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$$f = 2x_1^2 + 9x_1^2 + 3x_3^2 + 8x_1x_2 - 4x_1x_3 - 10x_2x_3$$

$$g = 2y_1^2 + 3y_1^2 + 6y_3^2 - 4y_1y_2 - 4y_1y_3 + 2y_2y_3$$

$$f = (2x_1^2 + 8x_1x_2 - 4x_1x_3) + 9x_1^2 + 3x_3^2 - 10x_2x_3 = \frac{(5\sqrt{2}x_1 + 25\sqrt{2}x_2 - 5\sqrt{2}x_3)^2}{2}$$

$$= (x_1^2 + 8x_1x_2 + 8x_1^2 - 4x_1x_3 + x_3^2 - x_2x_3) - 2x_1x_3 + x_1^2 + x_2^2 = (z_1^2) + z_2^2 \quad (x_3^2 = z_1)$$

$$g = (\sqrt{2}y_1 - \sqrt{2}y_2 - \sqrt{2}y_3)^2 = (2y_1^2 + 2y_2^2 + 2y_3^2 - 4y_1y_2 - 4y_1y_3 + 4y_2y_3) + 4y_2^2 + 4y_3^2 + y_1^2 = a_1^2 + a_2^2 \quad (y_3 = a_1)$$

$$\textcircled{1} x_1 = \frac{1}{\sqrt{2}} (z_1 - 25\sqrt{2}x_2 + 5\sqrt{2}x_3) = \frac{z_1}{\sqrt{2}} - 2x_2 + z_3 = \frac{z_1}{\sqrt{2}} - 2z_1 + 2z_2 + z_3 = \frac{z_1}{\sqrt{2}} = 2z_1 - z_3$$

$$x_2 = z_3$$

$$\textcircled{2} x_2 = \frac{5\sqrt{2}y_1 + \sqrt{2}y_2 + 5\sqrt{2}y_3}{\sqrt{2}} = 2(2y_1 + 2y_3) - y_3 = y_1 - 3y_2 - 6y_3$$

$$= y_1 - y_2 - y_3 - 2y_2 - 4y_3 - y_3 = y_1 - 3y_2 - 6y_3$$

$$x_1 = y_1 + 2y_2 + y_3 = y + 2y_3$$

$$4y x_3 = y_3$$

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$$x_1^2 + x_2^2 + 5x_3^2 - 6x_1x_2 - 2x_1x_3 + 2x_2x_3$$

$$A = \begin{vmatrix} 1 & -3 & -1 \\ -3 & 1 & 1 \\ -1 & 1 & 5 \end{vmatrix} \Rightarrow \begin{vmatrix} 1-\lambda & -3 & -1 \\ -3 & 1-\lambda & 1 \\ -1 & 1 & 5-\lambda \end{vmatrix} = \begin{vmatrix} 3-\lambda & -\lambda & 5-\lambda \\ -3 & 2-\lambda & 1 \\ -1 & 1 & 5-\lambda \end{vmatrix} =$$

$$= -\lambda^3 + 2\lambda^2 - 36 = -(\lambda+2)(\lambda-3)(\lambda-6) = 0$$

$$\lambda_1 = -2$$

$$\lambda_2 = 3$$

$$\textcircled{1} \begin{vmatrix} 3 & -3 & -1 \\ -3 & 3 & 1 \\ -1 & 1 & 2 \end{vmatrix} = \begin{vmatrix} 2 & -3 & -1 \\ -1 & 1 & 2 \\ 0 & 0 & 1 \end{vmatrix} = \begin{vmatrix} -1 & 1 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{vmatrix} = \begin{vmatrix} -1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{vmatrix}$$

$$\text{QCP: } \frac{x_1 | x_2 | x_3}{1 | 1 | 0} : a_1 = a_2 = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0 \right)$$

$$\textcircled{2} \begin{vmatrix} -2 & -3 & -1 \\ -3 & -2 & 1 \\ -1 & 1 & 2 \end{vmatrix} = \begin{vmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{vmatrix}$$

$$\text{QCP: } \frac{x_1 | x_2 | x_3}{1 | -1 | 1} : a_1 = a_2 = \left(\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right)$$

$$\textcircled{3} \begin{vmatrix} -5 & -3 & -1 \\ 3 & -5 & 1 \\ -1 & 1 & -1 \end{vmatrix} = \begin{vmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & -\frac{1}{2} \\ 0 & 1 & -\frac{1}{2} \end{vmatrix}$$

$$\text{QCP: } \frac{x_1 | x_2 | x_3}{1 | -1 | -2} : a_1 = a_2 = \left(\frac{1}{\sqrt{6}}, -\frac{1}{\sqrt{6}}, -\frac{2}{\sqrt{6}} \right)$$

$$Q = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{6}} \\ 0 & \frac{1}{\sqrt{3}} & -\frac{2}{\sqrt{6}} \end{pmatrix}$$

$$\begin{cases} x_1 = \frac{1}{\sqrt{2}} y_1 + \frac{1}{\sqrt{3}} y_2 + \frac{1}{\sqrt{6}} y_3 \\ x_2 = \frac{1}{\sqrt{2}} y_1 - \frac{1}{\sqrt{3}} y_2 - \frac{1}{\sqrt{6}} y_3 \\ x_3 = \frac{1}{\sqrt{3}} y_1 - \frac{2}{\sqrt{6}} y_3 \end{cases}$$

$$B = Q^T A Q = \begin{pmatrix} -2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 6 \end{pmatrix}$$

$$f(x) = -2y_1 + 3y_2 + 6y_3$$

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$$2x_1^2 + x_2^2 + 3x_3^2 + 2\lambda x_1 x_2 + 2x_1 x_3$$

$$A = \begin{vmatrix} 2 & \lambda & 1 \\ \lambda & 1 & 0 \\ 1 & 0 & 3 \end{vmatrix}$$

$$\Delta_1 = 2 > 0$$

$$\Delta_2 = 2 - \lambda^2 = (\sqrt{2} - \lambda)(\sqrt{2} + \lambda) > 0$$

$$\Delta_3 = (6 - \lambda + 0 - 0) = 5 > 0$$

$$\underline{\underline{\sqrt{2} > \lambda > -\sqrt{2}}}$$