

Q1) $\lim_{x \rightarrow 0} \frac{3x^2 - 5x^3 + o(x^3)}{x^3 + 4x^2 + o(x^3)}$

$$= \frac{-5x^2 + o(x^2)}{x^2 + o(x^2)} = -5$$

Q.6. $\lim_{x \rightarrow 1} \frac{\sqrt[3]{5+x^3} - \sqrt{3+x^2}}{x-1}$ (2)

$$\begin{cases} x \rightarrow 1 \\ x = 1+t \\ t \rightarrow 0 \end{cases}$$

$$\Rightarrow \lim_{t \rightarrow 0} \frac{\sqrt[3]{5+(1+t)^3} - \sqrt{3+(1+t)^2}}{t} = \frac{\sqrt[3]{8+t^3+3t^2+t} - \sqrt{3+t^2+2t+1}}{t}$$

$$= \frac{2(1+\frac{1}{3}t+o(t)) - 2(1+\frac{1}{2}t+o(t))}{t} = \frac{2(1+\frac{1}{3}t+o(t)) - 2(1+\frac{1}{2}t+o(t))}{t}$$

$$= \frac{\frac{1}{3}t - \frac{1}{2}t + o(t)}{t} = \frac{1}{4}$$

Q.12 $\lim_{x \rightarrow 0} \left(\frac{1+\tan x}{1+\sin x} \right)^{\frac{1}{\sin x}} = e^{\lim_{x \rightarrow 0} \frac{1}{\sin x} \left(\frac{1+\tan x}{1+\sin x} - 1 \right)}$

$$= e^{\lim_{x \rightarrow 0} \left(\frac{1}{\sin x} \cdot \frac{1+\tan x - 1 - \sin x}{1+\sin x} \right)}$$

$$\begin{cases} x \rightarrow 0 \\ x = \tan t \\ t \rightarrow 0 \end{cases}$$

$$\Rightarrow e^{\lim_{t \rightarrow 0} \frac{t + o(t) + t}{(t+o(t))(1+t+o(t))}} = e^{\lim_{t \rightarrow 0} \frac{2t+o(t)}{t+o(t)}} = e^2$$

$$= e^{\lim_{t \rightarrow 0} \frac{2t+o(t)}{o(t)}} \cdot e^0 = 1$$

$$\text{6.16} \quad \lim_{x \rightarrow 0} \frac{e^x}{x} = \lim_{x \rightarrow 0} \frac{e^x - e^{-x}}{x(e^x + e^{-x})} = \lim_{x \rightarrow 0} \frac{1+x+o(x) - 1-x+o(x)}{x(1+x+1-x+o(x))} = \frac{2x+o(x)}{2x+o(x)} = 1$$

$$\text{6.32} \quad \lim_{x \rightarrow 0} \frac{\sqrt[3]{220x} - \sqrt[4]{16-x} - e^x}{\ln(1+e^x - \cos x)} = \lim_{x \rightarrow 0} \frac{3(1+\frac{x}{4})^{\frac{1}{3}} - 2(1-\frac{x}{16})^{\frac{1}{4}} - e^x}{\ln(1+1+x+o(x) - 1 + \frac{x^2}{2} + o(x))} = \lim_{x \rightarrow 0} \frac{3(1+\frac{x}{4} + \frac{x^2}{12} + o(x)) - 2(1-\frac{x}{16} + o(x)) - 1-x-o(x)}{\ln(1+x+o(x))} = 1$$

$$= \lim_{x \rightarrow 0} (3 + \frac{x}{2} + o(x) - 2 + \frac{x}{8} + o(x) - 1 - x + o(x)) = \frac{0}{0}$$

$$= \lim_{x \rightarrow 0} \left(\frac{22x + 32x}{864} - x + o(x) \right) = \lim_{x \rightarrow 0} \left(\frac{54x - 864x}{864} + o(x) \right) =$$

$$= \lim_{x \rightarrow 0} \left(-\frac{810x}{864} + o(x) \right) = -\frac{810}{864}$$

$$\text{8.46} \quad f(x) = \cos x - 1, \quad g(x) = 1 - \cosh x \quad \frac{f(x)}{g(x)}$$

$$\lim_{x \rightarrow 0} \frac{\cos x - 1}{1 - \cosh x} = \lim_{x \rightarrow 0} \frac{1 - \frac{x^2}{2} + o(x^2)}{\frac{x^2}{2} + o(x^2)} =$$

$$\frac{1 - \frac{x^2}{2} + o(x^2)}{\frac{x^2}{2} + o(x^2)} = -1 \Rightarrow \text{fng}$$

$$\text{8.52} \quad f(x) = x^x, \quad g(x) = 1 \quad \frac{f(x)}{g(x)}$$

$$\lim_{x \rightarrow 0} x^x = \lim_{x \rightarrow 0} e^{x \cdot \ln x} = \lim_{x \rightarrow 0} e^0 = 1 \quad \text{fng}$$