$M_1 = \{\Theta\}$, $M_2 = \mathbb{R}^n$. Ці підпростори наз тривіальними

ДЗ ЕВ7

$$\begin{vmatrix} 1 & 2 & 3 & 4 & \dots & n \\ 2 & 2 & 3 & 4 & \dots & n \\ 3 & 3 & 3 & 4 & \dots & n \\ 4 & 4 & 4 & 4 & \dots & n \\ \dots & \dots & \dots & \dots & \dots & \dots \\ n & n & n & n & \dots & n \end{vmatrix} \begin{array}{l} -II \\ -III \\ -IV \\ -V \\ \dots \\ \end{array} = \left| \begin{array}{l} \text{від кожного рядка} \\ \text{віднімемо наступний} \end{array} \right| =$$

$$= \begin{vmatrix} -1 & 0 & 0 & 0 & \dots & 0 \\ -1 & -1 & 0 & 0 & \dots & 0 \\ -1 & -1 & -1 & 0 & \dots & 0 \\ -1 & -1 & -1 & -1 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ n & n & n & n & \dots & n \end{vmatrix} = (-1)^{n-1} \cdot n$$

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$$\begin{vmatrix} 0 & 1 & 2 & 3 & \dots & n-1 \\ 1 & 0 & 1 & 2 & \dots & n-2 \\ 2 & 1 & 0 & 1 & \dots & n-3 \\ 3 & 2 & 1 & 0 & \dots & n-4 \\ \dots & & & & & \\ n-1 & n-2 & n-3 & n-4 & \dots & 0 \end{vmatrix} \begin{matrix} \\ -I \\ -II \\ -III \\ \\ -(n-1) \end{matrix} \left| \begin{matrix} \text{від кожного} \\ \text{віднім. попередн.} \end{matrix} \right| =$$

$$= \begin{vmatrix} 0 & 1 & 2 & 3 & \dots & n-1 \\ 1 & -1 & -1 & -1 & \dots & -1 \\ 1 & 1 & -1 & -1 & \dots & -1 \\ 1 & 1 & 1 & -1 & \dots & -1 \\ \dots & & & & & \\ 1 & 1 & 1 & 1 & \dots & -1 \end{vmatrix} = \left| \begin{matrix} \text{до кожного стовпця} \\ \text{додамо останній} \end{matrix} \right| =$$

$+n \quad +n \quad +n \quad +n$

$$= \begin{vmatrix} n-1 & n & n+1 & n+2 & \dots & n-1 \\ 0 & -2 & -2 & -2 & \dots & -1 \\ 0 & 0 & -2 & -2 & \dots & -1 \\ 0 & 0 & 0 & -2 & \dots & -1 \\ \dots & & & & & \\ 0 & 0 & 0 & 0 & \dots & -1 \end{vmatrix} = (-2)^{n-2}(-1)(n-1) = (-2)^{n-2}(1-n)$$

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$$\begin{vmatrix} a_0 & a_1 & a_2 & \dots & a_n \\ a_0 & x & a_2 & \dots & a_n \\ a_0 & a_1 & x & \dots & a_n \\ \dots & & & & \\ a_0 & a_1 & a_2 & \dots & x \end{vmatrix} -I = \begin{vmatrix} a_0 & a_1 & a_2 & \dots & a_n \\ 0 & x-a_1 & 0 & \dots & 0 \\ a_0 & a_1 & x & \dots & a_n \\ \dots & & & & \\ a_0 & a_1 & a_2 & \dots & x \end{vmatrix} =$$

$$= (x-a_1)\begin{vmatrix} a_0 & a_2 & a_3 & \dots & a_n \\ a_0 & x & a_3 & \dots & a_n \\ \dots & \dots & \dots & \dots & \dots \\ a_0 & a_2 & a_3 & \dots & x \end{vmatrix} = g(x)\cdot(x-a_1) \;=$$

$f(x)$: $(x-a_1)$, $(x-a_2)$, ... $(x-a_3)$ ... $(x-a_{n-1})$

$$= (x-a_1)(x-a_2)\dots(x-a_{n-1})\cdot\begin{vmatrix} a_0 & a_n \\ a_0 & x \end{vmatrix} =$$

$$= (x-a_1)(x-a_2)\dots(x-a_{n-1})(a_0x-a_0a_n) = a_0(x-a_1)(x-a_2)\dots(x-a_{n-1})\cdot$$

$$\cdot (x-a_n).$$

2gy

$$\begin{vmatrix} 1+x & 1 & 1 & 1 \\ 1 & 1-x & 1 & 1 \\ 1 & 1 & 1+z & 1 \\ 1 & 1 & 1 & 1-z \end{vmatrix} = -\begin{vmatrix} 1 & 1-x & 1 & 1 \\ 1+x & 1 & 1 & 1 \\ 1 & 1 & 1+z & 1 \\ 1 & 1 & 1 & 1-z \end{vmatrix} =$$

$$= \begin{vmatrix} 1-x & 1 & 1 & 1 \\ 1 & 1+x & 1 & 1 \\ 1 & 1 & 1+z & 1 \\ 1 & 1 & 1 & 1-z \end{vmatrix}$$

При $x=0 \to \mathrm{I}=\mathrm{II}$; $z=0 \to \mathrm{III}=\mathrm{IV}$.

$$\left(\begin{vmatrix} \cancel{x} & 0 & 0 & z \\ 0 & -x & 0 & z \\ 0 & 0 & z & z \\ 1 & 1 & 1 & 1-z \end{vmatrix} = x\cdot\begin{vmatrix} -x & 0 & z \\ 0 & z & z \\ 1 & 1 & 1-z \end{vmatrix} - \begin{vmatrix} 0 & 0 & z \\ -x & 0 & z \\ 0 & z & z \end{vmatrix}\right) =$$

$$= x\begin{vmatrix} -x & 0 & 2 \\ 0 & z & z \\ 1 & 1 & 1-z \end{vmatrix} - z\begin{vmatrix} -x & 0 \\ 0 & z \end{vmatrix} = x\left(-x\begin{vmatrix} z & z \\ 1 & 1-z \end{vmatrix} + \begin{vmatrix} -z & 0 \\ z & z \end{vmatrix}\right) +$$

$$+ xz^2 = x(-x(-z^2) + (-z)^2) + xz^2 = x(xz^2 - z^2) + xz^2 =$$

$$= x^2z^2 - xz^2 + xz^2 = x^2z^2$$

Ex 8

$$D_n = \begin{vmatrix} 0 & 1 & 1 & \dots & 1 & 1 \\ 1 & a_1 & 0 & \dots & 0 & 0 \\ 1 & 0 & a_2 & \dots & 0 & 0 \\ 1 & 0 & 0 & \dots & a_{n-1} & 0 \\ 1 & 0 & 0 & \dots & 0 & a_n \end{vmatrix} =$$

$$= a_n D_{n-1} (-1)^{n+n} \begin{vmatrix} 1 & 1 & \dots & 1 & 1 \\ a_1 & 0 & \dots & 0 & 0 \\ 0 & a_2 & \dots & 0 & 0 \\ 0 & 0 & \dots & a_{n-1} & 0 \end{vmatrix} =$$

$$= a_n D_{n-1} (-1)^{n+1} \times (-1)^{n+1} \prod_{i=1}^{n-1} a_i = (-1) a_n D_{n-1} \cdot a_1 a_2 \dots a_{n-1}$$

Ex 9

$$D_n = \begin{vmatrix} 3 & 2 & 0 & 0 & \dots & 0 \\ 1 & 3 & 2 & 0 & \dots & 0 \\ 0 & 1 & 3 & 2 & \dots & 0 \\ 0 & 0 & 1 & 3 & \dots & 0 \\ 0 & 0 & 0 & \dots & & 3 \end{vmatrix} = 3\begin{vmatrix} 3 & 2 & 0 & \dots & 0 \\ 1 & 3 & 2 & \dots & 0 \\ 0 & 1 & 3 & \dots & 0 \\ 0 & 0 & 0 & \dots & 3 \end{vmatrix} -$$

$$-2\begin{vmatrix} 1 & 2 & 0 & \dots & 0 \\ 0 & 3 & 2 & \dots & 0 \\ 0 & 1 & 3 & \dots & 0 \\ 0 & 0 & 0 & \dots & 3 \end{vmatrix} = 3D_{n-1} - 2D_{n-2}$$

$x^2 - 3x + 2 = 0$

$x_1 = 2 \qquad x_2 = 1$

$D_n = C_1 2^n + C_2$

$D_1 = 3 = 2C_1 + C_2$

$D_2 = 7 = 4C_1 + C_2$

$D_n = 2^{n+1} - 3$

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$$\begin{vmatrix} 5 & 6 & 0 & 0 & 0 & \dots & 0 & 0 \\ 4 & 5 & 2 & 0 & 0 & \dots & 0 & 0 \\ 0 & 1 & 3 & 2 & 0 & \dots & 0 & 0 \\ 0 & 0 & 1 & 3 & 2 & \dots & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \dots & 3 & 2 \\ 0 & 0 & 0 & 0 & 0 & \dots & 1 & 3 \end{vmatrix} =$$

$=$

$3D_{n-1} - 2D_{n-2}$

$x^2 - 3x + 2 = 0$

$x_1 = 1 \qquad x_2 = 2$

$D_n = C_1 + C_2 2^n$

$D_1 = 5 = C_1 + 2C_2$

$D_2 = 1 = C_1 + 4C_2$

$D_n = 9 + 2^{n-1}$

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$$D_n = \begin{vmatrix} \alpha+\beta & \alpha\beta & 0 & 0 & \dots & 0 \\ 1 & \alpha+\beta & \alpha\beta & 0 & \dots & 0 \\ 0 & 1 & \alpha+\beta & \alpha\beta & \dots & 0 \\ 0 & 0 & 0 & 0 & \dots & \alpha+\beta \end{vmatrix} =$$

$$: (\alpha + \beta) D_{n-1} - \alpha\beta D_{n-2}$$

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

$$D_n = c_1 \alpha^n + c_2 \alpha_n^n$$