Домашня робота

№445 $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

1) $a=7$, $b=2$

$\frac{x^2}{49}+\frac{y^2}{4}=1$

2) $2a=10 \Rightarrow a=5$

$2c=8$

$c=4$

$b^2=a^2-c^2=25-16=9$

$b=3$

$\frac{x^2}{25}+\frac{y^2}{9}=1$

3) $2c=24$ $\quad \varepsilon=\frac{12}{13}$

$\varepsilon=\frac{c}{a} \Rightarrow a=\frac{c}{\varepsilon}=\frac{12\cdot 13}{12}=13$

$b^2=a^2-c^2=169-144=25$ $\qquad \frac{x^2}{144}+\frac{y^2}{25}=1$

4') $2b = 16 \quad \varepsilon = \frac{3}{5}$

$b^2 = a^2 - c^2$

$\frac{b^2}{a^2} = \frac{a^2}{a^2} - \frac{c^2}{a^2}$

$\frac{b^2}{a^2} = 1 - \varepsilon^2$

$\cancel{a^2 = \frac{b^2}{1-\varepsilon^2}}$

$\cancel{a^2 = \frac{256}{\frac{16}{25}}}$

$a^2 = \frac{b^2}{(1-\varepsilon)(1+\varepsilon)}$

$a^2 = \frac{256}{(\frac{2}{5})(\frac{8}{5})} = \frac{256}{\frac{16}{25}} = \frac{25 \cdot 256}{16} = 16 \cdot 25 = 400$

$a = 20$

$\frac{x^2}{400} + \frac{y^2}{256} = 1$

5) $2c = 6 \quad \frac{2a}{\varepsilon} = \frac{50}{3}$

$\frac{a}{\varepsilon} = \frac{25}{3} \quad) \quad \varepsilon = \frac{c}{a} \Rightarrow \frac{a}{\frac{c}{a}} = \frac{25}{3} \Rightarrow \frac{a^2}{c} = \frac{25}{3}$

$a = 5 \quad c = 3$

$b^2 = 25 - 9 = 16$

$\frac{x^2}{25} + \frac{y^2}{16} = 1$

6) $\varepsilon = \frac{3}{4} \quad \frac{2a}{\varepsilon} = \frac{32}{3} \Rightarrow \cancel{\frac{16}{3}}$

$\frac{a}{\varepsilon} = \frac{16}{3}$

$a = \frac{16\varepsilon}{3} = \frac{16 \cdot \frac{3}{4}}{3} = 4 \Rightarrow c = 3$

$b^2 = 16 - 9 = 7$

$\frac{x^2}{16} + \frac{y^2}{7} = 1$

~~$A(x_1 y_1)$~~ ~~x_0, y_0~~

~~$\frac{x^2}{b^2} = 1$~~

576 $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ еліпс

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ гіпербола

$(x_0; y_0)$ - точка перетину => дотичні $\perp$

$\frac{xx_0}{a_1^2} + \frac{yy_0}{b_1^2} = 1$

$\frac{yy_0}{b_1^2} = 1 - \frac{xx_0}{a_1^2}$

$y = \frac{b_1^2}{y_0} - x\frac{x_0 b_1^2}{y_0 a_1^2}$

$k_1 = -\frac{x_0 b_1^2}{y_0 a_1^2}$

$\frac{xx_0}{a_2^2} - \frac{yy_0}{b_2^2} = 1$

$\frac{yy_0}{b_2^2} = \frac{xx_0}{a_2^2} - 1$

$y = x \cdot \frac{x_0 b_2^2}{y_0 \cdot a_2^2} - \frac{b_2^2}{y_0}$

$k_2 = \frac{x_0 b_2}{y_0 \cdot a_2}$

$$\begin{cases} x_0^2 b_1^2 + y_0^2 a_1^2 = a_1^2 b_1^2 \\ x_0^2 b_2^2 - y_0^2 a_2^2 = a_2^2 b_2^2 \end{cases}$$

$\Delta = -a_2^2 b_1^2 - a_1^2 b_2^2$

$\Delta x = -a_1^2 a_2^2 b_1^2 - a_1^2 a_2^2 b_2^2$

$\Delta y = a_1^2 b_1^2 b_2^2 - a_2^2 b_1^2 b_2^2$

$x_0^2 = \frac{a_1^2 a_2^2 b_1^2 + a_1^2 a_2^2 b_2^2}{a_2^2 b_1^2 + a_1^2 b_2^2}$; $y_0^2 = \frac{a_2^2 b_1^2 b_2^2 - a_1^2 b_1^2 b_2^2}{a_2^2 b_1^2 + a_1^2 b_2^2}$

$$\frac{x_0^2}{y_0^2} = \frac{a_1^2 a_2^2 b_2^2 + a_1^2 a_2^2 b_1^2}{a_1^2 b_2^2 b_1^2 - a_2^2 b_1^2 b_2^2} = \frac{(a_1 a_2)^2 (b_1^2 + b_2^2)}{(b_1 b_2)^2 (a_1^2 - a_2^2)}$$

$$k_1 \cdot k_2 = -\frac{x_0^2}{y_0^2} \cdot \frac{b_1^2 b_2^2}{a_1^2 a_2^2} = -\frac{(b_1^2 + b_2^2)}{(a_1^2 - a_2^2)} = -1$$

<u>610</u> $y = kx + b$ $\qquad y^2 = 2px$

$(kx+b)^2 = 2px$

$k^2x^2 + 2kxb - 2px + b^2 = 0$

$x^2k^2 + x(2kb - 2p) + b^2 = 0$

$D = (2kb - 2p)^2 - 4 \cdot k^2 \cdot b^2 = 0$

$(kb - p)^2 - k^2b^2 = 0$

$\cancel{k^2b^2}$ $k^2b^2 - 2kbp + p^2 - k^2b^2 = 0$

<u>$p = 2kb$</u>

<u>612</u> $y^2 = px$ $\qquad M_1(x_1; y_1)$

$p = \frac{y^2}{2x_1}$ $\qquad (y = kx_1 + b)$

$$\begin{cases} b = y_1 - kx_1 \\ p = 2kb \\ y_1^2 = 2px_1 \end{cases}$$

$b = \frac{p}{2k} = y_1 - kx_1 \Rightarrow p = 2k(y_1 - kx_1)$

$2ky_1 - 2k^2x_1 - p = 0$

$k^2 2x_1 - k \cdot 2y_1 + p = 0$

$D = 4y_1^2 - 8x_1p = 8px_1 - 8px_1 = 0$

$k = \frac{2y_1}{2 \cdot 2x_1} = \frac{y_1}{2x_1}$

$b = \frac{p}{2y_1} = p \cdot \frac{x_1}{y_1}$

$y = x_2 \quad \frac{y_2}{2x_2} = \frac{px_2}{y_2}$

$2x_2 y_2 y = x y_2^2 + 2p x_2^2$

$2x_2 y_2 y = 2p x_2 \cdot x + 2p x_2^2$

$2x_2 y_2 y = 2p x_2 (x + x_2)$

$y \cdot y_2 = p(x + x_2)$

677 $\begin{cases} y^2 = 2p_1 x \\ x^2 = 2p_2 y \end{cases} \Rightarrow x = \frac{y^2}{2p_1}$

$\frac{y^4}{4p_1^2} = 2p_2 y$

$y^4 - 8p_1^2 p_2 y = 0$

$y^3 = 8p_1^2 p_2$

$y = 2\sqrt[3]{p_1^2 p_2}$

$x^2 = 4p_2 \sqrt[3]{p_1^2 p_2}$

$x = \pm 2p_2^{\frac{1}{2}} (p_1 p_2)^{\frac{1}{3}}$

677 (2,3) $11x^2 - 20xy - 4y^2 - 20x - 8y + 1 = 0$

$A = 11 \quad B = -10 \quad C = -4 \quad D = -10 \quad E = -4 \quad F = 1$

$\delta = \begin{vmatrix} A & B \\ B & C \end{vmatrix} = 11 \cdot (-4) - (-10) \cdot (-10) = -44 - 100 = -144$

$x_0 = \frac{\begin{vmatrix} B & D \\ C & E \end{vmatrix}}{\delta} = \frac{0}{\delta} = 0$

$y_0 = \frac{\begin{vmatrix} D & A \\ E & B \end{vmatrix}}{\delta} = \frac{\begin{vmatrix} -10 & 11 \\ -4 & -10 \end{vmatrix}}{\delta} = \frac{144}{-144} = -1.$

$x^* = x + 0 \qquad y^* = y - 1$

$11x^2 - 20x(y-1) - 4(y-1)^2 - 20x - 8(y-1) + 1 = 0$

$11x^2 - 20xy + 20x - 4y^2 + 8y - 4 - 20x - 8y + 8 + 1 = 0$

$11x^2 - 20xy - 4y^2 + 5 = 0$

$A = 11 \quad B = -10 \quad C = -4 \quad D = 5$

$$\begin{cases} x = x'\cos\alpha - y'\sin\alpha \\ y = x'\sin\alpha + y'\cos\alpha \end{cases}$$

$-10\,tg^2\alpha + 15\,tg\,\alpha + 10 = 0$

$tg = -\frac{1}{2} \qquad tg = 2$

$\sin\alpha = \frac{tg\,\alpha}{\pm\sqrt{1+tg^2\alpha}} = \frac{2}{\pm\sqrt{5}} \qquad \cos\alpha = \frac{1}{\pm\sqrt{1+tg^2\alpha}} = \frac{1}{\pm\sqrt{1+4}} =$

$= \frac{1}{\sqrt{5}}$

$11\left(x'\frac{1}{\sqrt5} - y'\frac{2}{\sqrt5}\right)^2 - 20\left(x'\frac{1}{\sqrt5} - y'\frac{2}{\sqrt5}\right)\left(x'\frac{2}{\sqrt5} + y'\frac{1}{\sqrt5}\right) -$

$- 4\left(x'\frac{2}{\sqrt5} + y'\frac{1}{\sqrt5}\right)^2 + 5 = 0$

$11\left(\frac{x'^2}{5} - 2x'y'\frac{2}{5} + y'^2\frac{4}{5}\right) - 20\left(x'^2\frac{2}{5} + x'y'\frac{1}{5} - x'y'\frac{4}{5} - y'^2\frac{2}{5}\right) -$

$- 4\left(x'^2\frac{4}{5} + 2x'y'\frac{2}{5} + y'^2\frac{1}{5}\right) + 5 = 0$

$\frac{11x'^2}{5} - \frac{44x'y'}{5} + \frac{44y'^2}{5} - \frac{40x'^2}{5} - 4x'y' + 16x'y' + 8y'^2$

$\Rightarrow \frac{16x'^2}{5} - \frac{16x'y'}{5} - \frac{4y'^2}{5} + 5 = 0$

$-y_1'^2 + 16y'^2 + 5 = 0$

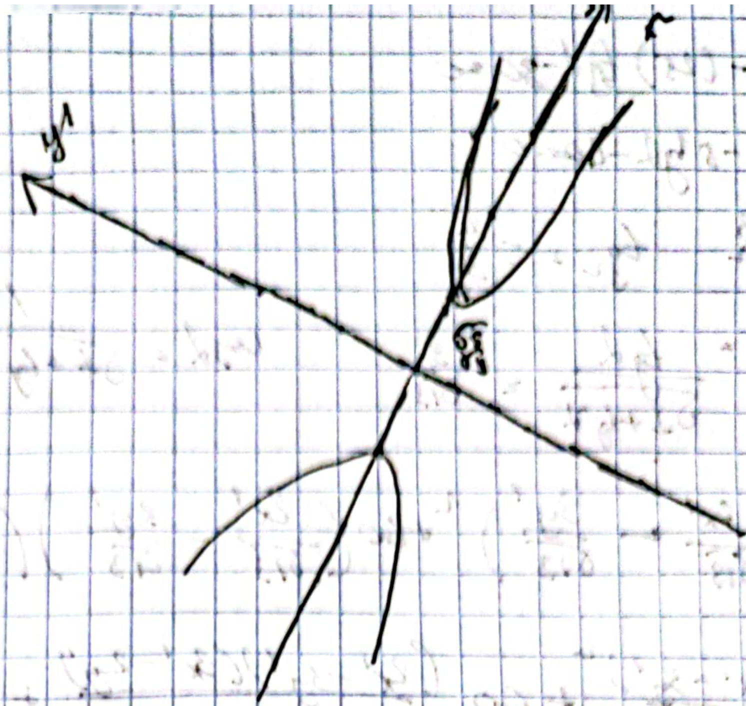
$-y_1'^2 + 16y'^2 = -5$

Scanned with CamScanner

$$\frac{9x^2}{5} - \frac{16y^2}{5} = 1$$

$$\frac{x'^2}{\frac{5}{9}} - \frac{y'^2}{\frac{5}{16}} = \cancel{1}$$

$$\frac{9x'^2}{5} - \frac{16y'^2}{5} = 1$$



3) $7x^2+60xy+32y^2-14x-\cancel{...}60y+7=0$

$A=7 \quad B=30 \quad C=32 \quad D=-7 \quad E=-30 \quad F=7$

$$\delta = \begin{vmatrix} 7 & 30 \\ 30 & 32 \end{vmatrix} = 7\cdot 32 - 30\cdot 30 = -676$$

$$x_0 = \frac{\begin{vmatrix} B & D \\ C & E \end{vmatrix}}{\delta} = \frac{\begin{vmatrix} 30 & -7 \\ 32 & -30 \end{vmatrix}}{\delta} = 1$$

$$y_0 = \frac{\begin{vmatrix} D & A \\ E & B \end{vmatrix}}{\delta} = \frac{0}{\delta} = 0$$

$\tilde{x} = x-1 \qquad \tilde{y} = y+0$

$$7(\tilde{x}+1)^2+60(\tilde{x}+1)\tilde{y}+32\tilde{y}^2-14(\tilde{x}+1)-60\tilde{y}+7=0$$

$$7\tilde{x}^2+14\tilde{x}+7+60\tilde{x}\tilde{y}+60\tilde{y}+32\tilde{y}^2-14\tilde{x}-14-60\tilde{y}+7=0$$

$$7\tilde{x}^2+60\tilde{x}\tilde{y}+32\tilde{y}^2=0$$

$$\begin{cases} \tilde{x} = x'\cos\alpha - y'\sin\alpha \\ \tilde{y} = x'\sin\alpha + y'\cos\alpha \end{cases}$$

$30\,tg^2\alpha - (25)\,tg\alpha - 30 = 0$

$6\,tg^2\alpha - 5\,tg\alpha - 6 = 0$

$tg_1 = \frac{3}{2} \qquad tg_2 = -\frac{2}{3}$

$\sin\alpha = \frac{tg\alpha}{\sqrt{1+tg^2\alpha}} = \frac{3}{\sqrt{13}} \qquad \cos\alpha = \frac{1}{\sqrt{1+tg^2\alpha}} = \frac{2}{\sqrt{13}}$

$$7\left(\frac{2x'}{\sqrt{13}} - \frac{3y'}{\sqrt{13}}\right)^2 + 60\left(\frac{2x'}{\sqrt{13}} - \frac{3y'}{\sqrt{13}}\right)\left(\frac{3x}{\sqrt{13}} + \frac{2y'}{\sqrt{13}}\right) + 32\left(\frac{3x'}{\sqrt{13}} + \frac{2y'}{\sqrt{13}}\right)^2$$

$$7\,\frac{(2x'-3y')^2}{13} + 60\,\frac{(2x'-3y')(3x'-2y')}{13} + 32\,\frac{(3x'+2y')^2}{13} = 0$$

$$7(4x'^2 - 12x'y' + 9y'^2) + 60(6x^2 - 5x'y' - 6y'^2) + 32(9x'^2 + 12x'y' + 4y'^2) = 0$$

$$28x'^2 - 84x'y' + 63y'^2 + 360x'^2 - 300x'y' - 360y'^2 + 288x'^2 + 384x'y' +$$
$$+ 128y'^2 = 0$$

$676x'^2 - 169y'^2 = 0$

$4x'^2 - y'^2 = 0$

$\frac{x'^2}{\frac{1}{4}} - y'^2 = 0$