6.21

$$\left|\begin{array}{ccccc} 24 & 19 & 36 & 72 & -38 \\ 49 & 40 & 73 & 147 & -80 \\ 73 & 59 & 98 & 219 & -118 \\ 47 & 38 & 71 & 143 & -72 \end{array}\right| \begin{array}{l} \\ -II \\ -3I \\ -2I \end{array} \sim$$

$$\sim \left|\begin{array}{ccccc} 24 & 19 & 36 & 72 & -38 \\ 2 & 4 & 2 & 6 & -8 \\ 1 & 2 & -10 & 3 & -10 \\ -1 & -2 & -1 & -3 & -4 \end{array}\right| \sim \left|\begin{array}{ccccc} 24 & 19 & 36 & 72 & -38 \\ 1 & 2 & -10 & 3 & -10 \\ -1 & -2 & -1 & -3 & -4 \\ 2 & 4 & 2 & 6 & -8 \end{array}\right| \begin{array}{l} \sim \\ +II \\ +2II \end{array}$$

$$\sim \left|\begin{array}{ccccc} 24 & 19 & 36 & 72 & -38 \\ 1 & 2 & -10 & 3 & -10 \\ 0 & 0 & -11 & 0 & -14 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right| \quad r(A)=3$$

6.44 $a_1 = (1, 0, 0, 2, 5)$

$a_2 = (0, 1, 0, 3, 4)$

$a_3 = (0, 0, 1, 4, 7)$

$a_4 = (2, -3, 4, 11, 12)$

$$\left|\begin{array}{ccccc} 1 & 0 & 0 & 2 & 5 \\ 0 & 1 & 0 & 3 & 4 \\ 0 & 0 & 1 & 4 & 7 \\ 2 & -3 & 4 & 11 & 12 \end{array}\right| \begin{array}{l} \\ \\ \\ -I+3II-4III \end{array} \sim \left|\begin{array}{ccccc} 1 & 0 & 0 & 2 & 5 \\ 0 & 1 & 0 & 3 & 4 \\ 0 & 0 & 1 & 4 & 7 \\ 0 & 0 & 0 & 0 & -14 \end{array}\right| \quad r=4$$

673 $a_1 = (1, 2, 0, 0)$

$a_2 = (1, 2, 3, 4)$

$a_3 = (3, 6, 0, 0)$

$$\begin{pmatrix} 1 & 2 & 0 & 0 \\ 1 & 2 & 3 & 4 \\ 3 & 6 & 0 & 0 \end{pmatrix} \cdot (-3) \sim \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad r = 2$$

Базис a_1, a_2; a_1, a_3

680 $a_1 = (2, -1, 3, 5)$

$a_2 = (4, -3, 1, 3)$

$a_3 = (3, -2, 3, 4)$

$a_4 = (4, -1, 5, 17)$

$a_5 = (7, -6, -7, 0)$

$$\left(\begin{array}{ccccc} 2 & 4 & 3 & 4 & 7 \\ -1 & -3 & -2 & -1 & -6 \\ 3 & 1 & 3 & 15 & -7 \\ 5 & 3 & 4 & 17 & 0 \end{array}\right) \begin{array}{l} \\ \cdot(-1) \\ \cdot(-3) \\ \cdot(-1) \end{array} \sim \left(\begin{array}{ccccc} 1 & 1 & 1 & 3 & 1 \\ -1 & -3 & -2 & -1 & -6 \\ 1 & -3 & 0 & 11 & -14 \\ 2 & 2 & 1 & 2 & 7 \end{array}\right) \begin{array}{l} \\ \cdot I \\ \cdot I \\ \cdot(-2) \end{array} \sim$$

$$\sim \left(\begin{array}{ccccc} 1 & 1 & 1 & 3 & 1 \\ 0 & -2 & -1 & 2 & -5 \\ 0 & -4 & -1 & 8 & -15 \\ 0 & 0 & -1 & -4 & 5 \end{array}\right) \cdot(-2) + IV \sim \left(\begin{array}{ccccc} 1 & 1 & 1 & 3 & 1 \\ 0 & -2 & -1 & -2 & 0 \\ 0 & 0 & -1 & 4 & 5 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

a_1, a_2, a_3 - базис

$a_4 = a_1 - a_2 + a_3$

689

$$\begin{cases} 2x_1 + 7x_2 + 3x_3 + x_4 = 6 \\ 3x_1 + 5x_2 + 2x_3 + 2x_4 = 4 \\ 9x_1 + 4x_2 + x_3 + 7x_4 = 2 \end{cases}$$

$$\left(\begin{array}{cccc|c} 2 & 7 & 3 & 1 & 6 \\ 3 & 5 & 2 & 2 & 4 \\ 9 & 4 & 1 & 7 & 2 \end{array}\right) \begin{matrix} \\ -I' \\ -3II \end{matrix} \sim \left(\begin{array}{cccc|c} 0 & 11 & 5 & -1 & 10 \\ 1 & -2 & -1 & 1 & -2 \\ 0 & -11 & -5 & 1 & -30 \end{array}\right) \begin{matrix} \\ \\ +I \end{matrix} \sim$$

$$\sim \left(\begin{array}{cccc|c} 0 & 11 & 5 & -1 & 10 \\ 1 & -2 & -1 & 1 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & 9 & 4 & 0 & 8 \\ 0 & 11 & 5 & -1 & 10 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

x_1, x_2 - glavne

$$11x_2 + 5x_3 - x_4 = 10$$

$$x_2 = \frac{-5x_3 + x_4 + 10}{11}$$

$$\begin{cases} x_1 + 9x_2 + 4x_3 = 8 \\ x_1 = 8 - 4x_3 - 9\dfrac{-5x_3 + x_4 + 10}{11} \end{cases}$$

$x_3 = 0 \quad x_2 = -\frac{1}{11}$

$x_4 = 0 \quad x_1 = \frac{10}{11}$

$$\begin{cases} x_2 = \dfrac{-5x_3 + x_4 + 10}{11} \\ x_1 = \dfrac{-2 + 4x_3 - 9x_4}{11} \end{cases}$$

$x_3 \in R \quad x_4 \in R$

691

$$\begin{cases} 3x_1 + 4x_2 + x_3 + 2x_4 = 3 \\ 6x_1 + 8x_2 + 2x_3 + 5x_4 = 7 \\ 9x_1 + 12x_2 + 3x_3 + 10x_4 = 13 \end{cases}$$

$$\left(\begin{array}{cccc|c} 3 & 4 & 1 & 2 & 3 \\ 6 & 8 & 2 & 5 & 7 \\ 9 & 12 & 3 & 10 & 13 \end{array}\right) \begin{matrix} \\ -2I \\ -3I \end{matrix} = \left(\begin{array}{cccc|c} 3 & 4 & 1 & 2 & 3 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 4 & 4 \end{array}\right) \sim$$

$$\sim \left(\begin{array}{cccc|c} 3 & 4 & 1 & 0 & 4 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 4 & 4 \end{array}\right)$$

$x_4 = 1$

$3x_1 + 4x_2 + x_3 = 1$

$$\begin{cases} x_3 = \dfrac{1 - 4x_2 - x_3}{3} \\ x_4 = 1 \\ x_1, x_2 \in \mathbb{R} \end{cases}$$

$x_2 = 0 \quad x_3 = 0 \quad x_2 = 1 \quad x_3 = 1$

№5 $\left(\begin{array}{cccc|c} 2 & -1 & 3 & 4 & 5 \\ 4 & -2 & 5 & 6 & 7 \\ 6 & -3 & 7 & 8 & 9 \\ 8 & -4 & 9 & 10 & 11 \end{array}\right)$ -2I -3I

$$\sim \left(\begin{array}{cccc|c} 2 & -1 & 3 & 4 & 5 \\ 0 & 0 & -1 & -2 & 0 \\ 0 & 0 & -2 & -4 & 9 \\ 8 & -4 & 9 & 10 & 11 \end{array}\right) -4\text{II} \sim$$

$$\sim \left(\begin{array}{cccc|c} 2 & -1 & 3 & 4 & 5 \\ 0 & 0 & -1 & -2 & 0 \\ 0 & 0 & 0 & 0 & 9 \\ 0 & 0 & 3 & 6 & -6 \end{array}\right) \Rightarrow \text{несовместна}$$

№5 $\left(\begin{array}{cccc} 2 & -4 & 5 & 3 \\ 3 & -6 & 4 & 2 \\ 4 & -8 & 17 & 11 \end{array}\right)$ $\begin{array}{c} \\ -\text{I} \\ -2\text{I} \end{array}$ $\sim \left(\begin{array}{cccc} 1 & -2 & -1 & -1 \\ 0 & 0 & 7 & 5 \\ 0 & 0 & 7 & 5 \end{array}\right) \sim$

$$\sim \left(\begin{array}{cccc} \textcircled{1} & -2 & -1 & -1 \\ 0 & 0 & \textcircled{7} & 5 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

x_2, x_3 – базисные

$k_3 = -\frac{5}{2} k_1$

$k_4 - 2k_2 + \frac{5}{2} k_1 - k_1 = 0$

$k_2 = k_4 + \frac{1}{2} k_1$

$\begin{cases} k_3 = k_1 + \frac{1}{2} k_1 \\ k_2 = \frac{5}{2} k_1 \\ k_1 k_4 \in R \end{cases}$

201 $\begin{pmatrix} 5 & 8 & -4 \\ 6 & 9 & -5 \\ 4 & 7 & -3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 5 \\ 4 & -1 & 3 \\ 9 & 6 & 5 \end{pmatrix} =$

$= \begin{pmatrix} 5\cdot 3 + 8\cdot 4 + (-4)\cdot 9 & 5\cdot 2 + 8\cdot(-1) + (-4)\cdot 6 & 5\cdot 5 + 8\cdot 3 + (-4)\cdot 5 \\ 6\cdot 3 + 9\cdot 4 + (-5)\cdot 9 & 6\cdot 2 + 9\cdot(-1) + (-5)\cdot 6 & 6\cdot 5 + 9\cdot 3 + (-5)\cdot 5 \\ 4\cdot 3 + 7\cdot 4 + (-3)\cdot 9 & 4\cdot 2 + 7\cdot(-1) + (-3)\cdot 6 & 4\cdot 5 + 7\cdot 3 + (-3)\cdot 5 \end{pmatrix} =$

$= \begin{pmatrix} 11 & -22 & 29 \\ 9 & -17 & 32 \\ 13 & -12 & 26 \end{pmatrix}$

Вар. $\begin{pmatrix} 17 & -6 \\ 35 & -12 \end{pmatrix}^5 = A$

$\begin{pmatrix} 2 & 3 \\ 5 & 7 \end{pmatrix} = B$

$\begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix} = C$

$\begin{pmatrix} -7 & 3 \\ 5 & -2 \end{pmatrix} = D$

$A^5 = ?$ $\quad A = BCD$

$A^5 = BCD \cdot BCD \cdot BCD \cdot BCD \cdot BCD$ (2)

$DB = \begin{pmatrix} -7 & 3 \\ 5 & -2 \end{pmatrix} \begin{pmatrix} 2 & 3 \\ 5 & 7 \end{pmatrix} = \begin{pmatrix} -7\cdot 2 + 3\cdot 5 & -7\cdot 3 + 3\cdot 7 \\ 5\cdot 2 - 2\cdot 5 & 5\cdot 3 - 2\cdot 7 \end{pmatrix} =$

$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = E$

$B_2^5 0$ ②

$C^5 = \begin{pmatrix} 2^5 & 0 \\ 0 & 3^5 \end{pmatrix} \cdot \begin{pmatrix} 32 & 0 \\ 0 & 243 \end{pmatrix}$

② $\begin{pmatrix} 2 & 3 \\ 5 & 4 \end{pmatrix} \begin{pmatrix} 32 & 0 \\ 0 & 243 \end{pmatrix} \cdot D = \begin{pmatrix} 64 & 729 \\ 160 & 1701 \end{pmatrix} \cdot \begin{pmatrix} -4 & 3 \\ 5 & -2 \end{pmatrix} =$

$= \begin{pmatrix} 3154 & -1161 \\ 7335 & -2702 \end{pmatrix}$