ДЗ 9

2) $p(x,y) = \begin{cases} xe^{-x(1+y)}, & x>0,\ y>0 \\ 0 & \text{в інш. випадках} \end{cases}$

$$F(x,y) = \int_0^x \int_0^y v e^{-v(1+u)} du\, dv = \int_0^x \int_0^y v e^{-v} e^{-vu} du\, dv =$$

$$= \int_0^x -e^{-v} e^{-vu} \Big|_0^y dv = \int_0^x -e^{-v(1+y)} + e^{-v} dv = -e^{-v} + \frac{e^{-v(1+y)}}{1+y} \Big|_0^x$$

$$= -e^{-x} + \frac{e^{-x(1+y)}}{1+y} - \frac{1}{y+1} + e^{-0} = \frac{e^{-x(1+y)} - 1}{1+y} + 1 - e^{-x} -$$

— сумісна ф-я розподілу

$$f_x = \int_0^{+\infty} x e^{-x(1+y)} dy = -e^{-x(1+y)} \Big|_0^{+\infty} = -e^{-x(1+y)} \Big|_0^{+\infty} \to e^{-x}$$

$$f_y = \int_0^{+\infty} x e^{-x(1+y)} dx = -\frac{x e^{-x(1+y)}}{y+1} \Big|_0^{+\infty} +$$

$$+ \int_0^{+\infty} \frac{e^{-x(1+y)}}{y+1} dx = \int_0^{+\infty} \frac{e^{-x(1+y)}}{y+1} dx = \frac{-e^{-x(1+y)}}{(y+1)^2} \Big|_0^{+\infty} =$$

$$= \frac{1}{(y+1)^2}$$ — маргінальна щільність

$$f_x f_y = \frac{e^{-x}}{(y+1)^2} \neq xe^{-x(1+y)} = \rho(x,y)$$

$$M(1,1) \to \quad \frac{e^{-1}}{4} \neq e^{-2}$$

3) $$f(x,y) = \begin{cases} 24x^2y(1-x), & 0<x<1, \quad 0<y<1 \\ 0 - \text{в інших випадках} \end{cases}$$

$$f(x) = \int_0^1 24x^2 y(1-x)\,dy = 12y^2x^2(1-x)\Big|_0^1 = 12x^2(1-x)$$

$$f_y = \int_0^1 24y(x^2-x^3)\,dx = 24y\left(\frac{x^3}{3} - \frac{x^4}{6}\right)\Big|_0^1 = 2y$$

$$f_x f_y = 12x^2(1-x)2y = 24x^2y(1-x) = f(x,y).$$

Випадкові вел. незалежні

4) $N_1(a, \sigma_1^2)$, $N_2(a_2, \sigma_2^2)$

$a_1 = MX_1$ $\qquad a_2 = MX_2$

$\sigma_1^2 = DX_1$ $\qquad \sigma_2^2 = DX_2$

$Y_1 = aX_1 - bX_2$, $Y_2 = aX_1 + bX_2$

$$\operatorname{cov}(Y_1, Y_2) = MY_1Y_2 - MY_1 MY_2$$

$$MY_1 = M(aX_1 - bX_2) = M(aX_1) - M(bX_2) = aMX_1 - bMX_2 =$$
$$= aa_1 - ba_2$$

$$MY_2 = M(aX_1 + bX_2) = aMX_1 + bMX_2 = aa_1 + ba_2$$

$MY_1Y_2 = M(aX_1 - bX_2)(aX_1 + bX_2) = a^2M(X_1^2) - b^2M(X_2^2) =$

$= a^2M(X_1^2) - a^2a_1^2 + a^2a_1^2 - b^2M(X_2^2) + b^2a_2^2 - b^2a_2^2 =$

$= a^2\sigma_1^2 - b^2\sigma_2^2 + a^2a_1^2 - b^2a_2^2$

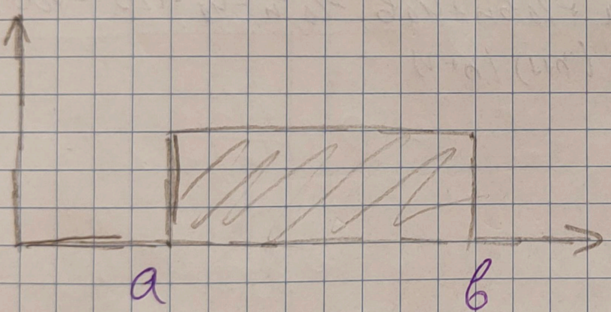
$cov(Y_1Y_2) = a^2d_1^2 - b^2\sigma_2^2 + a^2a_1^2 - b^2a_2^2 - (aa_1 - ba_2)(aa_1 + ba_2) =$

$= a^2\sigma_1^2 - b^2\sigma_2^2 + a^2a_1^2 - b^2a_2^2 - a^2a_1^2 + b^2a_2^2 = \underline{a^2\sigma_1^2 - b^2\sigma_2^2}$

5) $\eta = \min\{\xi_1, \xi_2 \dots \xi_n\}$

$P(\eta \le t) = 1 - P(\eta > t) = 1 - P(\xi_1 \ge t) \dots P(\xi_n \ge t)$

$$1 - P(\xi_i < t) = \begin{cases} 0, & t \le a \\ \frac{t-a}{b-a}, & a < t \le b \\ 1; & t \ge b \end{cases}$$




$$F_\eta = \begin{cases} 1 - (1-0)(1-0)\dots(1-0) = 1 - 1 = 0, & t \le a \\ 1 - \left(\frac{b-t}{b-a}\right)^n, & a < t \le b \\ 1 - (1-1)^n = 1, & t \ge b \end{cases}$$

$$f_\eta = \begin{cases} 0, & t \notin [a,b] \\ n(b-t)^{n-1} \cdot \frac{1}{(b-a)^n}, & t \in [a,b] \end{cases}$$

$$M\eta = \int_a^b t f(t)dt = \int_a^b \frac{tn(b-t)^{n-1}}{(b-a)^n} = \frac{n}{(b-a)^n} \int_a^b t(b-t)^{n-1}dt =$$

$$= \frac{n}{(b-a)^n}\left[-\frac{t(b-t)^n}{n}\Big|_a^b + \int_a^b \frac{(b-t)^n}{n}dt\right] =$$

$$= \frac{n}{(b-a)^n} = \left[\frac{a(b-a)^n}{n} - \frac{(b-t)^{n+1}}{n(n+1)}\Big|_a^b\right] = \frac{n}{(b-a)^n}\left[\frac{a(b-a)^n}{n} + \frac{(b-a)^{n+1}}{n(n+1)}\right] =$$

$$= a + \frac{(b-a)}{n+1} = \frac{an+b}{n+1}$$

$$\int_a^b \frac{t^2 n (b-t)^{n-1}}{(b-a)^n}\, dt = \frac{n}{(b-a)^n} \int_a^b t^2 (b-t)^{n-1}\, dt =$$

$$= \frac{n}{(b-a)^n}\left( -\frac{(b-t)^n}{n} t^2 \Big|_a^b + 2\int_a^b \frac{t(b-t)^n}{n}\, dt \right) =$$

$$= \frac{n}{(b-a)^n} \left| \frac{(b-a)^n}{n} a^2 - \frac{2t(b-t)^{n+1}}{n(n+1)} \right|_N^b + \int_a^b \frac{-(b-t)^{n+1}}{n(n+1)} =$$

$$= \frac{n}{(b-a)^n}\left( \frac{(b-a)^n a^2}{n} + \frac{2a(b-a)^{n+1}}{n(n+1)} + \frac{2(b-a)^{n+2}}{n(n+1)(n+2)} \right) =$$

$$= \frac{a^2(n+1)(n+2) + 2a(b-a)(n+2) + 2(b-a)^2}{(n+1)(n+2)} = \frac{a^2(n^2+3n+2) + 2a(bn+2b-an-2a)}{(n+1)(n+2)}$$

$$+ \frac{2(b^2-2ab+a^2)}{(n+1)(n+2)} = \frac{a^2n^2+3a^2n+2a^2+2abn+4ab-2a^2n-4a^2+2b^2-4ab+2a^2}{(n+1)(n+2)} =$$

$$= \frac{n^2a^2+a^2n+2abn+2b^2}{(n+1)(n+2)}$$

$$D_n = \frac{n^2a^2+a^2n+2abn+2b^2}{(n+1)(n+2)} - \frac{a^2n^2+2abn+b^2}{(n+1)^2} =$$

$$= \frac{(n^2a^2+a^2n+2a(n+2b^2)(n+1) - (a^2n^2+2abn+b^2)(n+2)}{(n+1)^2(n+2)} =$$

$$= \frac{2a^2n^2+(2ab+a^2)n+2b^2-2a^2n^2-4anb-2b^2}{(n+1)^2(n+2)} = \frac{a^2n-2abn}{(n+1)^2(n+2)} =$$

$$= \frac{an(a-2b)}{(n+1)^2(n+2)}$$