№3 №94

$A = \begin{pmatrix} 1 & -4 \\ 1 & 4 \end{pmatrix}$; $AA^T = \begin{pmatrix} 1 & -4 \\ 1 & 4 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ -4 & 4 \end{pmatrix} = \begin{pmatrix} 17 & -15 \\ -15 & 17 \end{pmatrix}$

$|AA^T - \lambda E| = \begin{vmatrix} 17-\lambda & -15 \\ -15 & 17-\lambda \end{vmatrix} = (17-\lambda)^2 - 225 =$

$= 289 - 34\lambda + \lambda^2 - 225 = 64 - 34\lambda + \lambda^2 = (\lambda - 32)(\lambda - 2)$

$\lambda_1 = 32 \quad \lambda_2 = 2$

$|AA^T - 2E| = \begin{pmatrix} 15 & -15 \\ -15 & 15 \end{pmatrix} \sim (1\ -1)$

$|AA^T - 32| = \begin{pmatrix} -15 & -15 \\ -15 & -15 \end{pmatrix} \sim (1\ 1)$

$q_1 = \begin{array}{c|c} x_1 & x_2 \\ \hline 1 & 1 \end{array}$

$p_1 = \left(\frac{1}{\sqrt{2}}; \frac{1}{\sqrt{2}}\right)$

$q_2 = \begin{array}{c|c} x_1 & x_2 \\ \hline 1 & -1 \end{array}$

$p_2 = \left(\frac{1}{\sqrt{2}}; -\frac{1}{\sqrt{2}}\right)$

$P = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$

$AA^T P = \begin{pmatrix} 17 & -15 \\ -15 & 17 \end{pmatrix}\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} \sqrt{2} & 16\sqrt{2} \\ \sqrt{2} & -16\sqrt{2} \end{pmatrix}$

$B = PDP^T = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}\begin{pmatrix} \sqrt{2} & 0 \\ 0 & 4\sqrt{2} \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{4}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{4}{\sqrt{2}} \end{pmatrix} =$

$= \begin{pmatrix} \frac{5\sqrt{2}}{2} & -\frac{3\sqrt{2}}{2} \\ -\frac{3\sqrt{2}}{2} & \frac{5\sqrt{2}}{2} \end{pmatrix}$

$Q = B^{-1} A$

$$B^{-1} = \begin{pmatrix} \frac{5\sqrt{2}}{16} & \frac{3\sqrt{2}}{16} \\ \frac{3\sqrt{2}}{16} & \frac{5\sqrt{2}}{16} \end{pmatrix}$$

$$Q = \begin{pmatrix} \frac{5\sqrt{2}}{16} & \frac{3\sqrt{2}}{16} \\ \frac{3\sqrt{2}}{16} & \frac{5\sqrt{2}}{16} \end{pmatrix} \begin{pmatrix} 1 & -4 \\ 1 & 4 \end{pmatrix} = \begin{pmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{pmatrix}$$

$$BQ = \begin{pmatrix} \frac{5\sqrt{2}}{2} & -\frac{3\sqrt{2}}{2} \\ -\frac{3\sqrt{2}}{2} & \frac{5\sqrt{2}}{2} \end{pmatrix} \begin{pmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{pmatrix}$$

№1276

$$x_3^2 - 2x_1^2 + x_3^2 + 2x_3 x_1 + 4x_2 x_3 + 2x_1 x_3 =$$

$$= (x_3^2 + 2x_3 x_3 + x_3^2 + x_1^2 + 2x_3 x_1 + x_2^2 + x_2^2 + 2x_2 x_3 + x_3^2) -$$

$$- x_3^2 + 4x_3 x_3 - x_3^2 - 2x_2^2 = (x_3 + x_1 + x_3)^2 - (x_3 - x_3)^2 - (\sqrt{2} x_2)^2 =$$

$$= z_1^2 - z_2^2 - z_3^2$$

№1277 $x_3^2 - 3x_3^2 - 2x_3 x_1 + 2x_3 x_3 - 6x_1 x_3 =$

$$= (x_3^2 - 2x_3 x_1 + 2x_3 x_3 + x_1^2 + x_3^2 - 2x_1 x_3) =$$

$$= x_1^2 - 4x_3^2 - 4x_1 x_3 = (x_1 - x_1 + x_3)^2 - (x_1 + 2x_3)^2 =$$

$$= y_3^2 - y_1^2$$

№1135

$x_1x_2 + x_2x_3 + x_3x_4 + x_4x_1$

$$\begin{cases} x_1 = y_1 - y_2 \\ x_2 = y_1 + y_2 \\ x_3 = y_3 \\ x_4 = y_4 \end{cases} = y_1^2 - y_2^2 + y_1y_3 + y_2y_3 + y_3y_4 + y_1y_4 -$$

$$- y_2y_1 = (y_1^2 + y_1y_3 + y_1y_4 + y_3^2 + y_4^2 + \frac{y_3y_4}{2}) -$$

$$- y_2^2 + y_2y_3 + \frac{y_3y_4}{2} - y_2y_3 \cdot \left| \begin{array}{l} y_1 + \frac{y_3}{2} + \frac{y_4}{2} = z_1 \\ y_2 = z_2 \\ y_3 = z_3 \\ y_4 = z_4 \end{array} \right| =$$

$$= z_1^2 - (z_2^2 - z_2z_3 + z_2z_4 + \frac{z_3}{2} + \frac{z_4}{2} - z_3z_4 =$$

№1136 $4x_1^2 + x_2^2 + x_3^2 - 4x_1x_2 + 4x_2x_3 - 2x_1x_3 =$

$$= 2(x_1^2 - 2x_1x_2 + 2x_2x_3 + x_2^2 + x_3^2 - 2x_1x_3) - x_2x_3 =$$

$$= (2x_1 - x_2 + x_3)^2 - x_2x_3 = \left| \begin{array}{l} 2x_1 - x_2 + x_3 = y_1 \\ x_2 = y_2 - y_3 \\ x_3 = y_2 + y_3 \end{array} \right| =$$

$$= y_1^2 + y_2^2 - y_3^2$$

$$\begin{cases} x_1 = y_1 - y_3 \\ x_2 = y_2 - y_3 \\ x_3 = y_2 + y_3 \end{cases}$$