

Q13 1536 $A = \begin{pmatrix} 17 & -8 & 4 \\ -8 & 17 & -4 \\ 4 & -4 & 11 \end{pmatrix}$

$$|A - \lambda E| = \begin{vmatrix} 17-\lambda & -8 & 4 \\ -8 & 17-\lambda & -4 \\ 4 & -4 & 11-\lambda \end{vmatrix} = \begin{vmatrix} 9-\lambda & -8 & 4 \\ 9-\lambda & 17-\lambda & -4 \\ 0 & -4 & 11-\lambda \end{vmatrix} =$$

$$= \begin{vmatrix} 9-\lambda & -8 & 4 \\ 0 & 25-\lambda & -8 \\ 0 & -4 & 11-\lambda \end{vmatrix} = (9-\lambda)(25-\lambda)(11-\lambda) - 24(9-\lambda) =$$

$$= \lambda_1 = 9, \lambda_2 = 24 \Rightarrow (-\lambda^3 + 45\lambda^2 - 567\lambda + 216) =$$

$$= (\lambda - 24)(\lambda^2 - 15\lambda + 81) = (\lambda - 24)(\lambda - 9)^2$$

$$|A - 9E| = \begin{vmatrix} 8 & -8 & 4 \\ 8 & 8 & -4 \\ 4 & -4 & 2 \end{vmatrix} = (8, -8, 4) = (2, -2, 1)$$

x_1	x_2	x_3	
1	1	0	$= a_2$
1	0	-1	$= a_3$

$$C_1 = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 0 \right)$$

$$C_2 = \left(-\frac{1}{\sqrt{5}}, 0, \frac{2}{\sqrt{5}} \right)$$

$$|A - 22E|_2 = \begin{vmatrix} -10 & -8 & 4 \\ -8 & -10 & -4 \\ 4 & -4 & -16 \end{vmatrix} = \begin{vmatrix} -10 & -8 & 4 \\ 0 & -18 & -36 \\ 0 & -2 & -4 \end{vmatrix} = \begin{vmatrix} -10 & -10 & 0 \\ 0 & -2 & -4 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \end{vmatrix} \Rightarrow C_3 = (0, -4, 1)$$

$$\begin{array}{c|c|c} x_1 & x_2 & x_3 \\ \hline 1 & 1 & 1 \end{array}$$

$$C_1 = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 0 \right)$$

$$C_2 = \left(-\frac{1}{\sqrt{5}}, 0, \frac{2}{\sqrt{5}} \right)$$

$$C_3 = (2, 2, 1)$$

$$x_1 = 0, x_2 = -\frac{\sqrt{2}}{2}$$

$$b_1 = 4$$

$$b_2 = 4$$

$$b_3 \Rightarrow x_3 = 0, x_4 = -\frac{\sqrt{2}}{10} \Rightarrow C_3 = \frac{\sqrt{2}}{10} \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 0 \right) =$$

$$= \left(-\frac{\sqrt{2}}{5}, 0, \frac{2\sqrt{2}}{5} \right) \text{ and } \left(\frac{2\sqrt{2}}{10}, \frac{2\sqrt{2}}{10}, 0 \right) =$$

$$= \left(-\frac{\sqrt{2}}{10}, \frac{\sqrt{2}}{10}, \frac{4\sqrt{2}}{5} \right)$$

$$B = (Q^{-1} A Q) \Rightarrow Q = \begin{pmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & \frac{2}{\sqrt{2}} \\ \frac{\sqrt{2}}{2} & 0 & \frac{2}{\sqrt{2}} \\ 0 & \frac{\sqrt{2}}{5} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$Q^{-1} = \begin{pmatrix} \frac{4\sqrt{2}}{9} & \frac{5\sqrt{2}}{9} & \frac{2\sqrt{2}}{9} \\ -\frac{5\sqrt{2}}{9} & \frac{5\sqrt{2}}{9} & \frac{4\sqrt{2}}{9} \\ \frac{2}{3} & -\frac{2}{3} & \frac{1}{3} \end{pmatrix}$$

$$B = \begin{pmatrix} 36 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 27 \end{pmatrix}$$