}{(\sqrt{1-x^2})^2} + \frac{6\arcsin x \cdot x^2}{\sqrt{1-x^2}^5}$$

$$f'(0) = \frac{2\arcsin 0}{(\dots)} = 0$$

$$f''(0) = \left(\frac{2}{\sqrt{1}}\right)^2 = 2$$

$$f'''(0) = 0$$

$$f^{(n)}(0) = \begin{cases} 0, & n\text{- непарне} \\ 2^{k-1} \cdot (k-1)!, & n\text{- парне} \end{cases}$$

B.26 $\lim\limits_{x\to 3} \dfrac{\cos x \cdot \ln(x-3)}{\ln(e^x-e^3)} = \lim\limits_{x\to 3} \underset{\to \cos(3)}{\cos x} \cdot \lim\limits_{x\to 3} \dfrac{\ln(x-3)}{\ln(e^x-e^3)}$

$\lim\limits_{x\to 3} \ln(x-3) = -\infty$

$\lim\limits_{x\to 3} \ln(e^x-e^3) = -\infty$

$\lim\limits_{x\to 3} \dfrac{\ln(x-3)}{\ln(e^x-e^3)} \overset{\frac{d}{dx}}{=} \lim\limits_{x\to 3} \dfrac{\frac{1}{x-3}}{\frac{e^x}{e^x-e^3}} = \lim\limits_{x\to 3} \dfrac{e^{-x}(e^x-e^3)}{x-3} =$

$= \lim\limits_{x\to 3} \underset{\to \frac{1}{e^3}}{e^{-x}} \cdot \lim\limits_{x\to 3} \dfrac{e^x-e^3 \to 0}{x-3 \to 0}$

$\lim\limits_{x\to 3} \dfrac{e^x-e^3}{x-3} = \lim\limits_{x\to 3} \dfrac{e^x}{1}$

$\lim\limits_{x\to 3} \cos x \cdot \lim\limits_{x\to 3} e^{-x} \cdot \lim\limits_{x\to 3} e^x = \cos(3) \cdot \dfrac{1}{\cancel{e^3}} \cdot \cancel{e^3} =$

$= \cos(3)$

B.24 $\lim\limits_{x\to\infty} \dfrac{\pi - 2\operatorname{arctg} x}{\sqrt[x]{e} - 1} = \dfrac{\lim\limits_{x\to\infty} \pi - 2\operatorname{arctg} x \to 0}{\lim\limits_{x\to\infty} \sqrt[x]{e} - 1 \to 0}$

$\lim\limits_{x\to\infty} \pi - 2\operatorname{arctg} x \overset{\frac{d}{dx}}{=} -\dfrac{2}{x^2+1}$

$\lim\limits_{x\to\infty} \sqrt[x]{e} - 1 \overset{\frac{d}{dx}}{=} -\dfrac{\sqrt[x]{e}}{x^2}$

$$\lim_{x\to+\infty} \frac{2x^2}{(x^2+1)\sqrt[x]{e}} = \lim_{x\to\infty} \frac{2x^2 \cdot e^{-\frac{1}{x}}}{x^2+1} = 2\lim_{x\to\infty} e^{-\frac{1}{x}} \cdot \lim_{x\to\infty}$$

$$= 2\lim_{x\to\infty} e^{-\frac{1}{x}} \cdot \lim_{x\to\infty} \frac{\frac{x^2}{x^2}}{\frac{x^2}{x^2}+\frac{1}{x^2}} = 2\lim_{x\to\infty} e^{-\frac{1}{x}} \cdot \lim_{x\to\infty} \frac{1}{1+\frac{1}{x^2}}$$

$$= 2\lim_{x\to\infty} e^{-\frac{1}{x}} \cdot 1 = 2\cdot 1\cdot 1 = 2$$

13.27 $\lim_{x\to 1}\left(\frac{{}^{(x-1)}1}{\ln x} - \frac{{}^{(\ln x)}1}{x-1}\right) = \lim_{x\to 1} \frac{-1+x-\ln x}{\ln x\cdot(x-1)} =$

$$\lim_{x\to 1} \frac{-1+x-\ln x}{\ln x\cdot(x-1)} = \frac{\lim_{x\to 1} -1+x-\ln x}{\lim_{x\to 1} \ln x\,(x-1)} =$$

$$\stackrel{\frac{d}{dx}}{=} \frac{\lim_{x\to 1} 1-\frac{1}{x}}{\lim_{x\to 1} \frac{x-1}{x}+\ln x} = \lim_{x\to 1} \frac{x-1}{-1+x+x\ln x} =$$

$$= \lim_{x\to 1} \frac{1}{x}\frac{x-1}{\ln x\left(\frac{x-1}{x\ln x}+1\right)} = \lim_{x\to 1}\frac{1}{x} \cdot \lim_{x\to 1}\frac{x-1}{\ln x} \cdot \lim_{x\to 1}\frac{1}{\frac{x-1}{x\ln x}+1}$$

$$\lim_{x\to 1} \frac{x-1}{\ln x} \stackrel{\frac{d}{dx}}{=} \lim_{x\to 1} \frac{1}{\frac{1}{x}} = \lim_{x\to 1} x$$

$$= \lim_{x\to 1} \frac{1}{\frac{x-1}{x\ln x}+1} \cdot \lim_{x\to 1} \frac{1}{x} = \lim_{x\to 1} \frac{1}{\frac{x-1}{x\ln x}+1} =$$

$$= \frac{1}{\lim_{x\to 1} \frac{x-1}{x\ln x}+1} = \frac{1}{\lim_{x\to 1}\frac{1}{x} \cdot \lim_{x\to 1}\frac{x-1}{\ln x}+1} =$$

$$\lim_{x\to 1}\frac{1}{x} = 1$$

$$\lim_{x \to 1} \frac{\overset{\to 0}{x-1}}{\underset{\to 0}{\ln x}} \overset{\frac{d}{dx}}{=} \lim_{x \to 1} \frac{1}{\frac{1}{x}} = \underline{\lim_{x \to 1} x = 1}$$

$$= \frac{1}{1+1} = \frac{1}{2}$$

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B. $\lim\limits_{x\to 0+0} x^x = \lim\limits_{x\to 0+0} e^{x\ln x} = \lim\limits_{x\to 0+0} e^{\frac{\ln x \Rightarrow \infty}{\frac{1}{x} \Rightarrow \infty}} =$

$e^{\frac{\ln x}{\frac{1}{x}}} \stackrel{\frac{d}{dx}}{=} e^{\frac{\frac{1}{x}}{-\frac{1}{x^2}}} = e^{\cancel{\frac{1}{x}} \cdot -\frac{\cancel{x^2}^{\,-x}}{1}} = e^{-x}$

$= \lim\limits_{x\to 0+0} e^{-x} = 1.$