Дз 3

23.8 $f(x,y)=\frac{xy}{1+x^2+y^2}$, $P_0(1,1)$, $P_1(2,2)$

$\overrightarrow{P_0P_1}=(1,1)=\vec{i}+\vec{j}$ $\quad |\overrightarrow{P_0P_1}|=\sqrt{1+1}=\sqrt{2}$

$e=\frac{\overrightarrow{P_0P_1}}{|\overrightarrow{P_0P_1}|}=\left(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right)$

$\cos\alpha=\frac{1}{\sqrt{2}}$ $\quad \cos\beta=\frac{1}{\sqrt{2}}$

$f'(x)=\frac{y(1+x^2+y^2)-xy\cdot 2x}{(1+x^2+y^2)^2}=\frac{y-yx^2+y^3}{(1+x^2+y^2)}=\frac{1}{9}$

$f'(y)=\frac{x-xy^2+x^3}{(1+x^2+y^2)^2}=\frac{1}{9}$

$\frac{\partial f}{\partial e}(P_0)=\frac{\partial f}{\partial x}(P_0)\cos\alpha+\frac{\partial f}{\partial y}(P_0)\cos\beta=\frac{1}{9}\cdot\frac{1}{\sqrt{2}}+\frac{1}{9}\frac{1}{\sqrt{2}}=$

$=\frac{2}{9\sqrt{2}}=\frac{\sqrt{2}}{9}$

$\operatorname{grad} f=\frac{\partial f}{\partial x}\vec{i}+\frac{\partial f}{\partial y}\vec{j}=\frac{1}{9}\vec{i}+\frac{1}{9}\vec{j}$

23.9 $f(x,y)=e^{x^2+y^2+z^2}$ $\quad P_0(1,0,0)$, $\vec{a}=(0,1,1)$

$\frac{\partial f}{\partial x}\Big|_{P_0}=e^{x^2+y^2+z^2}\cdot 2x\Big|_{P_0}=2e$

$\frac{\partial f}{\partial y}\Big|_{P_0}=e^{x^2+y^2+z^2}\cdot 2y\Big|_{P_0}=0$

$\frac{\partial f}{\partial z}\Big|_{P_0}=e^{x^2+y^2+z^2}\cdot 2z\Big|_{P_0}=0$

$|a|=\sqrt{2}$

$\cos\alpha = 0 \qquad \cos\beta = \frac{1}{\sqrt{2}} \qquad \cos\gamma = \frac{1}{\sqrt{2}}$

$$\frac{\partial f}{\partial a}\Big|_{P_0} = 2e \cdot 0 + 0 \cdot \frac{1}{\sqrt{2}} + 0 \cdot \frac{1}{\sqrt{2}} = 0$$

$$\operatorname{grad} f = \frac{\partial f}{\partial x}\vec{i} + \frac{\partial f}{\partial y}\vec{j} + \frac{\partial f}{\partial z}\vec{k} = 2e\vec{i} = (2e, 0, 0)$$

23.12 $\sqrt{3{,}98^2 + 3{,}01^2}$

$$f(x,y) = f(P_0) + f'_x(P_0)\cdot\Delta x + f'_y(P_0)\cdot\Delta y$$

$f(x,y) = \sqrt{x^2+y^2} \qquad P_0 = (x_0, y_0) = (4,3)$

$\Delta x = x - x_0 = -0{,}02$

$\Delta y = y - y_0 = 0{,}01$

$f'_x = \frac{x}{\sqrt{x^2+y^2}}, \quad f'_y = \frac{y}{\sqrt{x^2+y^2}}$

$f(x,y) = 5 - 0{,}02 \cdot \frac{4}{5} + 0{,}01 \cdot \frac{3}{5} = 5 - 0{,}01 = 4{,}99$

23.19 $F(x,y) = h(x+y, x-y)$

$u = x+y \qquad v = x-y$

$d^2u = 0 \qquad d^2v = 0$

$$d^2h = \left(\frac{\partial}{\partial u}du + \frac{\partial}{\partial v}dv\right)^2 h$$

$dF = h'_1 du + h'_2 dv$

$d^2F = h''_{11}du^2 + 2h''_{12}\,du\,dv + h''_{22}dv^2$

$du_1 = dx + dy$, $du_2 = dx - dy$

$dF = h'_1 (dx + dy) + h'_2 (dx - dy)$

$d^2F = h''_{11} (dx + dy)^2 + 2h''_{12} (dx^2 - dy^2) + h''_{22} (dx - dy)^2$

23.23 $f(x,y) = (2x-5) \cdot \cos \frac{1}{y^2}$, $y \neq 0$

$$\frac{\partial f}{\partial x} = 2 \cdot \cos \frac{1}{y^2}$$

$$\frac{\partial^2 f}{\partial x^2} = 0$$

$$\frac{\partial f}{\partial x^2 \partial y} = 0$$

23.26 $\frac{\partial^{m+n} f}{\partial x^m \partial y^n}$, $m, n \in \mathbb{Z}^+$, $f(x,y) = \sin(xy)$, $(x,y) \in R^2$

$$\frac{\partial^m f}{\partial x^m} = y^m \cdot \sin\left(xy + \frac{\pi m}{2}\right)$$

$$\frac{\partial^n f}{\partial y^n} = x^n \cdot \sin\left(xy + \frac{\pi n}{2}\right)$$

$$\frac{\partial^n}{\partial y^n} \left(\frac{\partial^m f}{\partial x^m}\right) = \frac{\partial^{m+n} f}{\partial x^m \partial y^n}$$

$\underline{<3.30}$ $x+y+z=e^{z}$, $z=z(x,y): \mathbb{R}^2 \to \mathbb{R}$

$dx+dy+dz=e^{z}dz$

$dx+dy=(e^{z}-1)dz$

$z'_x=\frac{1}{e^{z}-1}$ $\quad z'_y=\frac{1}{e^{z}-1}$

$z''_{xx}=-\frac{e^{z}}{(e^{z}-1)^3}=z''_{yy}$

$z''_{xy}=-\frac{e^{z}}{(e^{z}-1)^3}$