Домашняя работа 2.

1135 $f(x) = \arcsin\sqrt{x}$; $x_0 = \frac{1}{2}$

$\sqrt{x} = \sin y$

$x = \sin^2 y$

$y' = (\sin^2 y)' = \frac{1}{2\sin y \cos y}$

$= \frac{1}{2\sin\arcsin\sqrt{x} \cdot \cos\arcsin\sqrt{x}} = \frac{1}{2\sqrt{x}\cdot\sqrt{1-x}} = \frac{1}{2\sqrt{x-x^2}}$.

1132 $f(x) = y = \log_2 x$

$x = 2^y$

$y' = \frac{1}{2^y \cdot \ln 2} = \frac{1}{2^{\log_2 x}\cdot \ln 2} =$

$= \frac{1}{x \ln 2}$

№25

$$f(x) = \begin{cases} x-1, & x<1 \\ x^2-1, & x\geq 1 \end{cases} \qquad x_0 = 1$$

$$f'_л = \lim_{\Delta x \to 0-0} \frac{f(x_0) - f(x_0-\Delta x)}{\Delta x} = \frac{0 - f(1-\Delta x)}{\Delta x} =$$

$$= \frac{-(1-\Delta x-1)}{\Delta x} = \frac{1+\Delta x+1}{\Delta x} = 1.$$

$$f'_п = \lim_{\Delta x \to 0+0} \frac{f(x_0+\Delta x) - f(x_0)}{\Delta x} = \frac{(1+\Delta x)^2 - 1}{\Delta x} =$$

$$= \frac{2\Delta x + \Delta x^2 - 1}{\Delta x} = \frac{2\Delta x + \Delta x^2}{\Delta x} = \Delta x + 2 = 2$$

$f(x)$ не дифференцируема

11.51 $e^y + y = \ln x + x$, $P(1, 0)$

$F(x, y) = e^y + y - \ln x - x = 0$

$e^y \cdot y' + y' - \frac{1}{x} + 1$

$y'(e^y + 1) = \frac{1+x}{x}$

$dy' = \frac{1+x}{x(e^y+1)}$

$dy(P) = \frac{1+1}{1 \cdot (1+1)} = \frac{2}{1} = 2$

11.53 $x(t) = \ldots$

$f = u^v$

$\frac{df}{dx} = \ldots \cdot \frac{df}{du} \cdot \frac{du}{dx} + \frac{df}{dv} \cdot \frac{dv}{dx} +$

$df = v \cdot u^{v-1} \cdot \ldots \cdot dv \cdot du$

11.56 $f = \ln \operatorname{tg} \frac{v}{v}$

$\frac{df}{du} = \frac{1}{\operatorname{tg} \frac{v}{v}} \cdot \frac{1}{v} \cdot \frac{du}{dx} \cdot$

$\frac{df}{dv} = \ldots - \left(\frac{1}{\operatorname{tg} \frac{v}{v}}\right) \cdot \frac{v}{v^2}$

$df = \left(\frac{1}{\operatorname{tg} \frac{v}{v}} \cdot \frac{1}{v}\right) du - \left(\frac{v}{v^2} \cdot \frac{1}{\operatorname{tg} \frac{v}{v}}\right) dv$

12.3 $f(x) = \frac{e^x}{x}$, $n = 10$

$$f^{10}(x) = \sum_{k=0}^{10} C_{10}^k (e^x)^k \cdot \left(\frac{1}{x}\right)^{10-k} =$$

$$= e^x \cdot \sum_{k=0}^{10} C_{10}^k \cdot \frac{k!(-1)^k}{x^{k+1}}$$

12.6 $f(x) = \ln\frac{x^2-1}{x^2-4x+4} = \frac{x^2-1}{(x-2)^2}$

$$f'(x) = \frac{(x-2)^2}{x^2-1} \cdot \left(\frac{x^2-1}{(x-2)^2}\right)' = \frac{(x-2)^2}{x^2-1} \cdot \left(\frac{2x\cdot(x-2)^2 - (x^2-1)\cdot 2(x-2)}{(x-2)^4}\right) =$$

$$= \frac{2(x-2)(x^2-2x-x^2+1)}{(x^2-1)(x-2)^2} = \frac{2(1-2x)}{(x^2-1)(x-2)}$$

12.15. $f = u^3$

$$df = d(u^3) = 3u^2 \cdot du$$

$$d(df) = 3\cdot(2u\cdot du\cdot du + u^2\cdot d(du))$$

12.20. $f = u\ln v$

$$df = du\cdot\ln v + u\cdot d(\ln v) = du\cdot\ln v + u\cdot\frac{1}{v}\cdot dv$$

$$d(df) = d(du)\cdot\ln v + du\cdot\frac{1}{v}\cdot dv + du\cdot\frac{1}{v}\cdot dv + u\cdot\left(-\frac{1}{v^2}\cdot dv\right)\cdot dv +$$

$$+ u\cdot\frac{1}{v}\cdot d(dv)$$