

(D<sub>3</sub>)

1542

$$f_1 = (1, 2, 1)$$

$$f_2 = (1, 1, 1)$$

$$f_3 = (1, 1, 0)$$

$$\varphi = \begin{pmatrix} 1 & 1 & 3 \\ 0 & 5 & -1 \\ 1 & 2 & -3 \end{pmatrix}$$

$$T = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 1 & 1 \\ 1 & 2 & 0 \end{pmatrix} \Rightarrow T^{-1} = \left( \begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 2 & 1 & 1 & 0 & 1 & 0 \\ 1 & 2 & 0 & 0 & 0 & 1 \end{array} \right) \xrightarrow{R_2 - R_1}$$

$$\xrightarrow{R_3 - R_1} \left( \begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & -1 & -1 & -2 & 1 & 0 \\ 0 & 1 & -1 & -1 & 0 & 1 \end{array} \right) \xrightarrow{R_2 \leftrightarrow R_3} \left( \begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 1 & 0 \\ 0 & 1 & -1 & -2 & 1 & 0 \\ 0 & 0 & -2 & -3 & 1 & 1 \end{array} \right) \xrightarrow{R_3 \times (-1/2)}$$

$$\Rightarrow T^{-1} = \begin{pmatrix} -1 & 1 & 0 \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ \frac{3}{2} & -\frac{1}{2} & -\frac{1}{2} \end{pmatrix}$$



$$\textcircled{1} P = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 1 & 1 \\ 1 & 2 & 0 \end{pmatrix} \begin{pmatrix} 1 & 1 & 3 \\ 0 & 5 & 1 \\ 2 & 2 & -3 \end{pmatrix} \begin{pmatrix} -1 & 1 & 0 \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ \frac{3}{1} & -\frac{1}{2} & -\frac{1}{2} \end{pmatrix} =$$

$$= \begin{pmatrix} 3 & 13 & -1 \\ 4 & 14 & 2 \\ 1 & 11 & 1 \end{pmatrix} \begin{pmatrix} \swarrow & & \\ & \searrow & \\ & & \dots \end{pmatrix} = \begin{pmatrix} 2 & -3 & 7 \\ 6 & -4 & 6 \\ 6 & -5 & 5 \end{pmatrix}$$

$$\textcircled{2} B = \begin{pmatrix} -1 & 1 & 0 \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ \frac{3}{1} & -\frac{1}{2} & -\frac{1}{2} \end{pmatrix} \begin{pmatrix} 2 & 6 & 6 \\ -3 & -4 & -5 \\ 2 & 6 & 5 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 2 & 1 & 1 \\ 1 & 2 & 0 \end{pmatrix} =$$

$$= \begin{pmatrix} -5 & -10 & -11 \\ 6 & 6 & 6 \\ 1 & 2 & 9 \end{pmatrix} \begin{pmatrix} \downarrow & & \\ & \dots & \\ & & \dots \end{pmatrix} = \begin{pmatrix} -36 & -32 & -15 \\ 30 & 30 & 14 \\ 26 & 22 & 9 \end{pmatrix}$$

1556

$$f = 2x_1y_1 + 3x_1y_2 + x_3y_3 + 2x_2y_1 + 2x_1y_2 + x_1y_3 + x_2y_3 +$$

$$+ x_2y_3 + x_1y_2$$

$$A = \begin{pmatrix} 2 & 1 & 1 \\ -1 & -3 & 1 \\ 1 & 2 & -1 \end{pmatrix} \Rightarrow A^T = \begin{pmatrix} 2 & -1 & 1 \\ 1 & -3 & 2 \\ 1 & 1 & -1 \end{pmatrix}$$

$$U = \begin{pmatrix} 2 & 2 & 1 \\ 2 & 3 & 1 \\ 1 & 1 & 1 \end{pmatrix} \Rightarrow U^{-1} = \left( \begin{array}{ccc|ccc} 2 & 2 & 1 & 1 & 0 & 0 \\ 2 & 3 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{array} \right) =$$

$$\begin{pmatrix} 1 & 1 & 0 & 1 & 0 & -1 \\ 0 & 1 & -1 & 0 & 1 & -2 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 & 0 & 1 & 0 & -1 \\ 0 & 1 & -1 & 0 & 1 & -2 \\ 0 & 0 & 1 & -1 & 0 & 2 \end{pmatrix} \equiv$$

$$\begin{pmatrix} 1 & 0 & 0 & 2 & -1 & 1 \\ 1 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 2 \end{pmatrix}$$



$$A_1 \cdot U^{-1} \cdot A^T \cdot U = \begin{pmatrix} 2 & -1 & 1 \\ -1 & 1 & 0 \\ -1 & 0 & 2 \end{pmatrix} \begin{pmatrix} 2 & -1 & 1 \\ 1 & -3 & 2 \\ 1 & 1 & -1 \end{pmatrix} \begin{pmatrix} 2 & 2 & 1 \\ 2 & 3 & 1 \\ 1 & 1 & 1 \end{pmatrix} =$$

$$= \begin{pmatrix} 2 & 0 & 1 \\ -1 & -2 & 1 \\ 0 & 3 & -3 \end{pmatrix} \begin{pmatrix} 2 & 2 & 1 \\ 2 & 3 & 1 \\ 1 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 5 & 5 & 3 \\ -5 & -2 & -2 \\ 3 & 6 & 0 \end{pmatrix}$$

1556  $U = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{pmatrix}$   $A = \begin{pmatrix} 1 & 2 & -3 \\ 2 & -3 & 1 \\ 3 & 2 & -1 \end{pmatrix}$

$$U^{-1} = \left( \begin{array}{ccc|ccc} 2 & -1 & 0 & 1 & 0 & 0 \\ -1 & 2 & -1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 & 0 & 1 \end{array} \right) = \left( \begin{array}{ccc|ccc} 1 & 1 & -1 & 1 & 1 & 0 \\ -1 & 2 & -1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 & 0 & 1 \end{array} \right) =$$

$$= \left( \begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & 1 \\ -1 & 2 & -1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 & 0 & 1 \end{array} \right) = \left( \begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 2 & 2 \\ 0 & -1 & 1 & 0 & 0 & 1 \end{array} \right) =$$

$$= \left( \begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 2 & 2 \\ 0 & 0 & 1 & 1 & 2 & 3 \end{array} \right)$$

$$A^T = \begin{pmatrix} 1 & 2 & 3 \\ 2 & -3 & 2 \\ -3 & 1 & -1 \end{pmatrix}$$

$$A_1 = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 2 & -3 & 2 \\ -3 & 1 & -1 \end{pmatrix} \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{pmatrix},$$

$$= \begin{pmatrix} 0 & 0 & 4 \\ -1 & -2 & 5 \\ -4 & -1 & 4 \end{pmatrix} \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{pmatrix} = \begin{pmatrix} 0 & -4 & 4 \\ 0 & -3 & 7 \\ -7 & -2 & 5 \end{pmatrix}$$